INTER-BEAT (R-R) INTERVALS ANALYSIS USING A NEW TIME DELAY ESTIMATION TECHNIQUE

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ABSTRACT

In this paper, a new time delay estimation technique is applied for the R-R interval analysis. The key idea is about the R-R interval estimation of the ECG signal that is regarded as a time delay estimation in which we cast the change-point detection problem. This work aims to present an ad-hoc time delay technique that stems from operational calculus, differential algebra and non commutative algebra. So, we will be able to identify the change-point location and consequently the delay. Consequently, the R-R intervals estimation is deduced from a time delay estimation. The experimental results are provided and discussed to illustrate the efficiency of the proposed method under the real case. Tests are performed according to the recordings from MIT-BIH arrhythmia database. Moreover, the issues of the proposed algorithm robustness which is compared to literature are examined.

Index Terms— R wave detection, change-point, delay operator, operational calculus, detection rules.

1. INTRODUCTION

The electrocardiogram (ECG) is an electrical signal generated by the heart's muscular activity. A typical ECG beat mainly has different waves (P, Q, R, S and T). The time duration between consecutive R points is known by the R-R interval times series or inter-beat interval. After interpolating and resampling the Instantaneous Heart Rate (the reciprocal of each R-R interval in minutes), the resulting signal constitutes the Heart Rate Variability (HRV) [13]. This latter may be used as an indicator of some arrhythmias. For example, it is confirmed the HRV is a strong and an independent predictor of mortality following an acute myocardial infraction. Furthermore, HRV signals can be combined with information obtained from the electroencephalogram (EEG) to achieve a robust automatic newborn seizure detection during sleep.

The delays which are known as transport lags, dead times or latencies arise in signal processing where a time delay is also known as a time difference of arrival between two signals; like in a biomedicine. In this paper, we focus on the TDE in the ECG signals.

Because of its importance, the Time Delay Estimation (TDE) has attracted the attention of several researchers, we cite the Charles D. Woody's works [1], who proposed in 1967 an adaptive filter to estimate the latency of the neural signal by correlation. He calculated the cross correlation between each sweep and template. However, this method is suboptimal because the cross correlation is biased and all the estimated delays are taken into account in averaging process at the end of one iteration. Hence, A. Cabasson [2] proposed a new method that formalizes the Woody's one using an iterative Maximum Likelihood Estimator (MLE) to determine the delays which correspond to P-R interval. A common drawback of these methods is the use of the whole signal that contains the considered event which is the P wave in the case of Cabasson's studies. However, it is demonstrated that the methods using local criteria are more accurate [3].

To overcome such problem, we consider in this paper an approach that locally approximates the delay by using the principle of the sliding window and elementary model which is a smooth piecewise signal. Moreover, the estimated delay depends only on the observations on the sliding window. The originality of our idea relies on the fact that the estimation of the R wave occurrence is considered as a TDE problem. Indeed, to the best of our knowledge, this property has been rarely addressed to date. Also, an additional novelty consists in designing a new ad-hoc time delay estimator using a recent estimation technique of algebraic flavor (see [4] and the references therein). The algebraic technique we adopted allows parameters estimation, considering the delay as other unknown parameters.

The remainder of the paper is organized as follows: in section 2, we give an insight of the proposed approach and the proposed R wave detector. In section 3, we provide the method performances on real ECG signals. In section 4, some conclusions are drawn.

2. DESCRIPTION OF OUR APPROACH

The technique we used which is among the well-known time delay has been recently put in light and allowed to express the delay by a new philosophy. More precisely, we show that a discontinuity appears as an additive delayed signal. If we estimate the instant of discontinuities occurrence so it leads to the delay estimation. In addition, our approach is based on the use of the following notions introduced in [4] that we recall with some additions.

2.1. Time Delay Estimation

First, let's recall the heart period refers to the R-R interval which is defined by the following:

$$RR(i) = t_i - t_{i-1}$$

where t_i is the occurrence time of the i^{th} beat (or the R wave), $1 \leq i \leq N$ where N is the number of beats on one ECG recording. Either way, it's ultimately important to detect and localize the occurrence of the i^{th} R wave accurately and robustly. The variable estimation t_i, can be achieved by using the piecewise regular model

$$y(t) = \sum_{i \geq 0} H(t - t_{i-1}) f_i(t - t_{i-1}) + n(t) \tag{1}$$

Where $H(\bullet)$ is the Heaviside function (causality constraint), $n(t)$ is the observation's noise which blurring the discontinuities, $f_i(t)$ is assumed to be a polynomial. A change-point manifestation is translated by the passage from f_i to f_{i+1} at the instant t_i . We denote by $x(t) \triangleq y(t) - n(t)$ the non-observed signal. Consider an interval I_τ^T, with the origin τ and the length T: $I_\tau^T = (\tau - T, \tau)$, $\tau \geq T$. For $t \in I_0^T$, let set $x_\tau(t) \triangleq H(t)x(t + \tau)$. Next, choose T such that is at most one discontinuity point t_τ in I_τ^T. If t_τ is such an R wave location (a change-point) then T is chosen as the duration of an R wave occurrence. Now, redefine the discontinuity point t_τ relatively to I_τ^T:

- $t_\tau = 0$ if x_τ is smooth
- $0 < t_\tau \leq T$ otherwise

The representation of the signal in the sliding window can be expressed as

$$x_\tau(t) = a(t)H(t) + b(t)H(t - t_\tau) \tag{2}$$

where $a(t) = \sum_{i=0}^{p} a^i t^i$ and $b(t) = \sum_{i=0}^{q} b^i t^i$ are two polynomials of degree p and q respectively. We notify that the use of such a model is justified because it represents locally the signal during a small size window. The change-point estimation t_τ that corresponds to R wave occurrence time is regarded as a TDE. This section is devoted to the direct estimation of the delay.

We choose for the remainder of this section to represent the ECG signal by a simplest model of $x(t)$, viz. a piecewise constant model, i.e, $a(t) = a_0$ and $b(t) = b_0$ in equation (2). We assume a_0 and b_0 are unknown constant. Translating equation (2) to operational domain

$$X_\tau(s) = \frac{a_0}{s} + b_0 \frac{1}{s} e^{-t_\tau s} \tag{3}$$

where X_τ represents the Laplace transform of x_τ and s is its complex argument. The delay operator written with its classic exponential notation satisfies the differential equation $(\frac{d}{ds} + t_\tau)e^{-t_\tau s} = 0$, $(\frac{d}{ds} + t_\tau)$ is said to be the minimal annihilator of $e^{-t_\tau s}$ (see [4]). In that step we can proceed by two manners. First, we can carry out a simultaneous identification. We multiply on the left both sides by the minimal annihilator $II_1 = (\dfrac{d}{ds} + t_\tau).s$ this yields:

$$(sX_\tau(s) - a_0)t_\tau = -(1 + s\frac{d}{ds})X_\tau(s) \tag{4}$$

We end up with a linear system with the form $A\,\theta = b$ with $\theta = (t_\tau, -a_0\, t_\tau)$, where the entries of the 2×2 square matrix A , and the 2×1 vector b are differential operators of the form $\sum_{finite} s^\nu \frac{d}{ds^\alpha}$. More precisely, $A = \begin{bmatrix} s^{-1}Y_\tau(s) & s^{-2} \\ s^{-2}Y_\tau(s) & s^{-3} \end{bmatrix}$ and $b = -\begin{bmatrix} s^{-2}Y_\tau(s) + s^{-2}\dfrac{dY_\tau(s)}{ds} \\ s^{-3}Y_\tau(s) + s^{-3}\dfrac{dY_\tau(s)}{ds} \end{bmatrix}$ are obtained by using three orders of integration, i.e, we multiply both sides of equation (4) by $s^{-\nu}$, $\nu = 1, 2$ and we replace X_τ by its noisy version Y_τ. This operation allows also to avoid time derivative (positive power of s). Hence, the matrix A generated from this system is in general ill-conditioned and yields therefore poor estimates in a noisy setting [5]. Second, we do consider the individual estimation. The principle of this estimation method is simple: we minimize the number of parameters by subsequent elimination. More we reduce the number of parameters of the system, the more we reduce the estimation error on the parameters of interest, here t_τ. For more details see [5]. We multiply on the left both sides of equation (3) by s and we derive with respect to s to annihilate the a_0 parameter, this yields:

$$X_\tau(s) + s\frac{dX_\tau(s)}{ds} = -b_0 t_\tau e^{-t_\tau s} \tag{5}$$

Multiply both sides by the minimal annihilator of $e^{-t_\tau s}$ yields

$$\left(\frac{d}{ds} + t_\tau\right)\left(X_\tau(s) + s\frac{dX_\tau(s)}{ds}\right) = 0 \tag{6}$$

then we develop :

$$2\frac{dX_\tau(s)}{ds} + s\frac{d^2X_\tau(s)}{ds} + t_\tau\left(X_\tau(s) + s\frac{dX_\tau(s)}{ds}\right) = 0 \tag{7}$$

Estimation of the delay t_τ: the time derivative is avoided by multiplying both sides of equation (7) by $s^{-\nu}$, $\nu > o$. This operation allows to obtain iterated time integrals which is an operation more robust. For $\nu > 0$ and by replacing X_τ by its observation version Y_τ, the resulting equation is given by:

$$\begin{aligned} &2s^{-\nu}\frac{dY_\tau(s)}{ds} + s^{1-\nu}\frac{d^2Y_\tau(s)}{ds} \\ &+t_\tau\left(s^{-\nu}Y_\tau(s) + s^{1-\nu}\frac{dY_\tau(s)}{ds}\right) = 0 \end{aligned} \tag{8}$$

Finally, the above equation leads in the time domain to the linear estimator of t_τ (conversely to the above system linear on θ) denoted $\hat{t_\tau}$ ($\nu = 2$) which only depends on y_τ

$$\hat{t_\tau} = \frac{\displaystyle\int_0^T [2(T-\tau)\tau - \tau^2] y_\tau(\tau) d\tau}{\displaystyle\int_0^T (T - 2\tau) y(\tau) d\tau} \tag{9}$$

In the previous calculation, we used the following differential operator $II_2 = \frac{1}{s^2}.(\frac{d}{ds} + t_\tau).\frac{d}{ds}.s$ which permits to annihilate a_0 and b_0 which are unknown and to calculate t_τ by an integral. We notice that :

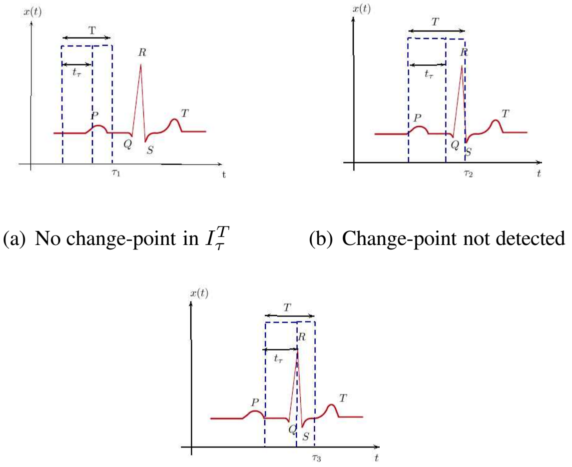
- Two different operators are used: $\frac{d^\alpha Y_\tau(s)}{ds^\alpha} \Leftrightarrow (-t)^\alpha y_\tau^\alpha(t)$ and $(k-1)! s^{-k} Y_\tau(s) \Leftrightarrow \displaystyle\int_0^T (t-\alpha)^{(k-1)} y_\tau(\alpha) d\alpha$ (Cauchy formula)
- The results above are obtained on the integral window $[0, T]$. But in practice, we have to extend this formula to work on a sliding window of length T and thus on the time integral window $[t - T, t]$. To implement the algorithm on a sliding window, we approximate the integral by a sum using the trapezoidal rule because of its lower complexity.
- $T > 0$ should be very small to guarantee fast estimation and the considered model approximation
- ν corresponds to number of iterated integrals which are simple examples of low pass filters. In noisy case, integration with $H(s) = \frac{1}{s}$ can be replaced by any strictly transfer function and particularly by low pass filters such as $T(s) = \frac{1}{\gamma s + 1}$.

By considering the implantation of the delay estimator as it is expressed in formula (9), it is clear that we can't handle an accurate value of the change-point and consequently the delay time. An issue of this problem is proposed and then adapted to our problem for R wave detection as it is shown in figure 1.

2.2. Proposed R wave detection procedure

After the implementation of the previous time delay estimator described in section 2.1, an R wave detection procedure is required to get back the R wave occurrences and consequently R-R interval series. The procedure is summarized by the following steps:

1. Detect peaks in the signal. Each time a peak is detected is it is classified as either as R wave or noise.
2. Ignore all peaks that precede or follow larger peaks by less than a waiting time equal to $Dist$ (*refractory period*).
3. If the peak is higher than the detection threshold we call it R wave, otherwise we call it noise.



(a) No change-point in I_τ^T (b) Change-point not detected

(c) Change-point detected

Fig. 1. Detector schema

4. If no R wave is detected within 1.5 R-to-R intervals so there is a peak higher than half the detection threshold, and the peak followed the preceding detection by at least $360ms$, we classify that peak as an R wave (*back search*).

For more details for rule one see [6]. The rules from step 3 to 4 and as outlined above are detailed in [7] and [8].
The refractory period used in step 2 means that an R wave is expected to appear after a waiting delay equal to $Dist$ following the latest detected R wave. It is updated further using a version of an empirical formula of the QT interval:

$$Dist = 0.4 * mRR \tag{10}$$

Here mRR represents a combination of averaged latest four interval RR_{mean} with the shortest one of them RR_{min}:

$$\frac{7RR_{mean} + RR_{min}}{8}$$

When a beat is located nearer than $Dist$ to the latest R wave, this candidate is ignored if its amplitude is smaller than the latest detected R wave.
The adaptive detection threshold used in 3 and 4 is calculated by using estimates of R wave peaks and noise peak heights. The detection threshold is set according to:

Detection_Threshold= Mean_Noise_Peak (11)
+TH*(Mean_Peak-Mean_Noise_Peak)

Where TH is the threshold coefficient.

3. EXPERIMENTAL RESULTS

Tests are carried out on a set of 48 records from the MIT-BIH Arrhythmia Database [9] via physionet website.The sampling rate is of 360 Hz. Every file contains a total of 60 minutes recording supported by another reference annotation file which indicates R beats marks.

For all the experiments, the detection performances are measured in terms of three benchmarks parameters: the Specificity (+P), the Sensitivity (Se) and the Detection Error Rate (DER) which are normally computed by:

$$Sp(\%) = 1 - \frac{FP}{TP+FP} = \frac{TP}{TP+FP}\%, \quad (12)$$

$$Se(\%) = 1 - \frac{FN}{TP+FN} = \frac{TP}{TP+FN}\%, \quad (13)$$

$$DER(\%) = \frac{FP+FN}{\text{Total number of QRS complex}}\%, \quad (14)$$

A false positive (FP) indicates the number of the false reported beats; whereas, a false negative (FN) is the number of a missed beats. A true positive (TP) stands for the number of a beats correctly detected. To be in agreement with literature, a "true positive" is defined as a maximum detected within 200ms of an annotated beat. Our method requires on

Record	N beats	FP	FN	Sp(%)	Se(%)	DER
104	2229	6	5	99.72	99.77	0.49
105	2572	27	36	98.78	98.38	2.79
108	1763	3	21	99.82	98.80	1.35
111	2124	0	0	100	100	0
203	2978	4	17	99.86	99.42	0.70
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Totals	**108476**	**144**	**338**	**99.86**	**99.68**	**0.47**

Table 1. Recapitulative results for the proposed time delay estimator in term of R wave detection for 48 MIT-BIH recordings.

the optimization of few parameters (T, TH). The size of the sliding window should be small to reduce the noise effect. Here, the choice of T depends on the noise level in the recording. We set $T = 25$ samples. This value is slowly increased in the case of the worst recordings such as recording 105 and 108.

To optimize the detection threshold as it's given by equation (11), we perform a set of 9 trials with TH ranging from 0.1 to 0.9. We interest in TH coefficient which minimizes the detection error rate of each record. The table 1 summarizes the detection scores of our algorithm for 48 half hour recordings. The results are $Sp = 99.86\%$, $Se = 99.68\%$ and $DER = 0.47\%$.

3.1. Performances in the presence of noise

Low frequency noise such as baseline shift and powerline interference is considered as an additive constant and is eliminated by the differential operator Π_2 which permits to annihilate the unknown constant a_0 and b_0 as mentioned in section

Comparaison criterion	FP	FN	DER(%)
Proposed algorithm	**144**	**338**	**0.47**
Wavelet transform [10]	65	112	0.15
Bandpass filtering I [7]	248	340	0.54
Bandpass filtering II [8]	507	277	0.68
Christov algorithm I [11]	215	294	0.46
Christov algorithm II [11]	239	240	0.43

Table 2. Comparison of performances for R wave detection using MIT-BIH arrhythmia database

2. In addition, high frequency noise with phase and bandwidth similar to that of the wanted signal may cause problems. One notable source of noise which affects R wave detection is muscle noise. However, our results do not show significant contribution of such noise. This property is originated from iterated time integrals involved in the calculation of the estimation delay (equation (9)) and which play the role of low pass filter attenuating the noise power. Records 104, 111 and 203 are annotated suffering from muscle noise. The table 1 shows that each of these records have Sp and Se scores greater than 98%.

3.2. Time complexity

Let $T(N)$ be the time required to execute N samples of data in order to estimate the time delay operator. Let B the time needed for a comparison, Z the time required for an assignment, A for an increment. The body loop requires $X + N * Y$ (initialization X + loop iteration $N * Y$)). Moreover, to compute an integral as it is expressed in formula (9) we use the trapezoidal rule with an integral window of a length $T = n * Te$ with $n = 40$ and Te is the sampling period. We prove that : $T(N) = N*(2n*(Z+Y)+2X+12Z+3B+A)+5Z$. By the definition of order it follows that $T(N) \in O(N)$. If we assume a computer can perform 1 million tasks per second (a reasonable speed for many machines) so our delay estimator requiring $T(N)$ finishes in a reasonable amount of time. This fact is deduced thanks to the polynomial form of the function time $T(N)$.

3.3. Comparison with other techniques

The table 2 shows the performances evaluation with four methods previously mentioned in literature for R wave detection: those by Li [10], Pan-Tompkins [8], Hamilton [7] and Chritov [11]. Therefore, our method is compared similarly to Christov's method. However, Li's method based on wavelet transform presents superior performances than those shown in table 2. Whilst, other R wave detection algorithms have reported best results (Christov algorithm II 0.43 % and Li 0.15%), they use preprocessing step (filtering) which has the disadvantage of introducing a time latency. Also, these later methods quoted in this paper use a lot of clinical informations of the ECG. Conversely, our method proves a satisfactory results without any preprocessing. In addition, our method

Comparison criterion	FP		FN		DER(%)	
	105	108	105	108	105	108
Proposed algorithm	**27**	**03**	**36**	**21**	**2.79**	**1.35**
Wavelet transform [10]	15	13	13	15	1.09	1.59
Bandpass filtering I [7]	53	50	22	47	2.95	5.67
Bandpass filtering II [8]	67	199	22	22	3.46	12.54
Christov algorithm I [11]	35	40	2	2	1.43	2.38
Christov algorithm II [11]	36	42	2	1	1.47	2.43

Table 3. Comparison of the performances for R wave detection for the most noisy records 105 and 108

uses a minimum of clinical property about the signal under analysis.

The algorithm is tested and compared to in poor instances against the two MIT-BIH records 105 and 108 which match the noisy case very well. The predominant feature of these records is a high grade of noise and artifacts. The comparison made between the performance of the proposed algorithm and literature are listed in the table 3. The reliability of the proposed detector is compared favorably with that of other published results especially concerning the most noisy record 105. In the case of record 108 our method gives the best results with $Sp = 99.82\%$ and $Se = 98.80\%$ and $DER = 1.35\%$.

Receiver Operator Characteristic (ROC) curves are used in the evaluation. We focus our attention on the signal record 118e06 from the MIT-noise Stress Database (see [6] for more details of the obtention of such record). The signal to the noise ratio, i.e $SNR = 10\lg_{10}\left(\frac{\sum |y(t_i|^2}{\sum |n(t_i)|^2}\right)$, corresponds to $SNR = 6dB$. ROC curves depicted in figure 2 plot the mean value of Se among all values of Se obtained by varying the threshold coefficient TH, versus the mean value of $(1 - Sp)$ obtained for different value of $(1 - Sp)$ related to different threshold TH. ROC curves compare the derivative method [6] against delay estimation technique results. By using this later method the area under curve (AUC) is closed to 1 than our previous work which is based on derivative method for R wave detection. This result shows that our actual method improves performances noticeably in the worst case.

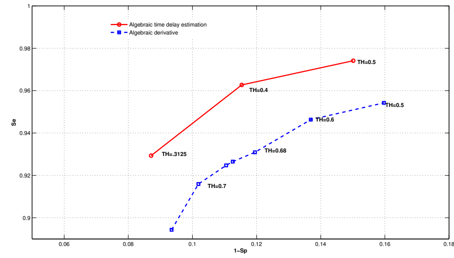


Fig. 2. ROC curves for the selected record 118e06 from the MIT-Noise-Stress Database (SNR=6dB).

4. CONCLUSION

This paper tackled the inter-beat intervals analysis with the new tool of estimation based on algebraic approach. Furthermore, the time delay estimation problem is substituted for the inter-beat interval analysis. The experimental results show that the statistical indices such as the sensitivity and the specificity are comparable to or higher than those cited in scientific literature. The mean issue to be explored concerns a model signal of a higher order. Hence, the robustness analysis with respect to the model order can result from this idea. In addition, the temporal relationship between the dynamic of the heart rate variability and the electroencephalographic activity during sleep, which was an object of a previous research [12], can be another possible direction.

5. REFERENCES

[1] C. D. Woody, "Characterization of an adaptative filter for the analysis of variable latency neuroelectric signals," *Med. & biol. Eng. Comp*, vol. 5, pp. 539–553, 1967.

[2] A. Cabasson and O. Meste, "Time delay estimation: A new insight into the woodys method.," *IEEE Signal Processing Letters*, vol. 15, pp. 573–576, 2008.

[3] Michel Fliess, Cédric Join, Mamadou Mboup, and Alexandre Sedoglavic, "Estimation des dérivées d'un signal multi-dimensionnel avec applications aux images et aux vidéos," in *GRETSI*, Louvain-la-Neuve Belgique, 2005, pp. 136–139.

[4] M. Fliess, C. Join, and M. Mboup, "Algebraic change-point detection," *Applicable Algebra in Engineering, Communication and Computing*, vol. 22, no. 2, pp. 131–143, 2010.

[5] M. Mboup, C. Join, and M. Fliess, "Numerical differentiation with annihilators in noisy environment," *Numerical Algorithms*, 2009.

[6] S. Rezk, C. Join, and S. Elasmi, "An algebraic dervative-based method for R wave detection," in *Proceeedings of EUSIPCO, 19*, 2011.

[7] P.S. Hamilton and W.J. Tompkins, "Quantitative investigation of QRS detection rules using the MIT-BIH arrhythmia database," *IEEE Trans. Biomed. Eng.*, vol. BME-33, pp. 1157–1165, 1986.

[8] J. Pan and W.J. Tompkins, "A real time QRS detection algorithm," *IEEE Trans. Biomed. Eng.*, vol. 32, pp. 230–236, 1985.

[9] Massachusetts Institute of Technology , MA, Cambrige, *MIT-BIH database distribution*, 1997.

[10] C. Li, C. Zheng, and C. Tai, "Detection of ECG characteristic points using wavelet transforms," *IEEE Trans. Biomed. Eng*, vol. 42, pp. 21–28, 1995.

[11] I. Christov, "Real time electrocardiogram qrs detection using combined adaptive threshold," *Biomed. Eng. Online*, vol. 3, no. 1, pp. p28, 2004.

[12] S. Rezk, C. Join, S. Elasmi, and M. Dogui, "Frequency change-point detection in physiological signals : an algebraic approach," *IJSTA*, vol. 2, no. 1, pp. 456–468, 2008.

[13] http://physionet.org/tutorials/hrv/.