NONCAUSAL ZERO-FORCING PRECODING SUBJECT TO INDIVIDUAL CHANNEL POWER CONSTRAINTS

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ABSTRACT

This paper investigates Zero Forcing (ZF) precoding in MIMO FB channels subject to individual channel power constraints. It shows that the optimal ZF precoder is noncausal in general and presents a general formulation for the design of optimal noncausal ZF precoder subject to individual channel power constraints. A convex optimization based method is derived from the formulation to design optimal noncausal precoder. Numerical example demonstrates the effectiveness of the design method and the advantage of the designed optimal noncausal ZF precoder.

***Index Terms*—** Zero-forcing precoding, oversampled filter banks, noncausal systems, convex optimization

1. INTRODUCTION

Zero-Forcing (ZF) precoding has proved to be an efficient approach to eliminating the inter-channel and inter-symbol interferences, and it has been extensively investigated in [1, 2, 3, 4, 5, 6, 7, 8]. In MIMO system, if the number of transmitters is greater that of receivers, the system is redundant. Unlike in non-redundant systems where ZF precoder is uniquely given by the channel inverse, the ZF precoder in redundant MIMO systems is not unique. This non-uniqueness allows for the incorporation of other design requirements, such as transmission power constraints, in the design of ZF precoder.

Transmission power is always bounded in real communication systems. There are two power constraint schemes in MIMO communication system - the total power constraint and the individual channel power constraint. In total power constraint design, the sum of all individual transmitter power is constrained, but it lacks the restriction on the power of each individual transmitter. The output power in this design may concentrate on a few transmitters, exceeding their output power limits. In practice, each transmitter has its unique output power limit, especially in distributed systems. Hence, the design under individual power constraints is more realistic and more important in practice.

ZF precoder design for constant matrix channel is thoroughly studied in [3] for both total power constraint and individual channel power constraint. However, the constant matrix channel are not applicable to wide band communication channels or high rate data transmission because of the frequency selective fading. In these situations, the MIMO communication channel behaves more like an MIMO linear dynamic system than a constant matrix [9], and can be modeled by FIR filter bank (FB) [1].

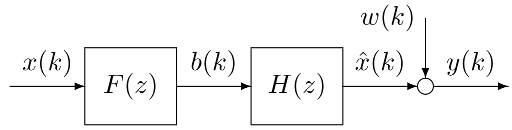

Fig. 1. MIMO communication system with precoder

The design of ZF precoder for FB channels with total power constraint is systematically studied in [4, 6]. All these works are confined to the causal precoders, and hence are restricted to the minimum phase FB channels. Due to the causality constraint, these designs generally achieve lower signal-to-noise ratio (SNR) than noncausal design. With the advance of fast DSP processors, the noncausal precoder can now be implemented for FIR FB channel, having the unknown initial conditions absorbed in finite taps. Noncausal precoder eliminates the minimum phase restriction on the FB channel required in the causal design [4], and can achieve the (theoretically) maximum SNR. Our recent work [7] has studied the design of noncausal ZF precoder with total power constraint, and has demonstrated the advantages of noncausal precoding. This paper extends the results of [7] to the noncausal ZF precoding subject to individual channel power constraints. It is motivated by the importance of individual channel power constraints in practice as discussed above.

2. PROBLEM FORMULATION

Fig. 1 shows the diagram of an MIMO communication system with precoding. In the diagram, $x(k) \in \mathbb{C}^M$, $b(k) \in \mathbb{C}^N$ and $y(k) \in \mathbb{C}^M$ are respectively the input, transmitted and received signals; $\hat{x}(k) \in \mathbb{C}^M$ is the channel output signal and $w(k) \in \mathbb{C}^M$ is the additive noise in transmission. $H(z) \in \mathbb{C}^{M\times N}$ is the polyphase matrix of FB channels with N transmitters and M receivers, $N > M$; $F(z) \in \mathbb{C}^{N\times M}$ is the precoder to be designed. The channel $H(z)$ is always causal but the precoder $F(z)$ is not necessarily causal.

Assume that the precoder $F(z)$ is designed to satisfy the ZF condition $H(z)F(z) = \alpha I$. Then the received signal $y(k)$ in Fig. 1 is given by

$$y(k) = \hat{x}(k) + w(k) = H(z)F(z)x(k) + w(k) \\ = \alpha x(k) + w(k).$$

Define the SNR of the received signal

$$SNR := \frac{P_{\hat{x}}}{P_w} = \frac{\alpha^2 P_x}{P_w}$$

and the SNR coefficient $:= \alpha$, where P_x and P_w are the power of input signal $x(k)$ and additive noise $w(k)$, respectively. The SNR between the input signal $x(k)$ and noise $w(k)$ is usually untunable, while the SNR coefficient α can be tuned by precoder design. The larger the α is, the higher the SNR of the received signal.

Denote $H^{-1}(z)$ the right inverse of $H(z)$ satisfying $H(z)H^{-1}(z) = I$. Then the ZF precoder $F(z)$ is given by

$$F(z) = \alpha H^{-1}(z). \quad (1)$$

Since $H(z) \in \mathbb{C}^{M\times N}$ and $N > M$, $H^{-1}(z)$ and hence $F(z)$ are not unique. The SNR coefficient α has an upper limit, because the transmitting power of $b(k)$ is always bounded in practice. Assume without loss of generality that the power spectral density of $S_{xx}(e^{j\omega}) = I$. Then, the total transmitting power of $b(k)$ is given by

$$\begin{aligned} P_b &= \frac{1}{2\pi}\int_0^{2\pi} Tr\{S_{bb}(e^{j\omega})d\omega\} \\ &= \frac{1}{2\pi}\int_0^{2\pi} Tr\{F(e^{j\omega})S_{xx}(e^{j\omega})F^*(e^{j\omega})d\omega\} \\ &= \|F\|_2^2 = \alpha^2\|H^{-1}(z)\|_2^2. \end{aligned}$$

Denote F_j and H_j^{-1} the jth rows of $F(z)$ and H^{-1}, respectively. It then follows from the definitions for the power of vector signal and the H_2 norm of transfer function matrix [10] that $P_b = \sum_{j=0}^{N-1} P_b(j) = \|F\|_2^2 = \sum_{j=0}^{N-1} \|F_j\|_2^2$ and

$$P_b(j) = \|F_j\|_2^2 = \alpha^2\|H_j^{-1}\|_2^2 \le \gamma_j, \forall j \in [0, N-1], \quad (2)$$

where $P_b(j)$ is the output power of the jth channel and γ_j is its upper limit.

Since $H^{-1}(z)$ is not unique, there are many α and $H^{-1}(z)$ satisfying (2). Thus, the design under individual channel power constraints is to find an $H^{-1}(z)$ that solves the following optimization problem:

$$\max_{\alpha, H^{-1}(z)} \alpha^2 \quad (3)$$

subject to

$$H(z)H^{-1}(z) = I \quad (4)$$

$$\alpha^2\|H_j^{-1}\|_2^2 \le \gamma_j, \forall j \in [0, N-1]. \quad (5)$$

3. EQUIVALENT DESIGN FORMULATION

3.1. State space expression of the set of right inverses

The non-unique $H^{-1}(z)$s are the dual frame FBs of the FB channel $H(z)$ [7], and the set of all $H^{-1}(z)$s can be represented in the state space for analysis and design computation by using the following notion from the linear system theory: A rational causal discrete-time transfer matrix $T(z)$ can be represented by $T(z) = D + C(zI - A)^{-1}B$. The quadruple (A, B, C, D) is called a state-space realization of $T(z)$ and is denoted as $T(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. Similarly, an anticausal $T(z)$ can be represented in state space by $T(z) = D + C(z^{-1}I - A)^{-1}B = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]_{ac}$.

Theorem 1 *Let* $H(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \in \mathbb{C}^{M\times N}$ *be the polyphase FB model of the channel with* $A \in \mathbb{R}^{n\times n}$ *and* $H(e^{j\omega})$ *being full row rank on* $\omega \in [0, 2\pi)$. *Then the set of all* $H^{-1}(z)$*s is given by*

$$H^{-1}(z) = H^{\dagger}(z) + \tilde{N}_{\perp}(z)V(z) \quad (6)$$

where

$$H^{\dagger}(z) = \tilde{H}(z)[H(z)\tilde{H}(z)]^{-1} \in \mathbb{C}^{N\times M} \quad (7)$$

is the canonical dual frame FB of $H(z)$, $V(z) \in \mathbb{C}^{(N-M)\times M}$ *is an arbitrary stable noncausal transfer matrix and* $\tilde{N}_{\perp}(z) \in \mathbb{C}^{N\times M}$ *is given by*

$$\tilde{N}_{\perp}(z) = \left[\begin{array}{c|c} (A+LC)^* & C_{\perp}^* \\ \hline (B+LD)^* & D_{\perp}^* \end{array}\right]_{ac}. \quad (8)$$

In (8),

$$L = -(AXC^* + BD^*)P^{-1}, \quad (9)$$

where

$$P = P^* = DD^* + CXC^* \quad (10)$$

and $X \ge 0$ *is the unique solution to the algebraic Riccati equation*

$$AXA^* - X + BB^* + L(CXA^* + DB^*) = 0; \quad (11)$$

$C_{\perp} \in \mathbb{C}^{(N-M)\times n}$ *and* $D_{\perp} \in \mathbb{C}^{(N-M)\times N}$ *satisfy*

$$\begin{bmatrix} C_{\perp} & D_{\perp} \end{bmatrix} \begin{bmatrix} XA^* & XC^* \\ B^* & D^* \end{bmatrix} = 0, \quad (12)$$

$$C_{\perp}XC_{\perp}^* + D_{\perp}D_{\perp}^* = I, \quad (13)$$

and can be obtained by QR decomposition [11]

$$\begin{bmatrix} C_{\perp} & D_{\perp} \end{bmatrix} = \begin{bmatrix} 0 & I_{(N-M)} \end{bmatrix} Q^* \begin{bmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & I_N \end{bmatrix},$$

where the matrix Q is the orthogonal matrix in QR decomposition

$$\begin{bmatrix} X^{\frac{1}{2}}A^* & X^{\frac{1}{2}}C^* \\ B^* & D^* \end{bmatrix}. \quad (14)$$

Proof: The right inverse is the analysis dual frame of $H(z)$ and can be represented by [12]

$$H^{-1}(z) = H^{\dagger}(z) + (I - H^{\dagger}(z)H(z))U(z), \qquad (15)$$

where $H^{\dagger}(z)$ is the para-pseudo inverse of $H(z)$ and $U(z)$ is the free parameter. It has been proved in [7] that $H^{\dagger}(z)$ can be written as $H^{\dagger}(z) = \tilde{N}(z)M(z)$, where $N(z)$ and $M(z)$ is the left copirme factorization of $H(z)$ satisfying $H(z) = M(z)^{-1}N(z)$ [10]. Then, (15) can be written as

$$\begin{aligned} H^{-1}(z) &= H^{\dagger}(z) + (I - \tilde{N}(z)M(z)M(z)^{-1}N(z))U(z) \\ &= H^{\dagger}(z) + (I - \tilde{N}(z)N(z))U(z). \end{aligned} \qquad (16)$$

Because $[\tilde{N}(z), \tilde{N}_{\perp}(z)]$ is unitary, $(I - \tilde{N}(z)N(z))U(z)$ in (16) is given by $\tilde{N}_{\perp}(z)N_{\perp}(z)U(z)$. Letting free parameter $V(z) = N_{\perp}(z)U(z)$ gives (6). □

3.2. Equivalent optimization for design

Theorem 1 reveals that the right inverses (dual frame FBs) of the FB channel $H(z)$ are noncausal in general. It has been proved in [7] that $H^{-1} = H^{\dagger}(z)$, obtained by setting $V(z) = 0$ in (6), has the minimum H_2 norm and hence is the optimal precoder that maximizes α under the total power constraint. But $H^{\dagger}(z)$ may not satisfy the individual channel power constraints in general. The solution for individual channel power constraints can be found by searching the solution set (6). The result is presented in Theorem 2 below by using the following notations.

Define $\Lambda := diag[\gamma_j]_{j=0,1,\cdots,N-1}$ and $Q_D :=$ the diagonal of Q, where γ_j is the individual power constraint for the j-th channel as given in (2).

Theorem 2 *Let* $F(z) = \alpha H^{-1}(z) = \left[\begin{array}{c|c} A_{Fc} & \alpha B_{Fc} \\ \hline C_{Fc} & 0 \end{array}\right] + \left[\begin{array}{c|c} A_{Fac} & \alpha B_{Fac} \\ \hline C_{Fac} & \alpha D_{Fac} \end{array}\right]_{ac}$ *be the state-space realization of the ZF precoder given in (1). Then the design problem (3)-(5) can be solved by the following optimization.*

$$\max_{\alpha, A_{Fc}, B_{Fc}, C_{Fc}, A_{Fac}, B_{Fac}, C_{Fac}, D_{Fac}, P_c, P_{ac}, Q_c, Q_{ac}} \alpha^2 \qquad (17)$$

subject to

$$\begin{bmatrix} P_c & P_c A_{Fc} & P_c B_{Fc} \\ A_{Fc}^T P_c & P_c & 0 \\ B_{Fc}^T P_c & 0 & \alpha^{-2} \end{bmatrix} > 0, \qquad (18)$$

$$\begin{bmatrix} P_{ac} & P_{ac} A_{Fac} & P_{ac} B_{Fac} \\ A_{Fac}^T P_{ac} & P_{ac} & 0 \\ B_{Fac}^T P_{ac} & 0 & \alpha^{-2} \end{bmatrix} > 0, \qquad (19)$$

$$\begin{bmatrix} Q_c & C_{Fc} \\ C_{Fc}^T & P_c \end{bmatrix} > 0, \qquad (20)$$

$$\begin{bmatrix} Q_{ac} & C_{Fac} & D_{Fac} \\ C_{Fac}^T & P_{ac} & 0 \\ D_{Fac}^T & 0 & \alpha^{-2} \end{bmatrix} > 0, \qquad (21)$$

$$\Lambda - (Q_c + Q_{ac})_D > 0. \qquad (22)$$

Proof: For the causal part of $F(z) = \alpha H^{-1}(z)$, suppose that there exist matrices $P_c = P_c^T > 0$ and $Q_c > 0$ such that (18) and (20) hold simultaneously. Then by Schur complement [13], (18) implies that there exists $G_c = G_c^T = P_c^{-1} > 0$ such that

$$G_c - A_{Fc} G_c A_{Fc}^T - B_{Fc} \alpha^2 B_{Fc}^T > 0. \qquad (23)$$

According to [14] and §2.7 in [13], (23) holds for $G_c = G_c^T > 0$ if and only if there exits a real matrix $G_{oc} = G_{oc}^T > 0$ satisfying Lyapunov equation

$$G_{oc} - A_{Fc} G_{oc} A_{Fc}^T - B_{Fc} \alpha^2 B_{Fc}^T = 0. \qquad (24)$$

Define $G_{\Delta} := G_c - G_{oc}$. It then follows from (23) and (24) that

$$\begin{aligned} & (G_c - A_{Fc} G_c A_{Fc}^T - B_{Fc} \alpha^2 B_{Fc}^T) - \\ & (G_{oc} - A_{Fc} G_{oc} A_{Fc}^T - B_{Fc} \alpha^2 B_{Fc}^T) \\ = \; & G_{\Delta} - A_{Fc} G_{\Delta} A_{Fc}^T > 0. \end{aligned} \qquad (25)$$

Notice that the spectral radius $\rho(A_{Fc}) < 1$ since $F_c(z)$ is stable. Consequently, the inequality (25) implies that $G_{\Delta} = G_{\Delta}^T > 0$ and $G_c = G_{oc} + G_{\Delta} > G_{oc} > 0$.

Because G_{oc} satisfies Lyapunov equation (24), it follows from [10] that $\|F_{cj}\|_2^2 = C_{Fc}(j) G_{oc} C_{Fc}(j)^T$, where $C_{Fc}(j)$ is the jth row of C_{Fc} and F_{cj} is the the jth channel of the casual part of F_j. Further, from $G_c > G_{oc}$ it follows that

$$C_{Fc}(j) G_c C_{Fc}(j)^T > C_{Fc}(j) G_{oc} C_{Fc}(j)^T = \|F_{cj}\|_2^2. \qquad (26)$$

Now decompose Q_c into

$$Q_c = \begin{bmatrix} Q_{c00} & \cdots & Q_{c0(N-1)} \\ \vdots & \ddots & \vdots \\ Q_{c(N-1)0} & \cdots & Q_{c(N-1)(N-1)} \end{bmatrix}.$$

Using above decomposition and Schur complement, (20) can be written as (27). Since all the leading principle minors of a positive definite matrix are positive definite, (27) implies that

$$\begin{aligned} & Q_{cjj} - (C_{Fc}(j) G_c C_{Fc}(j)^T > 0, \\ & Q_{cjj} > 0, \quad j = 0, 1, \cdots, N-1, \end{aligned} \qquad (28)$$

where $G_c = P_c^{-1}$ has been used to obtain (28). Since P_c satisfies (23) and (18), G_c in (28) satisfies (26). It follows from (28) and (26) that

$$\begin{aligned} Q_{cjj} &> C_{Fc}(j) G_c C_{Fc}(j)^T > C_{Fc}(j) G_{oc} C_{Fc}(j)^T \\ &= \|F_{cj}\|_2^2, \quad j = 0, 1, \cdots, N-1. \end{aligned} \qquad (29)$$

Similarly for the anticausal part, it gives

$$\begin{aligned} Q_{acjj} &> C_{Fac}(j) G_{ac} C_{Fac}(j)^T + D_{Fac}(j) \alpha^2 D_{Fac}(j)^T \\ &> C_{Fac}(j) G_{oac} C_{Fac}(j)^T + D_{Fac}(j) \alpha^2 D_{Fac}(j)^T \\ &= \|F_{acj}\|_2^2, \quad j = 0, 1, \cdots, N-1. \end{aligned} \qquad (30)$$

$$\begin{bmatrix} Q_{c00} & \cdots & Q_{c0(N-1)} \\ \vdots & \ddots & \vdots \\ Q_{c(N-1)0} & \cdots & Q_{c(N-1)(N-1)} \end{bmatrix} - \begin{bmatrix} C_{Fc}(0) \\ \vdots \\ C_{Fc}(N-1)) \end{bmatrix} P_c^{-1} \begin{bmatrix} C_{Fc}(0)^T & \dots & C_{Fc}(N-1)^T \end{bmatrix} > 0. \quad (27)$$

From the definitions of Λ and S_D and the inequalities (29) and (30), (22) is equivalent to the $\|F_j\|_2^2 < \gamma_j$. Thus, the matrix inequalities constraint from (17) to (22) guarantees the satisfaction of the inequalities (5), $\|F_j\|_2^2 = \alpha^2 \|H_j^{-1}\|_2^2 \leq \gamma_j, \forall j \in [0, N-1]$. □

4. SOLUTION TO DESIGN PROBLEM

According to (6), $H^{-1}(z) = H^\dagger(z) + \tilde{N}_\perp(z)V(z)$. So the seven state space parameter matrices of $H^{-1}(z)$ in the optimization problem (17)-(22) are functions of $H^\dagger(z)$, $\tilde{N}_\perp(z)$ and $V(z)$, and a complete state space representation of $H^{-1}(z)$ in terms of the state space realizations of $H(z)$, $H^\dagger(z)$, $\tilde{N}_\perp(z)$ and $V(z)$ is needed for solving the optimization problem.

Denote the minimal state space realizations of $H(z)$, $H^\dagger(z)$, $\tilde{N}_\perp(z)$ and $V(z)$ as follows.

$$H(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right], \quad (31)$$

$$H^\dagger(z) = \left[\begin{array}{c|c} A_{H^\dagger c} & B_{H^\dagger c} \\ \hline C_{H^\dagger c} & D_{H^\dagger c} \end{array}\right] + \left[\begin{array}{c|c} A_{H^\dagger ac} & B_{H^\dagger ac} \\ \hline C_{H^\dagger ac} & 0 \end{array}\right]_{ac}, \quad (32)$$

$$\tilde{N}_\perp(z) = \left[\begin{array}{c|c} A_{\tilde{N}_\perp} & B_{\tilde{N}_\perp} \\ \hline C_{\tilde{N}_\perp} & D_{\tilde{N}_\perp} \end{array}\right]_{ac}, \quad (33)$$

and

$$V(z) = \left[\begin{array}{c|c} A_v & B_v \\ \hline C_v & D_v \end{array}\right]_{ac}. \quad (34)$$

Given the channel FB $H(z)$, it is a routine to obtain its state space realization (31). With (31), the state space realization (32) for $H^\dagger(z)$ can then be computed using the procedure given in [7], and the state space realization (33) for $\tilde{N}_\perp(z)$ can be computed according to (8)-(14). The A_v, B_v, C_v and D_v in the state space realization (34) for $V(z)$ are the parameter matrices to be searched in the optimization.

Note that by (8), $\tilde{N}_\perp(z)$ has only anticausal part and so is its state space realization in (33). From (6), $H^{-1}(z) = H^\dagger(z) + \tilde{N}_\perp(z)V(z)$, where $H^\dagger(z)$ and $\tilde{N}_\perp(z)$, according to (7)-(14), are fixed for a given channel $H(z)$ and only $V(z)$ is free for design. The dimension of $V(z)$ is $(N-M) \times M$, which is the dimension of freedom available for the design. As $H^\dagger(z)$ is generally noncausal, so should be $V(z)$ in order to make full use of the design freedom of $V(z)$. However, if $V(z)$ contains a causal part, the product of $\tilde{N}_\perp(z)$ and $V(z)$ would be associated with the Sylvester equation [15], which makes the design problem (17)-(22) hard to solve by linear matrix inequality (LMI) approach. To avoid this difficulty, $V(z)$ is restricted to be anticausal only in this paper.

Using (32)-(34) and Lemma 2, it is easy, after some operations, to show that a complete state space representation of $H^{-1}(z)$ in terms of the state space realizations of $H(z)$, $H^\dagger(z)$, $\tilde{N}_\perp(z)$ and $V(z)$ is given by

$$\begin{aligned} H^{-1}(z) &= \left[\begin{array}{c|c} A_{Fc} & B_{Fc} \\ \hline C_{Fc} & D_{Fc} \end{array}\right] + \left[\begin{array}{c|c} A_{Fac} & B_{Fac} \\ \hline C_{Fac} & 0 \end{array}\right]_{ac} \\ &= \left[\begin{array}{c|c} A_{H^\dagger c} & B_{H^\dagger c} \\ \hline C_{H^\dagger c} & D_{H^\dagger c} \end{array}\right] + \\ & \left[\begin{array}{ccc|c} A_{H^\dagger ac} & 0 & 0 & B_{H^\dagger ac} \\ 0 & A_{H^\dagger ac} & B_{\tilde{N}_\perp} C_v & B_{\tilde{N}_\perp} D_v \\ 0 & 0 & A_v & B_v \\ \hline C_{H^\dagger ac} & C_{H^\dagger ac} & D_{\tilde{N}_\perp} C_v & D_{\tilde{N}_\perp} D_v \end{array}\right]_{ac}, \end{aligned} \quad (35)$$

where the A_v, B_v, C_v and D_v for the state space realization of $V(z)$ are the parameter matrices to be searched in the optimization. Substituting (35) into (17)-(22) gives a convex optimization subject to LMI constraints that can be solved by Matlab LMI toolbox or similar convex optimization software such as CVX.

5. NON-CAUSAL PRECODER IMPLEMENTATION

The anti-causal system cannot be processed in real-time, but it can be implemented by block transmission. A possible transmission scheme is to pack the source data of each input into blocks, reverse the time indices of the blocks, and then pass the blocks through the anticausal system. Although the block transmission results in transmission delay, it makes anti-causal implementation feasible. The unknown initial condition may result in the inaccuracy of the anticausal part. If the FB channel is FIR, the inaccuracy of the anticausal output could be limited in finite data samples. The following theorem gives the details

Theorem 3 *For an FIR FB channel $H(z)$ in block transmission, at most the first K reconstructed samples of each block are affected by the unknown initial condition, where K is the maximum order of the subband filters of $H(z)$.*

Proof: A noncausal ZF precoder $F(z)$ can be written as the sum of causal part and anti-causal part $F(z) = F_c(z) + F_{ac}(z)$. Since precoder $F(z)$ and channel model $H(z)$ satisfy the ZF condition, their product satisfy $H(z)F(z) = H(z)F_c(z) + H(z)F_{ac}(z) = I$ Because of the causality of $H(z)$ and $F_c(z)$, there is no initial condition problem for this part. Due to the ZF condtion, the product of $H(z)$ and anti-causal part $F_{ac}(z)$ only generate the causal output. Suppose $H(z)$ and $F_{ac}(z)$ with state-space realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$

and $\left[\begin{array}{c|c} A_{Fac} & B_{Fac} \\ \hline C_{Fac} & D_{Fac} \end{array}\right]_{ac}$, respectively. The causal product is given by [15]

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \left[\begin{array}{c|c} A_{ac} & B_{ac} \\ \hline C_{ac} & D_{ac} \end{array}\right]_{ac} = \left[\begin{array}{c|c} A & AYB_{ac} + BD_{ac} \\ \hline C & DD_{ac} + CYB_{ac} \end{array}\right] \quad (36)$$

where Y is the solution of the Sylvester equation $AYA_{Fac} - Y + BC_{Fac} = 0$. It can be seen from (36) that the product $H(z)F_{ac}(z)$ has the same order as $H(z)$. Thus, the distortion caused by unknown initial condition are restricted to the first K samples of each block since the maximum order of each subband filter is K. □

6. NUMERICAL EXAMPLE

Consider the redundant FIR FB channel with $N = 3$, $M = 2$ and polyphase matrix $H(z) = [H_{ij}(z)]_{i=1,2;\ j=1,2,3}$, where $H_{11} = 0.3467 + 0.6228z^{-1} + 0.7966z^{-2}$, $H_{12} = 0.7459 + 0.1255z^{-1}$, $H_{13} = 0.8224 + 0.0252z^{-1} + 0.4144z^{-2}$, $H_{21} = 0.7314 + 0.7814z^{-1} + 0.3673z^{-2}$, $H_{22} = 0.7449 + 0.8923z^{-1}$, $H_{23} = 0.2426 + 0.1296z^{-1}$. Assume the power of input signal $\sigma_x^2 = 1$. The individual channel power constraints for three channels are 2, 2, and 2. The maximum SNR coefficient α_c achieved by the causal design [8] is $\alpha_c = 0.6288$. By using Theorem 3, a noncausal precoder is designed and it achieves $\alpha_{nc} = 1.1462$. The SNR is almost doubled.

7. CONCLUSION

ZF precoder design with individual channel power constraints has been investigated in this paper. A general state space representation of all right inverses of the FB channel has been presented and used to show that for the general (nonminimum phase) FB channels, the optimal ZF precoder is noncausal in general. The design of ZF precoder has been cast into a constrained optimization problem, and the solution has been obtained by converting the optimization problem into that of a convex optimization subject to linear matrix constraints. The numerical example has demonstrated the effectiveness and advantage of the obtained solution.

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