DYNAMIC RATE ADAPTATION AND MULTIUSER DOWNLINK BEAMFORMING USING MIXED INTEGER CONIC PROGRAMMING

Yong Cheng† *Anne Philipp*§ *Marius Pesavento*†

†Communication Systems Group, Technische Universität Darmstadt, Germany
§ Department of Mathematics, Technische Universität Darmstadt, Germany

ABSTRACT

This paper considers jointly optimized rate adaptation and beamforming (JRAB) to achieve maximum weighted sum-rate in a multiuser downlink network. In our approach the rate adaptation consists in assigning modulation and coding schemes (MCSs) for the users which are modeled by multiple-choice constraints. The challenge of the problem lies in its combinatorial nature of the MCS selection process. We address the JRAB problem within the mixed integer second order cone programming (MI-SOCP) framework. We propose a convenient MI-SOCP reformulation that is specifically suitable to reduce the run-time in branch-and-bound (BB) methods. Further, a preprocessing step and a novel branching prioritizing principle (BPP) are introduced to speed up the BB type solutions. To facilitate applications in large systems, a fast heuristic algorithm is developed. We show via simulations that the improved MI-SOCP formulation and the BPP can significantly reduce the run-time for solving the JRAB problem with BB. Numeric results also show that the heuristic algorithm yields weighted sum-rates that are larger than that computed by the state-of-the-art MI-SOCP solver IBM ILOG CPLEX under the given run-time limitations.

Index Terms— Dynamic rate adaptation, Adaptive modulation and coding, Multiuser downlink beamforming, Joint optimization, Mixed integer conic programming

1. INTRODUCTION

Dynamic rate adaptation in the form of adaptive modulation and coding (AMC), which matches MCSs to channel conditions, has long been identified as an effective mean to improve bit-error-rate and throughput performance of wireless systems. AMC has been extensively studied in the literature (see, e.g., [1, 2, 3] and references therein) and has made its way into wireless standards (e.g., 3GPP LTE [4, Chap. 5]).

Advanced smart antenna technology is considered as another key enabler of future wireless systems. With multiple antennas at the basestation (BS), multiuser downlink beamforming can be employed to multiplex data transmissions to mobile users (MUs) in space, which improves dramatically the spectral efficiency of cellular networks. This technology has also been intensively investigated in the wireless research community (see, e.g., [5, 6, 7] and references therein) and has recently been adopted in wireless standards [4, Chap. 5].

In this paper we consider the problem of joint rate adaptation and multiuser downlink beamforming (JRAB), in which optimal data rates are assigned to mobile users (MUs) based on the channel and interference conditions of all MUs to maximize the weighted sum-rate of the downlink system. As the data rates associated with specific MCSs are discrete and the MUs, which are jointly served with beamformers, are coupled by cochannel interference (CCI), the JRAB problem involves mixed integer nonlinear optimization. In the JRAB problem, a MU may be allocated a zero data rate meaning that the MU is not scheduled. However, if a MU is assigned a non-zero data rate, a corresponding minimum received SINR level is required [1, 2]. As a result, the JRAB problem that we address implicitly includes joint user scheduling and beamforming [5], and joint admission control and beamforming [6], as well as joint MCS selection and power control [3].

We address the JRAB problem using a MI-SOCP approach. With the MCS selection and beamforming modeled respectively as multiple-choice constraints and second order cone constraints, the JRAB problem is formulated as a MI-SOCP. The branch-and-bound (BB) methods, which are widely adopted for solving MI-SOCPs [8], yield optimal solutions in reasonable run-time only for small-scale configurations of the JRAB problem. To reduce the run-time required for solving the JRAB problem with BB, a significantly improved MI-SOCP formulation is proposed. In addition, a preprocessing step and a branching prioritizing principle (BPP) are introduced to further speed up the BB solutions. Furthermore, a fast heuristic algorithm is developed for applications in medium to large scale systems to compute close-to-optimal solutions of the JRAB problem.

The simulations confirm that the improved MI-SOCP formulation and the BPP can significantly reduce the run-time for solving the JRAB problem with BB. Numeric results also demonstrate that the fast algorithm yields weighted sum-rates (WSRs) that are larger than that computed by the solver CPLEX [8] under the practical run-time limitations.

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2. SYSTEM MODEL

2.1. Multiuser Downlink System

Consider a cellular downlink system comprising one BS that is equipped with $M > 1$ transmit antennas, and $K > 1$ single-antenna MUs, as illustrated in Fig. 1.

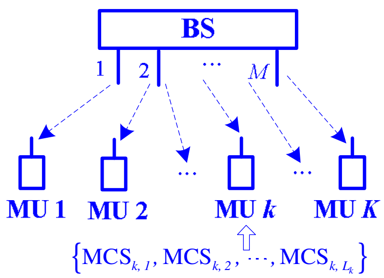

Fig. 1. Illustration of a multiuser downlink network

Let $\mathbf{h}_k \in \mathbb{C}^M$ denote the frequency-flat channel vector between the BS and the kth MU, $\forall k \in \mathcal{K} \triangleq \{1, 2, \cdots, K\}$. We define accordingly $\mathbf{w}_k \in \mathbb{C}^M$ as the beamformer used at the BS for transmitting data to the kth MU, $\forall k \in \mathcal{K}$. The received signal $y_k \in \mathbb{C}$ at the kth MU can be written as:

$$y_k = \mathbf{h}_k^H \mathbf{w}_k x_k + \sum_{j=1, j\neq k}^{K} \mathbf{h}_k^H \mathbf{w}_j x_j + z_k, \forall k \in \mathcal{K}, \qquad (1)$$

with $x_k \in \mathbb{C}$ representing the *unit-power* data symbol, i.e., $\mathrm{E}\left\{x_k x_k^H\right\} = 1$, designated for the kth MU, and $z_k \in \mathbb{C}$ denoting the additive noise at the receiver of the kth MU, with zero mean and variance σ_k^2, $\forall k \in \mathcal{K}$.

Similar to [5, 6, 7], it is assumed that the data symbols for different MUs are mutually statistically independent and also independent from the receiver noise. It is further assumed that single user detection is performed at the MUs, i.e., the CCI is treated as noise at the MUs. The received signal-to-interference-plus-noise-ratio (SINR) at the kth MU, denoted by SINR_k, is then given by [5, 6, 7]:

$$\text{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k}{\sum_{j=1, j\neq k}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2}, \forall k \in \mathcal{K}. \qquad (2)$$

2.2. Dynamic Rate Adaptation

In modern cellular communication systems like LTE, with dynamic rate adaptation in the form of AMC, the data rate of a MU, in bits-per-symbol or bits-per-channel-use (bpcu), takes discrete values determined by specific MCSs assigned to the MU, see Tab. 1 [4, Chap. 5]. Corresponding to each MCS and data rate, a minimum received SINR level is required to ensure the target block-error-rate (BLER) and/or bit-error-rate (BER) [1, 2, 3]. While the candidate MCSs and corresponding data rates are defined in the wireless standards [4, Chap. 5], the associated minimum received SINR requirements are equipment and implementation dependent. For instance, typical SINR requirements for different MCSs in LTE systems are listed in Tab. 1. These SINR values are obtained from extensive simulations for a target BLER of 10% [1, 2].

In this paper, we consider the scenario that the BS selects one specific MCS out of $L_k > 1$ candidate MCSs $\{\text{MCS}_{k,1}, \text{MCS}_{k,2}, \cdots, \text{MCS}_{k,L_k}\}$ for the kth MU, $\forall k \in \mathcal{K}$. If no appropriate MCS is selected for the kth MU, it is not served in the current time interval. To assign the lth candidate MCS to the kth MU, it is required that [1, 2]:

$$\text{SINR}_k \geq \Gamma_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k, \qquad (3)$$

with the constant $\Gamma_{k,l} > 0$ (in linear scale) representing the minimum SINR requirement corresponding to the lth candidate MCS of the kth MU, the set $\mathcal{L}_k \triangleq \{1, 2, \cdots, L_k\}$, the cardinality $L_k \triangleq |\mathcal{L}_k|$, and SINR_k defined in (2).

Throughout this paper, it is assumed that the BS has full knowledge of the CSI $\{\mathbf{h}_k, \forall k \in \mathcal{K}\}$, which is a common assumption made in the literature, see, e.g., [5, 6, 7]. The BS jointly assigns the MCSs and computes the beamformers $\{\mathbf{w}_k, \forall k \in \mathcal{K}\}$ for all K MUs.

Table 1. Candidate MCSs, data rates, and minimum received SINR requirements of LTE systems [1, 2]

Mod. Order	Code Rate ($\times$1024)	Data Rate $R_{k,l}$ (bits/symbol)	SINR Level $\widetilde{\Gamma}_{k,l}$ (dB)
QPSK	78	0.1523	−9.478
QPSK	120	0.2344	−6.658
QPSK	193	0.3770	−4.098
QPSK	308	0.6010	−1.798
QPSK	449	0.8770	0.399
QPSK	602	1.1758	2.424
16QAM	378	1.4766	4.489
16QAM	490	1.9141	6.367
16QAM	616	2.4063	8.456
64QAM	466	2.7305	10.266
64QAM	567	3.3223	12.218
64QAM	666	3.9023	14.122
64QAM	772	4.5234	15.849
64QAM	873	5.1152	17.786
64QAM	948	5.5547	19.809

3. RATE ADAPTATION AND BEAMFORMING

3.1. Problem Formulation

The problem of interest is to jointly optimize the MCS assignments and beamformer design of all K MUs with the objective of maximizing the WSR in bpcu. To model the assignment of MCSs, we introduce binary variables $\{a_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k\}$ and assume that the variable $a_{k,l}$ takes the value $a_{k,l} = 1$ if the kth MU is scheduled to be served with the lth candidate MCS, and $a_{k,l} = 0$ otherwise. The JRAB problem can then be formulated as:

$$\max_{\{\mathbf{w}_k, a_{k,l}\}} \quad \sum_{k=1}^{K} \lambda_k \sum_{l=1}^{L_k} a_{k,l} R_{k,l} \tag{4a}$$

$$\text{s. t.} \quad a_{k,l} \in \{0,1\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k, \tag{4b}$$

$$\sum_{l=1}^{L_k} a_{k,l} \leq 1, \forall k \in \mathcal{K}, \tag{4c}$$

$$\sum_{k=1}^{K} \|\mathbf{w}_k\|_2^2 \leq P^{(\max)}, \tag{4d}$$

$$\text{SINR}_k \geq \sum_{l=1}^{L_k} a_{k,l} \Gamma_{k,l}, \forall k \in \mathcal{K}, \tag{4e}$$

with the constant $\lambda_k \geq 0$ characterizing the priority weight of the kth MU, the constant $P^{(\max)} > 0$ denoting the maximum allowable transmit power (in linear scale) of the BS, and SINR_k defined in (2).

In the JRAB problem formulation (4), the so-called multiple-choice constraint in (4c) ensures that either the kth MU is served with one of its L_k MCSs (i.e., $\sum_{l=1}^{L_k} a_{k,l} = 1$), or the kth MU is currently not served (i.e., $\sum_{l=1}^{L_k} a_{k,l} = 0$). As a result, multiuser scheduling and admission control are included implicitly in formulation (4). The JRAB problem in the form of (4) is nonconvex even if the binary variables $\{a_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k\}$ are relaxed to be continuous, i.e., if constraint (4b) is replaced by $0 \leq a_{k,l} \leq 1$. We derive in the next subsection an equivalent MI-SOCP reformulation that can be efficiently solved, e.g., using BB methods [8].

3.2. An Equivalent MI-SOCP Formulation

Without loss of generality, it is assumed that the data rates and the corresponding SINR requirements associated with the MCSs are ordered as:

$$R_{k,1} \leq R_{k,2} \leq \cdots \leq R_{k,L_k}, \forall k \in \mathcal{K}, \tag{5a}$$

$$\Gamma_{k,1} \leq \Gamma_{k,2} \leq \cdots \leq \Gamma_{k,L_k}, \forall k \in \mathcal{K}. \tag{5b}$$

Substituting eq. (2) into eq. (4e), we obtain the following equivalent representation of the SINR constraints:

$$\left(\sum_{j=1}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2 \right) \sum_{l=1}^{L_k} a_{k,l} \Gamma_{k,l} \leq \left(1 + \sum_{l=1}^{L_k} a_{k,l} \Gamma_{k,l} \right) \mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k, \forall k \in \mathcal{K}. \tag{6}$$

The constraints in (6) represent the major difficulty in solving problem (4), as it involves the products of binary variables $\{a_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k\}$ and quadratic terms of beamformers $\{\mathbf{w}_k, \forall k \in \mathcal{K}\}$. However, under the constraints in (4b) and (4c), and with the ordering in (5), it can be proved by Cauchy-Schwarz inequality that the constraints in (6) are equivalent to the following more tractable constraints:

$$\sum_{j=1}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2 \leq \left(1 - \sum_{l'=l}^{L_k} a_{k,l'} \right) U_k^2 + \gamma_{k,l}^2 \mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k, \tag{7}$$

with the constants $U_k > 0$ and $\gamma_{k,l} \geq 1$ defined as:

$$U_k \triangleq \sqrt{P^{(\max)} \|\mathbf{h}_k\|_2^2 + \sigma_k^2}, \forall k \in \mathcal{K}, \tag{8}$$

$$\gamma_{k,l} \triangleq \sqrt{1 + 1/\Gamma_{k,l}}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k. \tag{9}$$

Further, it is well known that the SINR constraints (7) can be rewritten as [6, 7]:

$$\text{Im}\left\{\mathbf{h}_k^H \mathbf{w}_k\right\} = 0, \ \text{Re}\left\{\mathbf{h}_k^H \mathbf{w}_k\right\} \geq 0, \ \forall k \in \mathcal{K}, \tag{10a}$$

$$\left\| \left[\mathbf{h}_k^H \mathbf{W}, \ \sigma_k \right] \right\|_2 \leq \left(1 - \sum_{l'=l}^{L_k} a_{k,l'} \right) U_k + \gamma_{k,l} \text{Re}\left\{\mathbf{h}_k^H \mathbf{w}_k\right\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k, \tag{10b}$$

with the matrix $\mathbf{W} \triangleq [\mathbf{w}_1, \ \mathbf{w}_2, \ \cdots, \ \mathbf{w}_K] \in \mathbb{C}^{M \times K}$.

The constraints in (10b) become second order cone (SOC) constraints [9, Sec. 4.4.2] when the binary variables $\{a_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k\}$ originally defined on the discrete set $\{0,1\}$ are relaxed to be continuous and confined in the closed interval $[0, \ 1]$. We further introduce the auxiliary variables $\{c_k \geq 0, \forall k \in \mathcal{K}\}$ to reformulate the SINR constraints in (10b) with fewer quadratic constraints. The term c_k^2 models the received signal power of the kth MU when it is scheduled (i.e., $\sum_{l=1}^{L_k} a_{k,l} = 1$), and it models the upper bound on the received interference-plus-noise power of the kth MU when it is not scheduled (i.e., $\sum_{l=1}^{L_k} a_{k,l} = 0$), $\forall k \in \mathcal{K}$. The $\sum_{k=1}^{K} L_k$ quadratic constraints in (10b) can then be reduced into K quadratic constraints plus $\sum_{k=1}^{K} L_k$ linear constraints as given in eq. (11) below:

$$\left\| \left[\mathbf{h}_k^H \mathbf{W}, \ \sigma_k \right] \right\|_2 \leq c_k, \forall k \in \mathcal{K}, \tag{11a}$$

$$c_k \leq \left(1 - \sum_{l'=l}^{L_k} a_{k,l'} \right) U_k + \gamma_{k,l} \text{Re}\left\{\mathbf{h}_k^H \mathbf{w}_k\right\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k. \tag{11b}$$

We remark that, in contrast to (10b), the SINR constraints in (11) are favorable for BB implementations as the number of quadratic constraints in (11) is reduced, resulting in a reduced computational complexity at each node in the BB methods.

Replacing the SINR constraints in (4e) with their equivalences in (10a) and (11), we obtain the following MI-SOCP reformulation of the JRAB problem (4):

$$\max_{\{\mathbf{W}, \ a_{k,l}, c_k\}} \quad \sum_{k=1}^{K} \lambda_k \sum_{l=1}^{L_k} a_{k,l} R_{k,l} \tag{12a}$$

$$\text{s. t.} \quad \text{(4b), (4c), (4d), (10a), and (11).} \tag{12b}$$

4. OPTIMAL SOLUTIONS VIA BB METHODS

Small-scale JRAB problems (12) can be solved to global optimality with the BB methods in reasonable run-time. The computational complexity of the BB method depends mainly on the tightness of the associated continuous relaxation, the number of binary variables, and the branching priorities of the binary variables [8]. To reduce the run-time required for solving the JRAB problem (12) with BB methods, we propose in this section an improved MI-SOCP reformulation, a preprocessing step, and a branching prioritizing principle (BPP).

Introducing the auxiliary variable $t_k \geq 0$ to model the allocated power of the kth MU, $\forall k \in \mathcal{K}$, it can readily be verified that the JRAB problem formulations in (4) and (12) are equivalent to the following MI-SOCP:

$$\max_{\{\mathbf{W}, a_{k,l}, c_k, t_k\}} \quad \sum_{k=1}^{K} \lambda_k \sum_{l=1}^{L_k} a_{k,l} R_{k,l} \tag{13a}$$

$$\text{s. t.} \quad \|\mathbf{w}_k\|_2^2 \leq t_k \sum_{l=1}^{L_k} a_{k,l}, \tag{13b}$$

$$t_k \geq 0, \;\; t_k \leq P^{(\max)} \sum_{l=1}^{L_k} a_{k,l}, \tag{13c}$$

$$\sum_{k=1}^{K} t_k \leq P^{(\max)}, \forall k \in \mathcal{K}, \tag{13d}$$

$$\text{Re}\left\{\mathbf{h}_k^H \mathbf{w}_k\right\} \geq \sigma_k \sum_{l=1}^{L_k} a_{k,l} \sqrt{\Gamma_{k,l}}, \tag{13e}$$

$$\text{(4b), (4c), (10a), and (11),} \tag{13f}$$

where the constraints (13b) can be rewritten as [9, Exer. 4.26]:

$$\left\| \left[2\mathbf{w}_k, \; t_k - \sum_{l=1}^{L_k} a_{k,l} \right] \right\|_2 \leq t_k + \sum_{l=1}^{L_k} a_{k,l}, \forall k \in \mathcal{K}. \tag{14}$$

We remark that, due to the constraints in (13b), (13c), (13d), and (13e), the continuous relaxation of formulation (13) provides a much tighter upper bound on the objective function of (4) than that obtained from the continuous relaxation of (12). This is because for the continuous relaxation of (12), the reformulation in (11) can result in a lose approximation of (6). This is the case if the terms $\left\{\sum_{l=1}^{L_k} a_{k,l}, \forall k \in \mathcal{K}\right\}$ become close to one (but not exactly one). However, in the continuous relaxation of (13), the constraints in (13e) dominate the constraints (11) and limit the values of the terms $\left\{\sum_{l=1}^{L_k} a_{k,l}, \forall k \in \mathcal{K}\right\}$, resulting in a tighter upper bound.

To further speed up the BB methods, a preprocessing step can be included to reduce the number of candidate MCSs in the JRAB problem (4) if some of the MCSs cannot be supported even in the ideal case of no multiuser interference. Specifically, the variable $a_{k,l}$ is fixed to zero if the maximum received signal-to-noise-ratio (SNR) of the kth MU, i.e., $\|\mathbf{h}_k\|_2^2 P^{(\max)}/\sigma_k^2$, is smaller than the SINR requirement $\Gamma_{k,l}$.

In addition to preprocessing, we propose an effective branching prioritizing principle (BPP) that can be easily incorporated into the BB methods: (i) map the priorities of the binary variables to the corresponding data rates, i.e., a higher priority for a larger data rate; (ii) prioritize according to channel gains *when two binary variables correspond to the same data rate*, i.e., a higher priority for a larger channel gain.

5. A LOW-COMPLEXITY ALGORITHM

To facilitate applications in large networks, we propose in this section a fast heuristic algorithm to compute close-to-optimal solutions of the JRAB problem in (13). The solutions found by the heuristic algorithm can also serve as good initialization points for the BB type methods.

We define the integers $\Upsilon_{k,l}$, $k \in \mathcal{K}, l \in \mathcal{L}_j$, according to

$$\Upsilon_{k,l} \triangleq 1 + \sum_{j \in \mathcal{K}, m \in \mathcal{L}_j} \mathcal{I}\left(\lambda_j R_{j,m} < \lambda_k R_{k,l}\right), \tag{15}$$

with the indicator function $\mathcal{I}\left(x < y\right)$ defined as

$$\mathcal{I}\left(x < y\right) = \begin{cases} 1, & \text{if } x < y, \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Denote $\overline{\mathcal{K}}$ as the set of MUs that have been assigned a data rate in a given iteration, with $\overline{\mathcal{K}} = \emptyset$ in the initial iteration. After obtaining the supportable candidate MCS sets $\overline{\mathcal{L}}_k$, $\forall k \in \mathcal{K}$, via preprocessing, the following step

$$\left(\overline{k}, \, \overline{l}\right) = \underset{k \in \mathcal{K} \setminus \overline{\mathcal{K}}, \; l \in \overline{\mathcal{L}}_k}{\text{argmax}} \left(\Upsilon_{k,l} \max_{j \in \mathcal{K}} \|\mathbf{h}_j\|_2^2 + \|\mathbf{h}_k\|_2^2 \right). \tag{17}$$

is carried out in each iteration to select *tentatively* the $\overline{l}$th candidate MCS for the $\overline{k}$th MU if $\Upsilon_{\overline{k},\overline{l}} \geq 1$. Otherwise, if $\Upsilon_{\overline{k},\overline{l}} \leq 0$, the algorithm stops. Then, the following convex feasibility problem:

$$\text{find:} \;\; \mathbf{W}, \tag{18a}$$

$$\text{s. t.} \;\; \text{Im}\left\{\mathbf{h}_j^H \mathbf{w}_j\right\} = 0, \forall j \in \overline{\mathcal{K}} \cup \{\overline{k}\}, \tag{18b}$$

$$\left\| \left[\mathbf{h}_j^H \mathbf{W}, \; \sigma_j\right] \right\|_2 \leq \text{Re}\left\{\mathbf{h}_j^H \mathbf{w}_j\right\} \sqrt{1 + 1/\overline{\Gamma}_j}, \forall j \in \overline{\mathcal{K}} \cup \{\overline{k}\}, \tag{18c}$$

$$\sum_{j \in \overline{\mathcal{K}} \cup \{\overline{k}\}} \|\mathbf{w}_j\|_2^2 \leq P^{(\max)}, \tag{18d}$$

is solved, with $\overline{\Gamma}_j$ denoting the minimum SINR requirement corresponding to the MCS selected for the jth MU, $\forall j \in \overline{\mathcal{K}}$, and $\overline{\Gamma}_{\overline{k}} \triangleq \Gamma_{\overline{k},\overline{l}}$. If problem (18) is feasible, assign the $\overline{l}$th candidate MCS to the $\overline{k}$th MU and add $\overline{k}$ into the set $\overline{\mathcal{K}}$. This procedure is carried out iteratively. To prevent loops, we set $\Upsilon_{\overline{k},\overline{l}} = 0$ in each iteration. The algorithm is summarized in Alg. 1. The worst-case computational complexity of the proposed low-complexity algorithm consists in solving $\sum_{k=1}^{K} L_k$ times the feasibility problem in (18).

Initialization: Initialize the set $\overline{\mathcal{K}}$ to be empty set.
Step 1: Compute $\left(\overline{k}, \overline{l}\right)$ as in (17). If $\Upsilon_{\overline{k},\overline{l}} \leq 0$, exit.
Step 2: Check the feasibility of problem (18).
Step 3: If problem (18) is feasible, record $\overline{l}$th MCS for the $\overline{k}$th MU and update the set: $\overline{\mathcal{K}} = \overline{\mathcal{K}} \cup \{\overline{k}\}$.
Step 4: Set $\Upsilon_{\overline{k},\overline{l}} = 0$, go to **Step 1** and repeat.

Alg. 1: The proposed low-complexity algorithm

6. NUMERIC RESULTS AND DISCUSSIONS

In the simulations, we use the following channel model [4, Chap. 9]: (i) 3GPP LTE pathloss mode: $PL = 148.1 + 37.6 \log(d)$ (in dB), with d (in km) being the BS-MS distance; (ii) log-norm shadowing with zero mean, 8 dB variance; (iii) Rayleigh fading with zero mean and unit variance; and (iv) transmit antenna power gain of 9 dB, receiver noise figure of 7 dB, and system bandwidth of 1.4 MHz. The same candidate MCSs in Tab. 1 are used for the K MUs. The distances between the BS and the K MUs are randomly generated in the interval $[0.2, 1]$ km. The maximum transmit power $P^{(\max)}$ varies from 8 dB to 16 dB with a stepsize of 2 dB. The MU priority weights are set to be identical, i.e., $\lambda_k = 1, \forall k \in \mathcal{K}$.

We first simulate a small network with $K = 5$ MUs and $M = 4$ antennas at the BS. CPLEX is configured to compute optimal solutions and the associated optimality certificates. Tab. 2 lists the run-time averaged over 100 Monte Carlo runs and the 5 values of $P^{(\max)}$. Comparing the second and the first rows of Tab. 2, we observe that the average run-time consumed in finding optimal solutions of the formulation (13) is about 26% of that for the formulation (12). Comparing the second and the third rows of Tab. 2, we observe that the average run-time for solving the formulation (13) with BPP is about 35% of that without BPP.

Table 2. Average run-time, with $M = 4$ and $K = 5$

Methods	CPU Time
CPLEX on form (12)	24.40 seconds
CPLEX on form (13)	6.38 seconds
CPLEX on form (13) w/o BPP	18.31 seconds

In the second example, a system with $K = 12$ and $M = 4$ is simulated. The solver CPLEX is configured to find feasible solutions of the JRAB problem (13) under a practical run-time limitation of 60 seconds. Fig. 2 displays WSR vs. $P^{(\max)}$, with the results averaged over 100 Monte Carlo runs.

It can be observed from Fig. 2 that the proposed Alg. 1 yields WSRs that are larger than that computed by CPLEX under the given rum-time limit, while Alg. 1 only consumes about 8% of the run-time of CPLEX. Also, when CPLEX is initialized with the solutions found by Alg. 1, new solutions with larger WSRs can be reached by CPLEX. Finally, we observe that the solutions yielded by the proposed Alg. 1 are very close to the upper bound computed by CPLEX and therefore the low-complexity algorithm is indeed close-to-optimal.

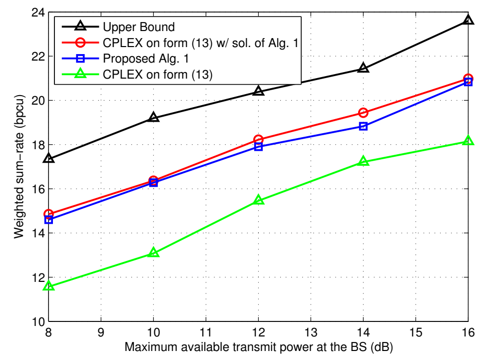

Fig. 2. The curves from top to bottom correspond respectively to the upper bound computed by CPLEX, the WSRs of CPLEX initialized with the solutions of Alg. 1, the WSRs of Alg. 1, and the WSRs of CPLEX without initialization.

7. REFERENCES

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