AN ITERATIVE METHOD FOR THE MAXIMIZATION OF THE SUM-RATE OF MULTICELL COOPERATIVE SYSTEMS

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ABSTRACT

In this paper, we consider a downlink cooperative multicell Base stations (BSs) system where the BSs are coordinating their transmitted signals to communicate with Mobile Stations (MSs) and both BSs and MSs are equipped with multiple antennas. We model this multicell cooperative system under the presence of channel estimation errors and we consider a general framework where the power that is transmitted from the BSs to the MSs is not constrained to be uniform. We propose to perform a modified version of the classical ZF beamforming where we decouple the power control and beamforming processes and we derive its corresponding cooperative system sum-rate. In order to maximize the system sum-rate, we propose a "projected gradient" based iterative algorithm and we show that using our proposed algorithm as a power allocation policy leads to significantly enhance the reachable multicell cooperative system sum-rate compared to the known situation where the power of the BSs is uniformly distributed. Simulation results are given to support our claims.

Index Terms— Cooperative base stations, Zero-Forcing beamforming, channel estimation errors, projected gradient, system sum-rate.

1. INTRODUCTION

Recently, multi-cell Base Stations (BSs) cooperation has been introduced as a promising direction for LTE Advanced which is deemed to be four generation communication systems [1]. While in previous systems Mono-Site Multiple Input Multiple Output (MIMO) has been used, in LTE Advanced, BSs cooperation requires Multi-Site MIMO to be deployed. This technique has the potential to mitigate Inter-Cell Interference (ICI) and increase data rates without sacrificing additional spectrum. To reach the optimal capacity of these systems, [2] showed that the mitigation of ICI can be performed thanks to the Dirty paper coding (DPC). However, DPC is found to be rather a complicated scheme to be implemented in practical systems. This motivates investigation of linear and feasible precoding techniques such as, Tomlinson-Harashima Precoding (THP) [3], Zero-Forcing (ZF) and Block-Diagonalization (BD) beamforming [4] and [5]. In the literature, both BD and ZF techniques are widely considered in a full CSI case at the transmitter and when the power is evenly distributed between the users (see [5], [6] and [7]). For example, in [5], the authors investigated a BD scheme in mono-cellular Multi-User MIMO (MU-MIMO) system, in order to eliminate the interference and derived the optimal power allocation based on the water-filling technique. In [7], the authors assumed a perfect CSI knowledge in a BS cooperative system where the BSs are subject to per-antenna power constraints and performed a BD processing. In practice, the perfect CSI may not be available because of the channel estimation errors that can be due to the channel fading caused by the motion of the Mobile Stations (MSs) in a multipath propagation environment.

In this paper, we propose to derive and then maximize the sum rate expression in a cooperative multicell BSs system in the presence of channel estimation errors.

2. NOTATIONS

The boldface lower case letters denote vectors and boldface upper case letters to denote matrices. The superscripts $.^H$, $.^T$ and $.^*$, denote the conjugate transpose, the transpose and the element wise conjugation, respectively. $\mathbf{I}_M$, $diag(\mathbf{a}_1, \ldots, \mathbf{a}_K)$ and tr refers to the identity matrix of order M, a diagonal matrix and trace, respectively. The notation $\mathbb{C}^{a \times b}$ denotes a complex matrix with a rows and b columns, $\|\mathbf{x}\|$ refers to the Euclidean distance and $\mathcal{E}$ is the expectation operator.

3. SYSTEM MODEL

We consider a multicell downlink MU-MIMO system, where several cooperative BSs coordinate their transmitted signals to the MSs via a preprocessing matrix, in order to cancel the ICI. The two widely used downlink preprocessing algorithms in MU-MIMO systems are: BD and ZF beamforming (see [5],[6] and [7]). The transmission scheme includes: M BSs (each BS is equipped with N_t transmit antennas), K MSs (each MS is equipped with N_r receive antennas). All BSs inter-cooperate to transmit signals to the existing MSs via a whole set of MN_t BS antennas. Each of these transmissions is defined as a link, over which a data stream is transmitted.

The channel matrix from the m^{th} BS to the k^{th} MS, denoted by $\mathbf{H}_k^{(m)}$, includes the contribution of: a small-scale fading component that is also known as fast Rayleigh fading, a medium-scale fading component known as shadowing and a large-scale fading component known as pathloss.

Small-scale fading is characterized here by a matrix $\mathbf{C}_k^{(m)} \in \mathbb{C}^{N_r \times N_t}$ where the $(i,j)^{th}$ entry of $\mathbf{C}_k^{(m)}$ denotes the path gain from the j^{th} antenna of the m^{th} BS to the i^{th} antenna of the k^{th} MS and

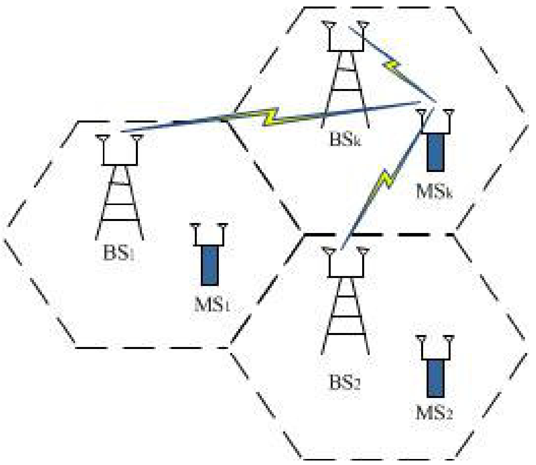

Fig. 1. Multicell downlink MU-MIMO system: $M = 3$, $N_t = 2$, $K = 3$ and $N_r = 2$.

are assumed to be i.i.d. zero-mean and unit-variance complex Gaussian random variables. The $(i,j)^{th}$ element of the channel matrix $\mathbf{H}_k^{(m)}$ is given by $\mathbf{H}_k^{(m)}(i,j) = \alpha_{i_k,j_m}\mathbf{C}_k^{(m)}(i,j)$ where α_{i_k,j_m} encompasses both medium- and large-scale fading from the m^{th} BS to the k^{th} MS.

3.1. Perfect CSI case

Let N_s denote the number of data streams for the k^{th} user, where $N_s < min(MN_t, N_r)$. In a cooperative multicell BSs system, the equivalent discrete-time received signal, $\mathbf{y}_k \in \mathbb{C}^{N_r\times 1}$, at the k^{th} MS can be expressed as:

$$\begin{aligned}\mathbf{y}_k &= \underbrace{\sum_{m=1}^{M}\mathbf{H}_k^{(m)}\mathbf{W}_k^{(m)}(\mathbf{P}_k^{(m)})^{\frac{1}{2}}\mathbf{x}_k}_{\text{useful signal}} \\ &+ \underbrace{\sum_{j=1,j\neq k}^{K}\sum_{m=1}^{M}\mathbf{H}_k^{(m)}\mathbf{W}_j^{(m)}(\mathbf{P}_j^{(m)})^{\frac{1}{2}}\mathbf{x}_j}_{\text{interference term}} + \underbrace{\mathbf{n}_k}_{\text{noise}}, \quad (1)\end{aligned}$$

where:

- $\mathbf{x}_k \in \mathbb{C}^{N_s\times 1}$ is the transmitted data for the k^{th} MS, that is assumed to be i.i.d. and $\mathcal{E}\{\mathbf{x}_k\mathbf{x}_k^H\} = \mathbf{I}_{N_s}$,
- $\mathbf{n}_k \in \mathbb{C}^{N_r\times 1}$ denotes the complex Additive White Gaussian Noise (AWGN) with zero-mean and a covariance matrix that is equal to $\sigma^2\mathbf{I}_{N_r}$,
- $\mathbf{P}_k^{(m)} = diag(p_{k,1}^{(m)},\ldots,p_{k,N_s}^{(m)})$, where $p_{k,i}^{(m)}$ refers to the downlink power allocated to the i^{th} stream of the k^{th} MS for the m^{th} BS,
- $\mathbf{W}_k^{(m)} \in \mathbb{C}^{N_t\times N_s}$ represents the transmit beamforming matrix for the k^{th} MS from the m^{th} BS.

In [4] and [5], the authors assumed that each BS allocates the same power level to the same MS. In this paper, we consider a more general approach where the allocated power levels from the set of BSs to the same MS are not necessary identical. This is why the term $\mathbf{P}_k^{(m)}$ exists in expression (1). Consider $\mathbf{u_k}^{(m)} = \mathbf{W}_k^{(m)}(\mathbf{P}_k^{(m)})^{\frac{1}{2}}\mathbf{x}_k$, $k = 1\ldots K$, as the transmitted signal from the m^{th} BS to the MSs and define $\mathbf{U}^{(m)} = [(\mathbf{u_1}^{(m)})^T,\ldots,(\mathbf{u_K}^{(m)})^T]^T$ and where $\mathbf{x}_k$ is assumed i.i.d. with zero-mean and unit variance. For each m^{th} BS, the total transmitted power is equal to:

$$\mathcal{E}\left\{(\mathbf{U}^{(m)})^H\mathbf{U}^{(m)}\right\} = \sum_{k=1}^{K} tr\left(\mathbf{P}_k^{(m)}(\mathbf{W}_k^{(m)})^H\mathbf{W}_k^{(m)}\right).$$

In the sequel, we assume the following power constraints:

$$\sum_{k=1}^{K} tr(\mathbf{P}_k^{(m)}\,(\mathbf{W}_k^{(m)})^H\,\mathbf{W}_k^{(m)}) \leq P_T,\ m = 1,\ldots,M, \quad (2)$$

where P_T is defined as the maximum total transmitted power constraint for each BS. This means that the allocated power to each MS'antenna can vary from one BS to another. For the setup simplicity, we assume also in the sequel that $N_r = N_s$. The key problem of joint transmit processing among cooperative BSs is to define a joint transmit beamforming matrix to cancel the ICI term of expression (1).

In the previous work ([4] and [5]), the beamforming matrix $\tilde{\mathbf{W}}_k = [(\mathbf{W}_k^{(1)})^T,\ldots,(\mathbf{W}_k^{(M)})^T]^T \in \mathbb{C}^{M\,N_t\times N_r}$ for the k^{th} MS, satisfies $\tilde{\mathbf{H}}_k\tilde{\mathbf{W}}_j = \mathbf{0}, \forall k \neq j$, where $\tilde{\mathbf{H}}_k = [\mathbf{H}_k^{(1)},\ldots,\mathbf{H}_k^{(M)}] \in \mathbb{C}^{N_r\times M\,N_t}$. The authors of [7] proposed $\tilde{\mathbf{W}}_k = [(\mathbf{W}_k^{(1)}(\mathbf{P}_k^{(1)})^{\frac{1}{2}})^T,\ldots,(\mathbf{W}_k^{(M)}(\mathbf{P}_k^{(M)})^{\frac{1}{2}})^T]^T$ as a beamforming matrix, which would lead to couple the beamforming and the power control processes. This approach is not the one that we are looking for in our paper: we are searching for the optimal power allocation that verifies the constraint (2). The solution that we propose, consists in conceiving normalized beamforming matrices $\mathbf{W}_k^{(m)}$ such that:

$$\mathbf{H}_k^{(m)}\mathbf{W}_j^{(m)} = \mathbf{0}, \forall k \neq j. \quad (3)$$

Contrarily to the previous referred work, we are reasoning on each matrix $\mathbf{W}_k^{(M)}$ and not on the block matrix $\tilde{\mathbf{W}}_k$ in order to decouple the normalized beamforming and the power control matrices, which would result in further degrees of freedom in the design of the normalized beamforming scheme.

3.2. Imperfect CSI case

In practical situations, usually the perfect knowledge of the CSI could not be available at the transmitter side. As such, the channel matrix $\mathbf{H}_k^{(m)}$ can be expressed as [4]:

$$\mathbf{H}_k^{(m)} = (\hat{\mathbf{H}}^e)_k^{(m)} + \mathbf{\Delta}_k^{(m)}, \quad (4)$$

where $(\hat{\mathbf{H}}^e)_k^{(m)}$ and $\mathbf{\Delta}_k^{(m)}$ are the estimated channel and estimation error matrix, from m^{th} BS to k^{th} MS, respectively. The estimation error matrix $\mathbf{\Delta}_k^{(m)}$ is due to either the thermal noise that befoul the channel estimation process when pilot symbols are transmitted or the resulting prediction errors. In this paper, the channels are assumed to be not highly frequency-selective and then the second estimation error source that is prediction oriented, can be neglected. As a consequence, the entries of $\mathbf{\Delta}_k^{(m)}$ can be assumed to be i.i.d. complex Gaussian of zero-mean with a variance σ_e^2 that is quasi-constant and independent from the shadowing and the pathloss levels. The entries of $(\hat{\mathbf{H}}^e)_k^{(m)}$ can also be assumed to be i.i.d., zero mean and uncorrelated with those of $\mathbf{\Delta}_k^{(m)}$. Hence, the received signal at the k^{th} MS becomes:

$$\begin{aligned}\mathbf{y}_k &= \underbrace{\sum_{m=1}^{M}(\mathbf{H^e})_k^{(m)}(\mathbf{W^e})_k^{(m)}(\mathbf{P}_k^{(m)})^{\frac{1}{2}}\mathbf{x}_k}_{\text{useful signal}} \\ &+ \underbrace{\sum_{j=1,j\neq k}^{K}\sum_{m=1}^{M}(\mathbf{H^e})_k^{(m)}(\mathbf{W^e})_j^{(m)}(\mathbf{P}_j^{(m)})^{\frac{1}{2}}\mathbf{x}_j}_{\text{interference term}} \\ &+ \underbrace{\underbrace{\sum_{j=1}^{K}\sum_{m=1}^{M}\mathbf{\Delta}_k^{(m)}(\mathbf{W}^e)_j^{(m)}(\mathbf{P}_j^{(m)})^{\frac{1}{2}}\mathbf{x}_j}_{\mathbf{n}_k^e} + \mathbf{n}_k}_{\mathbf{b}_k:\text{ noise}}. \end{aligned} \quad (5)$$

As we can conclude, the useful signal in (5) has the same form as the one in the perfect CSI case (see the expression (1)). The only entity that changes compared to the perfect CSI case, is the noise term. For the same reasons described in Section 3.1, we should first cancel the interference term in (5). To do so, we conceive a beamforming matrix $(\mathbf{W^e})_j^{(m)}$ that satisfies the following zero-ICI condition:

$$(\mathbf{H}^e)_k^{(m)}(\mathbf{W}^e)_j^{(m)} = \mathbf{0}, \forall k \neq j. \quad (6)$$

4. COOPERATIVE ZF BEAMFORMING

In this section, we consider the case of perfect and imperfect knowledge of the CSI at the transmitter. We first propose to adequately modify the existing ZF beamforming of [5] and [6] to our context in order to mitigate the ICI effect in expression (1) and then derive the global system sum rate expression.

4.1. Downlink ZF technique in the perfect CSI case

In order to satisfy the condition given by (2), we first define an $MKN_r \times N_t$ global channel matrix as the following: $\hat{\mathbf{H}} = [(\mathbf{H}_1^{(1)})^T, \ldots, (\mathbf{H}_1^{(M)})^T, \ldots, (\mathbf{H}_K^{(1)})^T, \ldots, (\mathbf{H}_K^{(M)})^T]^T$ and $\hat{\mathbf{W}} = [\mathbf{W}_1^{(1)}, \ldots, \mathbf{W}_1^{(M)}, \ldots, \mathbf{W}_K^{(1)}, \ldots, \mathbf{W}_K^{(M)}]$ is the $N_t \times MKN_r$ global beamforming matrix. To verify the zero-ICI condition (3), one possible choice of $\hat{\mathbf{W}}$ is the pseudo-inverse of $\hat{\mathbf{H}}$ that can exist only if $MKN_r \leq N_t$:

$$\bar{\mathbf{H}} = \hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H)^{-1} \in \mathbb{C}^{N_t \times MKN_r}.$$

Then, each unit-norm column of the global beamforming matrix is defined as:

$$\hat{\mathbf{W}}_i = \frac{\bar{\mathbf{H}}_i}{\sqrt{(\hat{\mathbf{H}}\hat{\mathbf{H}}^H)_{i,i}}}, i = 1, \ldots, KMN_r. \quad (7)$$

Consequently, the equivalent received signal after selecting this beamforming matrix (7) is:

$$\mathbf{y}_k = \underbrace{[\xi_k^{(1)}, \ldots, \xi_k^{(M)}]}_{\bar{\mathbf{H}}_{eff,k} \in \mathbb{C}^{N_r \times MN_t}} \underbrace{\begin{pmatrix} (\mathbf{P}_k^{(1)})^{\frac{1}{2}}\mathbf{x}_k \\ \vdots \\ (\mathbf{P}_k^{(M)})^{\frac{1}{2}}\mathbf{x}_k \end{pmatrix}}_{\tilde{\mathbf{x}}_k \in \mathbb{C}^{MN_t \times N_r}} + \mathbf{n_k} \quad (8)$$

where the matrix $\xi_k^{(m)} = \mathbf{H}_k^{(m)}\mathbf{W}_k^{(m)}$ is by construction of the beamforming matrix, a diagonal one. Then the k^{th} MS rate [7] is represented by:

$$\mathcal{R}_{ZF,k} = log_2\, det\left(\mathbf{I}_{N_r} + \bar{\mathbf{H}}_{eff,k}\mathbf{Q}_k\bar{\mathbf{H}}_{eff,k}^H\mathbf{K}^{-1}\right), \quad (9)$$

where $\mathbf{K} = \sigma^2\mathbf{I}_{MN_r}$ denotes the background noise covariance and $\mathbf{Q}_k = \mathcal{E}(\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^H)$. As $\mathcal{E}(\mathbf{x}_k\mathbf{x}_k^H) = \mathbf{I}_{N_r}$, it results that:

$$\mathbf{Q}_k = \begin{pmatrix} (\mathbf{P}_k^{(1)})^{\frac{1}{2}} \\ \vdots \\ (\mathbf{P}_k^{(M)})^{\frac{1}{2}} \end{pmatrix} \left((\mathbf{P}_k^{(1)})^{\frac{1}{2}}, \ldots, (\mathbf{P}_k^{(M)})^{\frac{1}{2}}\right). \quad (10)$$

Let $\mathbf{D}_k = \bar{\mathbf{H}}_{eff,k}\mathbf{Q}_k\bar{\mathbf{H}}_{eff,k}^H$. Therefore,

$$\mathbf{D}_k = \left(\sum_{m=1}^{M}\xi_k^{(m)}(\mathbf{P}_k^{(m)})^{\frac{1}{2}}\right)\left(\sum_{l=1}^{M}\xi_k^{(l)}(\mathbf{P}_k^{(l)})^{\frac{1}{2}}\right)^H$$

where $\mathbf{D}_k$ is a diagonal matrix with diagonal elements that are equal to: $\mathbf{D}_k(i,i) = \| \sum_{m=1}^{M}\left(\xi_k^{(m)}(\mathbf{P}_k^{(m)})^{\frac{1}{2}}\right)(i,i)\|^2$. We deduce that the sum rate for the K MSs is:

$$\mathcal{R}_{ZF} = \sum_{k=1}^{K}\mathcal{R}_{ZF,k} = \sum_{k=1}^{K}\sum_{i=1}^{N_r} log_2\left(1 + \frac{\mathbf{D}_k(i,i)}{\sigma^2}\right). \quad (11)$$

Our main objective is to maximize the system sum rate under the power constraint given in (2). Taking into account the construction of the BD beamforming matrix, $\mathbf{W}_k^{(m)}$, it is easy to deduce that $(\mathbf{W}_k^{(m)})^H\mathbf{W}_k^{(m)} = \mathbf{I}_{N_r}$ and this implies that the power constraint (2) becomes the following:

$$\sum_{k=1}^{K} tr(\mathbf{P}_k^{(m)}) = \sum_{k=1}^{K}\sum_{i=1}^{N_r} p_{k,i}^{(m)} \leq P_T,\ m = 1, \ldots, M. \quad (12)$$

Let $C = \{p_{k,i}^{(m)} \in \mathbb{R}_+$ such that $\sum_{k=1}^{K}\sum_{i=1}^{N_r}\mathbf{p}_{k,i}^{(m)} \leq P_T,\ m = 1, \ldots, M\}$ be the set of admissible solutions. our main objective becomes to find the adequate power allocation solution that belongs to the set C and that maximizes the system sum rate: maximize 11 subject to 12.

Solving analytically this optimization problem, isn't straightforward. As the optimization constraint is linear, we propose to solve this optimization problem using a "projected gradient" based iterative approach.

4.2. Downlink ZF technique in the imperfect CSI case

In this section, we assume that the CSI is not perfectly known at the transmitter. We propose to consider a BD beamforming acheme in order to cancel the ICI under the same zero-ICI condition given by (6). The transmission scenario is identical to the one of Section 4.1: the power allocated from one BS to any MS is different from one BS to another.

Using the same previous reasoning in order to construct the beamforming matrix $(\mathbf{W^e})_k^{(m)}$, the same derivations of the CSI perfect case can be performed. In this case, the received signal at the k^{th} MS, after using the BD technique, can be expressed as follows:

$$\mathbf{y}_k = \sum_{m=1}^{M} (\xi^e)_k^{(m)} (\mathbf{P}_k^{(m)})^{\frac{1}{2}} \mathbf{x}_k \\ + \underbrace{\underbrace{\sum_{j=1}^{K}\sum_{m=1}^{M} \mathbf{\Delta}_j^{(m)} (\mathbf{W}^e)_k^{(m)} (\mathbf{P}_j^{(m)})^{\frac{1}{2}} \mathbf{x}_j}_{\mathbf{n}_k^e} + \mathbf{n}_k}_{\mathbf{b}_k\text{: noise}} \tag{13}$$

where $(\xi^e)_k^{(m)} = (\mathbf{H}^e)_k^{(m)} (\mathbf{W}^e)_k^{(m)}$ and superscript $.^e$ refers to the non-perfect CSI case as the used entities are the estimates instead of their true values. The matrix $\bar{\mathbf{H}}^e_{eff,k}$ has the same expression form as for $\bar{\mathbf{H}}_{eff,k}$ (see expression (8)). Let $\mathbf{b}_{k,i}$ be the i^{th} component of the vector $\mathbf{b}_k$. As the components of $\mathbf{x}_k$ and $\mathbf{n}_k$ are i.i.d. with zero-mean, the covariance matrix of the noise vector $\mathbf{b}_k$ is a diagonal one with diagonal terms defined as follows:

$$\mathcal{E}\{\mathbf{b}_{k,i}\mathbf{b}_{k,i}^*\} = \sigma^2 + \mathcal{E}\left\{\mathbf{e}_i \mathbf{n}_k^e (\mathbf{n}_k^e)^H \mathbf{e}_i^T\right\}$$

where the vector $\mathbf{e}_i = [0,\ldots, \underbrace{1}_{i^{th}\text{ component}}, 0,\ldots,0]$ represents the i^{th} vector of the canonical base. Furthermore, the entries of $\mathbf{\Delta}_k^{(m)}$ are i.i.d., zero-mean and of variance σ_e^2 and are assumed to be independent from those of $\mathbf{x}_k$, then $\mathcal{E}\{\mathbf{b}_{k,i}\mathbf{b}_{k,i}^*\} =$ $\sigma^2 + \sigma_e^2 tr\left(\sum_{m=1}^{M}\sum_{j=1}^{K}((\mathbf{W}^e)_j^{(m)})^H (\mathbf{W}^e)_j^{(m)} \mathbf{P}_j^{(m)}\right)$. For the same reasons explained in the Section 4.1, we obtain $((\mathbf{W}^e)_j^{(m)})^H (\mathbf{W}^e)_j^{(m)} = \mathbf{I}_{N_r}$. Consequently,

$$\mathcal{E}\{\mathbf{b}_{k,i}\mathbf{b}_{k,i}^*\} = \sigma^2 + \sigma_e^2 tr\left(\sum_{m=1}^{M}\sum_{j=1}^{K} \mathbf{P}_j^{(m)}\right). \tag{14}$$

This means that the covariance matrix of the vector noise $\mathbf{b}_k$ is equal to: $\sigma^2 + \sigma_e^2\, tr\left(\sum_{m=1}^{M}\sum_{j=1}^{K} \mathbf{P}_j^{(m)}\right) \mathbf{I}_r$. According to Section 4.1, the rate of the k^{th} MS is expressed as:

$$\mathcal{R}^e_{ZF,k} = log_2\, det\left(\mathbf{I}_{N_r} + \bar{\mathbf{H}}^e_{eff,k} \mathbf{Q}_k (\bar{\mathbf{H}}^e_{eff,k})^H (\mathbf{K}^e)^{-1}\right),$$

where $\bar{\mathbf{H}}^e_{eff,k} = [(\xi^e)_k^{(1)}, \ldots, (\xi^e)_k^{(M)}]$ and $\mathbf{K}^e$ is the noise covariance matrix: $\mathbf{K}^e = \left(\sigma^2 + \sigma_e^2\, tr\left(\sum_{m=1}^{M}\sum_{j=1}^{K} \mathbf{P}_j^{(m)}\right)\right) \mathbf{I}_{N_r}$. We deduce that the sum rate has the following expression:

$$\mathcal{R}^e_{ZF} = \sum_{k=1}^{K}\sum_{i=1}^{N_r} log_2\left(1 + \frac{\| \sum_{m=1}^{M}\left((\xi^e)_k^{(m)}(i,i)\sqrt{p_{k,i}^{(m)}}\right)\|^2}{\sigma^2 + \sigma_e^2 tr\left(\sum_{j=1}^{K}\sum_{m=1}^{M} p_{j,i}^{(m)}\right)}\right). \tag{15}$$

Our main objective keeps to optimize the system sum rate under the power constraint C: maximize (15) subject to (12).

Again, solving analytically this optimization problem, isn't straightforward. As the optimization constraint is linear, we propose a "projected gradient" based iterative algorithm to solve this maximization problem.

5. PROPOSED SYSTEM SUM-RATE MAXIMIZATION ALGORITHM

In this section, we describe our "projected gradient" based iterative algorithm to maximize the multicell cooperative BSs system sum rate in the case where the channel estimation error exist. The case of perfect channel estimation can be obtained by putting $\sigma_e^2 = 0$ in the obtained results. The optimal power allocation scheme is the solution of the following optimization problem:

$$\min_{\mathbf{p}}(-\mathcal{R}^e_{ZF}) \quad \text{such that } \mathbf{p} \in C, \tag{16}$$

where $\mathcal{R}^e_{ZF} : \mathbb{R}_+^{MKN_r} \to \mathbb{R}_+$ is continuously and differentiable function and the set C is closed convex and constitutes linear constraint. The convexity of C makes possible to use the orthogonal projection onto C, denoted by $P_C : \mathbb{R}_+^{KN_rM} \to C$ for obtaining directions which are also descent ones; namely a step is taken from $\mathbf{p}^k$ in the direction of $\bigtriangledown\mathcal{R}^e_{ZF}(\mathbf{p}^k)$ (where $\bigtriangledown$ denotes the gradient and $\mathbf{p}^k$ is the obtained power allocation vector at the k^{th} iteration). The resulting vector is projected onto C. However, the considered constraint presented by the set C is equivalent to: $\mathbf{g} = \mathbf{A}\mathbf{p} - P_T[1,\ldots 1]^T$ where $\mathbf{g}$ is the vector of active constraints and the columns of the matrix $\mathbf{A} \in \mathbb{R}_+^{M\times KN_rM}$ are their gradients. The projection matrix is $\mathbf{V}\mathbf{V}^H$ where $\mathbf{V}$ consists of the last $(KN_rM - M)$ rows of the $\mathbf{Q}$ factor in the QR factorization of $\mathbf{A}^T$. The principle of the proposed algorithm consists in the following main steps:

1. **Initialization** ($k = 0$): Take $\mathbf{p}^0 \in C$,
2. **Iterative step** (increment k):
   - if $\mathbf{p}^k$ is stationary, then stop.
   - Otherwise $\mathbf{p}^{k+1} = \mathbf{p}^k + t_k\mathbf{d}_k$ and consider $\mathbf{d}_k = \mathbf{V}\mathbf{V}^H \bigtriangledown\mathcal{R}^e_{ZF}(\mathbf{p}^k)$, where t_k is a positive stepsize that is defined adequately.

6. SIMULATION RESULTS

For the simulations setup, we consider a multicell cooperative BSs system composed of $M = 2$ BSs that are equipped with $N_t = 8$ transmit antennas and that are coordinating their transmitted signals to $K = 2$ MSs equipped with $N_r = 2$ receive antennas. The channels from the BSs to the different MSs are generated independently and following a Gaussian distribution and taking into account the combined effect of the pathloss and the shadowing so that the channel estimation error σ_e^2 belongs to the set of values 0.01, 0.05, 0.1, 0.5. In Fig.2, for each channel estimation error σ_e^2 value, we consider two situations of the considered multicell cooperative scheme: the first situation corresponds to a uniform power distribution to the MSs and the second one is a power allocation policy according to our "projected gradient" based iterative algorithm. We plot the curves of the obtained system sum-rate in (bits/s)/Hz versus the Signal to Noise Ratio (SNR). From the obtained performance curves, we can conclude that for both situations, the system sum-rate decreases as the channel estimation error increases. More importantly, the simulation results do highlight that the use of our proposed algorithm as power allocation policy, enhances the obtained multicell cooperative system sum-rate compared to the frequently considered situation where the power is uniformly distributed: $p_k^{(m)} = \frac{P_T}{MN_rK}$.

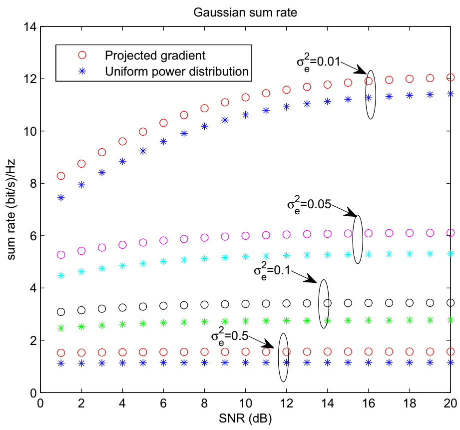

Fig. 2. Cooperative system sum-rate.

7. CONCLUSIONS

In this paper, we investigated an efficient power allocation policy that leads to significantly enhance the reachable downlink multicell cooperative system sum-rate. This policy is performed through the use of our proposed "projected gradient" based iterative algorithm that was derived in a general framework where the power is not allocated identically by each BS for the same MS and where the channel estimation errors due to the shadowing and the propagation pathloss are considered. The obtained simulation results did highlight the significant enhancement of the cooperative system reachable sum-rate when using our proposed algorithm. Future perspectives include the investigation of the extension of the obtained results in the context of downlink-uplink transmission.

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