DISTRIBUTED PUNCTURED LDPC CODING SCHEME USING NOVEL SHUFFLED DECODING FOR MIMO RELAY CHANNELS

Nanfan Qiu,Xiao Peng,Yichao Lu, Satoshi Goto

Graduate School of Information,Production and System, Waseda University,Fukuoka,Japan
Email:qiunanfan@moegi.waseda.jp;goto@waseda.jp

ABSTRACT

We propose a distributed punctured LDPC coding scheme to approach the theoretical limit for half-duplex MIMO relay channels. Distributed punctured LDPC coding scheme only requires the design of mother code which avoids complicated joint signal design in previous LDPC schemes [6],[7]. For punctured LDPC decoding, shuffled decoding algorithm proposed in [4] can provide good performance, however traditional BP decoding algorithm should also be used for calculating recoverability. Therefore we propose a low complexity algorithm to calculate the recoverability of punctured variable nodes which can reduce nearly 90% computational time compared with [4]. Furthermore, soft information relaying protocol is adopted to allow relay forwarding soft messages to destination. By this method, LLR values received at the destination will become more reliable. The simulation results verify that our proposed methods can achieve better performance comparing with previous reported methods in literature.

Index Terms— Punctured LDPC Codes, MIMO communication, soft information relaying, shuffled decoding.

1. INTRODUCTION

Low Density Parity Check (LDPC) codes were firstly invented in early 1960s by Gallagar [1] and recovered by Mackay [2] in 1999. Nowadays LDPC codes have been widely studied due to their powerful error-correcting capacity and low decoding complexity. Rate-compatible punctured LDPC codes are one kind of LDPC codes which can offer high flexibility in terms of code rate at a relatively small cost in complexity. In [3], layered BP decoding algorithm is proposed to accelerate the decoding convergence. However, this scheme cannot ensure updating the unpunctured variable nodes at the 1st iteration. [4] proposes a scheduling method of the shuffled BP decoding algorithm. This method requires only one iteration to recover whole punctured variable nodes and improve the reliability of unpunctured variable nodes at the 1st iteration. But this method uses traditional BP decoding algorithm to calculate the recoverability of punctured variable nodes, resulting in that two decoding methods should be used in this algorithm. In this paper, we propose a new method to calculate the recoverability of punctured variable nodes. The proposed method generates the recoverability by checking whether connected variable nodes are recovered or not. By avoiding complicated calculation, the complexity can be reduced. Furthermore, simulation results clarify that this method can reduce about 90% computational time and keep the same performance with shuffled BP decoding in [4].

Due to the potential of providing diversity gain and robustness against fading, multiple-input multiple-output (MIMO) technology is being seriously considered in future high data rate wireless communication systems. In particular, LDPC coded MIMO technology can provide high rate transmission. The application of LDPC codes to spectrally efficient MIMO signaling has been proposed in wireless local area network [5].

Relay networks can enhance the channel capacity compared to the direct link from the source to the destination. Distributed coding which performs joint signal design at the source and destination can further optimize the performance. [6] proposes distributed LDPC code designs for the half-duplex relay channel. The source transmission is decoded with the help of side information in the form of additional parity bits from the relay. In [7], Bilayer LDPC codes are proposed to approach the theoretically promised rate of the decode-and-forward (DF) relaying strategy by incorporating relay-generated parity bits in specially designed bilayer graphical code structures. Distributed LDPC coding scheme based on puncturing which requires much less design complexity is presented in [8]. This scheme is sensitive to the quality of the source-relay link. Motivated by these ideas, we employ the novel shuffled sum product decoding algorithm for punctured LDPC. Furthermore, we propose a new algorithm to calculate the recoverability of each punctured node with computational time reduction. We also employ soft information relaying protocol to re-encode additional parity check bits into soft bits instead of traditional hard decision. These soft parity check bits can convey more reliable messages to destination.

This work was supported by CREST Program and a grant of Program for Fostering Regional Innovation(Global Type 2nd Stage) implemented by MEXT, Japan.

The rest of this paper is organized as follows. The system model and general LDPC code distributing strategy is introduced in Section II. Two proposed algorithms are described in details in Section III. One is soft information relaying based relay protocol. The other is a fast algorithm to calculate recoverability for punctured LDPC shuffled decoding algorithm. Section IV shows that our proposed method can achieve a superior performance when compared to previous methods in literature. The paper is concluded in Section V.

2. SYSTEM DESCRIPTION

2.1. System Model

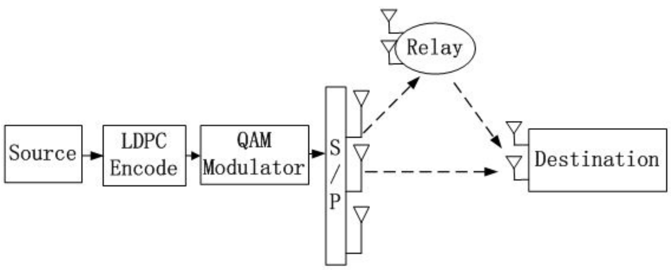

Fig. 1. System Model

Figure 1 shows the block diagram of LDPC coded MIMO communication system, where one source (S) sends independent messages to the destination (D), and in doing so it is aided by the relay (R). Two time slots are needed as the relay operates in a half-duplex mode. In the first time slot, S transmits messages to both R and D. While in the second time slot, R transmits additional parity check bits to D. We denote the source transmitted signal, relay received signal, relay transmitted signal and destination received signal by using X, V, W and Y, respectively. Then we can easily have the following channel model

$$V_1 = H_{SR}X_1 + N_{R1} \quad (1)$$
$$Y_1 = H_{SD}X_1 + N_{D1} \quad (2)$$
$$Y_2 = H_{RD}W_2 + N_{D2} \quad (3)$$

Where N_{R1}, N_{D1} and N_{D2} are Gaussian noises with zero mean and unit variance. We normalize the distance between S and D to unity, and d denotes the position of R relative to S with $0 \leq d \leq 1$. Here we assume that $d = 0.5$. In a MIMO system with n_T transmit antennas and n_R receive antennas, channel matrix H is an $n_R \times n_T$ complex matrix, in which the real and imaginary components of each element follow a Gaussian distribution with zero mean and variance $\frac{1}{2d^\alpha}$, where α is the path-loss exponent. In this paper, we choose $\alpha = 2$ in our simulation. We assume that perfect channel knowledge is known at the receiver, but unknown at the transmitter.

Here we use an iterative MIMO sphere detector to decode MIMO signals. By using the max-log approximation, the extrinsic LLR of the ith bit in x can be approximated as

$$L_E(x_i) \approx \frac{1}{2} \max_{x \in \mathcal{X}_{i,+1}} \{-\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{Hs}\|^2 + \mathbf{x}^T \cdot \mathbf{L}_A\}$$
$$- \frac{1}{2} \max_{x \in \mathcal{X}_{i,-1}} \{-\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{Hs}\|^2 + \mathbf{x}^T \cdot \mathbf{L}_A\} - L_A(x_i) \quad (4)$$

Where $\mathbf{x}$ is LDPC encoded bits and $\mathbf{s}$ is obtained by modulating $\mathbf{x}$ using QPSK. $\mathbf{L}_A$ is the vector of a priori LLR values. The basic idea of the sphere decoding algorithm is that rather than search over the entire lattice, one should search only lattice points in a hypersphere of radius round. Depth-first tree is used to find the points inside a sphere of radius and dimension. Fig.2 illustrates the diagram of iterative sphere detector.

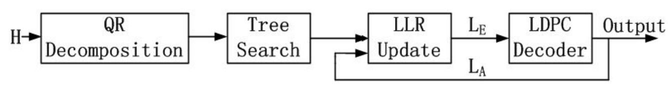

Fig. 2. Iterative Sphere Detector

2.2. LDPC Code Distributing Strategy

Here we will present the general LDPC code distributing strategy. The information vector $\mathbf{m}$ at the source is encoded using LDPC generator matrix $\mathbf{G}_1$ to yield the codewords $\mathbf{x}_s = \mathbf{m} \otimes \mathbf{G}_1$. LDPC coded bits are further modulated using QPSK modulation. The MIMO symbol vector mapper maps these symbols to multiple transmit antennas. Then signal X_1 is transmitted to the relay and the destination simultaneously in the first time slot. Here the rate of this codeword is

$$R_{SR1} = I(X_1; V_1) \quad (5)$$

To approach the maximum rate, we carefully design that R_{SR1} equals the capacity of the SR link. So the relay can decode X_1 reliably. However, the destination cannot decode X_1 since the capacity of the SD link is less than that of the SR link. So the destination stores received codewords for decoding at the end of the second time slot. We assume that total of N symbols are transmitted and durations of two time slots are respectively t and $t^{'}$. The destination receives $tNI(X_1; Y_1)$ bits of information in the first time slot. For reliable decoding, it stills needs additional $tN(I(X_1; V_1) - I(X_1; Y_1))$ bits. So in the second time slot,the relay transmits a codeword W_2 of rate

$$R_{RD} = \frac{t}{t^{'}}(I(X_1; V_1) - I(X_1; Y_1)) \quad (6)$$

The relay, after receiving the message V_1, performs MIMO sphere detection and LDPC decoding to recover codeword $\mathbf{x}_s$, additional parity check bits are generated by another parity check matrix H_2, where $x_r = x_s \otimes H_2^T$. These codewords which will aid the decoding are further modulated and transmitted to destination. At last destination performs LDPC decoding for the overall code using the extended check matrix H_{all}, where

$$H_{all} = \begin{bmatrix} H_1 & 0 \\ H_2 & I \end{bmatrix} \quad (7)$$

The check equation at the destination can be explained as

$$[x_s \; x_r] \otimes H_{all}^T = [x_s \; x_r] \otimes \begin{bmatrix} H_1^T & H_2^T \\ 0 & I \end{bmatrix} \quad (8)$$

$$= [x_s \otimes H_1^T \; x_s \otimes H_2^T \oplus x_r] \quad (9)$$

As H_1 is the parity check matrix of x_s, so the first term is zero. The second term should also be zero, thus additional parity bits transmitted by the relay should be chosen as $x_r = x_s \otimes H_2^T$.

3. PROPOSED METHODS

3.1. Soft Information Relaying Based Punctured LDPC Coding Scheme For Relay Channels

Two distributed LDPC coding schemes have been proposed for relay channels respectively in [6],[7]. Based on density evolution, these two schemes jointly design source and relay parity check matrices, at the cost of complicated derivation and calculation. Inspired by [8], we design distributed LDPC coding scheme using puncturing to approach the capacity in a lower complexity. Furthermore we employ soft information relaying protocol in relay to improve the performance.

Firstly we design a mother code H with rate R_c to suit the channel condition of overall relay channel. After LDPC encoding, the code rate is raised to R_{SR} due to puncture out a proportion of parity bits. In the relay, unpunctured variable nodes receive LLR$_{SR}$ from the MIMO sphere detector while punctured variable nodes receive zero LLR values. Soft output LLR after LDPC decoding in turn feed back to the MIMO sphere detector as a priori LLR. Then relay re-encodes additional parity check bits and sends them to the destination. Note that different parts of the received codeword suffer with different SNRs, where SNR$_{SR}$ for the source code and SNR$_{RD}$ for the additional parity bits. The accuracy of decoding in relay will have a large effect on the overall performance. In order to improve the reliability of additional parity check bits, we transmit soft bits instead of hard bits which means soft information relaying protocol. The critical problem is how to re-encode these additional parity check bits using soft bits. In the following we will explain it in details.

The first step is to compute soft bits using LLR values after punctured LDPC decoder.

$$\widehat{x}_s = \tanh(\frac{LLR}{2}) \quad (10)$$

The second step required for soft information relaying is generating additional parity check bits x_r from the soft bits. The soft bits in vector x_r are given by:

$$x_{r,j} = sgn_j \prod_{i \in S(H_{2,j})} |\widehat{x}_{s,i}| \quad \text{for } j = 1, \ldots, N_r \quad (11)$$

where $S(H_{2,j})$ is the set of message nodes incident to check node j in matrix H_2. In order to decide the sign of generated parity check bits x_r, hard decision output as x_s from the punctured LDPC decoder is needed. So the sign bits are given by

$$sgn_j = \begin{cases} 1 & \prod_{i \in S(H_{2,j})} |x_{s,i}| = 1 \\ -1 & \prod_{i \in S(H_{2,j})} |x_{s,i}| = 0 \end{cases} \quad (12)$$

Decode-and-forward protocol decodes the received signals using hard decision for eliminating noise effect. This protocol can achieve a good performance with the channel quality in the link between source and relay being high. However, if relay cannot decode the signals correctly, error propagation will occur and degrade the performance. Compared with decode-and-forward protocol, the proposed soft information relaying method uses soft bits instead of making hard decisions to provide additional information. By this proposal, the additional parity check bits can be more reliable especially when relay fails to decode the received signals, so the performance can be improved.

3.2. Novel Shuffled Decoding For Punctured LDPC Codes

In the standard BP decoding algorithm, the number of iteration required to recover whole punctured variable nodes is increasing with the number of punctured nodes. However, by using shuffled BP decoding method proposed in [4], only one iteration is needed to recover whole punctured nodes regardless of the puncturing algorithm and the number of punctured variable nodes. Furthermore, by assigning unpunctured nodes to the last scheduling group, this method can improve the reliability of unpunctured nodes at the first iteration. But this method employs traditional BP decoding algorithm to calculate the recoverability, resulting in that two decoding methods should be used. In this paper, we propose a new method to compute the recoverability, which can reduce about 90% computational time and keep the merits of shuffled BP decoding in [4].

Firstly we will introduce some notations and definitions. If a punctured variable node receives a nonzero LLR message from one of its neighboring check nodes, it is said to be recovered. In general, a punctured variable node is called k-step recoverable(k-SR) if it will be recovered after exactly k iterations. Let ε^i_{mn} and z^i_{mn} be the LLR which is sent from check node m to variable node n, and sent from variable node n to check node m, respectively. We denote the set of variable nodes that participate in check node m by $N(m) = \{n : H_{mn} = 1\}$, and the set of check nodes that participate in variable node n as $M(n) = \{m : H_{mn} = 1\}$. Then we can define k-SR node by the following equation:

$$SR(n) = k \text{ if } z^{k-1}_n = 0 \;\&\; z^{k-1}_{mn'} \neq 0, n' \in N(m)/n \quad (13)$$

Then we will calculate the recoverability of punctured variable nodes using the following proposed algorithm:

- Initialize: Set iteration number loop to 1,set variable node id i to 1.
- Step 1: For current variable node i, check whether it is recovered or not. if it is already recovered, go to step 4. Else go to step 2.
- Step 2: Find out all the check nodes connected to variable node i.Go to step 3.
- Step 3: For check node j, get the variable nodes connected to this check node. If all the variable nodes are recovered, we get the recoverability of variable node i is current iteration number loop, go to step 4. Else process next check node, do step 3 again. When all the check nodes are processed, go to step 4.
- Step 4: $i = i + 1$, if $i <= n$,go to step 1. Else $i = 1$, $loop = loop + 1$, if $loop > maxloop$, stop. Else, check whether all the variable nodes are recovered. If yes, Stop. Else, go to step 1.

After calculating the recoverability of each punctured variable node, we can minimize the number of shuffled sum product decoding iteration required to recover whole punctured variable nodes to one by a proper scheduling algorithm. The scheduling method is summarized as follows:

for variable node $n (1 \leq n \leq N)$,
if $SR(n) = i (\neq 0)$, assign to S_i
else if $SR(n) = 0$, assign to S_{L+1}

Here L represents the number of iteration required to recover whole punctured variable nodes. We assume that G represents the number of scheduling group which equals to $L+1$. N bits of a codeword are divided into scheduling group $S_g (1 \leq g \leq G)$. Then shuffled sum product algorithm is carried out as follows:

- Initialization: Set $i = 1$, the maximum number of iteration to I_{max}. For each m,n, set $z^0_{mn} = L_n$.
- For $1 \leq g \leq G$, process jointly the following two steps.

Horizontal step, for $n \in S_g$ and each $m \in M(n)$, process

$$r^i_{mn} = \sum_{n' \in N(m)/n, n' \in \bigcup_{k<g} S_k} \log\tanh(z^i_{mn'}/2) + \sum_{n' \in N(m)/n, n' \in \bigcup_{k\geq g} S_k} \log\tanh(z^{i-1}_{mn'}/2) \quad (14)$$

$$\varepsilon^i_{mn} = \log[(1 + r^i_{mn})/(1 - r^i_{mn})] \quad (15)$$

Vertical step, for $n \in S_g$ and each $m \in M(n)$, process

$$z^i_{mn} = L_n + \sum_{m' \in M(n)/m} \varepsilon^i_{mn} \quad (16)$$

$$z^i_n = L_n + \sum_{m' \in M(n)} \varepsilon^i_{mn} \quad (17)$$

- Hard decision and stopping criterion test.

Compared with [4], the proposed algorithm only checks whether the LLR values of variable nodes are zero or not, instead of complicated computation for message passing in LDPC decoding. So this method for calculating recoverability of each variable node can greatly reduce the computational time while ensuring the same performance.

4. SIMULATION RESULTS

In this section, we will present the simulation results of proposed shuffled LDPC decoding algorithm and distributed punctured LDPC scheme using soft information relaying for MIMO relay channels. Firstly we evaluate the performance of our proposed shuffled LDPC decoding algorithm using 1030×2048 LDPC code ($R = 0.497$). We raise the code rate to 0.75 by random puncturing. Table 1 shows the time comparison for calculating the recoverability of whole punctured variable nodes. We can see that our proposed algorithm can reduce about 90% computational time.

Table 1. comparison of time for calculating the recoverability

SNR(dB)	1	1.5	2	2.5	3
[3]	0.4911	0.4919	0.4944	0.4934	0.4951
Proposal	0.0481	0.0532	0.0495	0.0473	0.0489

Fig 3 shows the BER performance comparison of different decoding algorithms. As our proposal is a new algorithm to calculate recoverability for scheduling, shuffled decoding part keeps the same with [4]. So proposed algorithm has the same performance with [4] and outperforms traditional sum product decoding algorithm for punctured LDPC.

We consider the multiantenna system with $N_t = 4$ transmit antennas in the source and $N_r = 4$ in relay and destination. A 4 by 4 MIMO link is considered between relay and

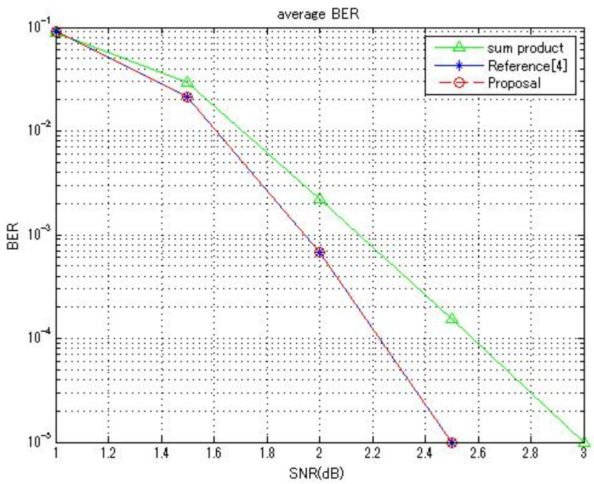

Fig. 3. BER of punctured LDPC decoding algorithm

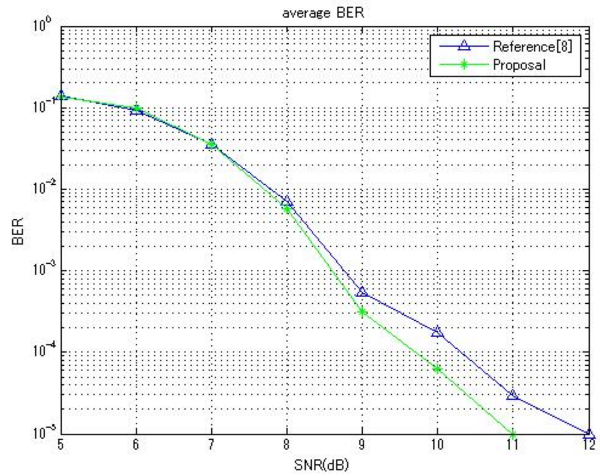

Fig. 4. BER of MIMO relay channels

destination. A 1008×1512 mother LDPC code is designed for the overall relay channel and the decoding iteration number is set to be 20. We assume that the relay locates in the middle between source and destination where $d = 0.5$. Fig 4 shows the BER performance of two distributed LDPC coding schemes. We can see that our proposed scheme can achieve a better performance than proposal in [8] by employing novel punctured LDPC decoding algorithm and soft information relaying. Furthermore, compared with distributed LDPC schemes in [6] and [7], only one LDPC code has to be optimized as a mother code. The same punctured LDPC decoder can be used at the relay and the destination. Therefore, lower design complexity can be achieved by using proposed punctured LDPC scheme.

5. CONCLUSION

In this paper, we mainly describe the distributed punctured LDPC scheme for MIMO relay channels. Firstly we introduce previous decoding algorithms for punctured LDPC code and different distributed LDPC coding schemes for relay channels. Then we describe the system model and sphere detection for MIMO signals in details. Furthermore, we propose two methods to improve the system performance. One proposal is employing soft information relaying protocol to increase the reliability of additional parity check bits. The other proposal is a fast algorithm to calculate the recoverability of punctured variable nodes. Clarified by the analysis and the simulation results, we demonstrate that the proposed distributed punctured LDPC coding scheme can achieve better performance in terms of BER and complexity as compared to previous reported methods in literature.

6. REFERENCES

[1] R. G. Gallager, "Low-Density Parity-Check Codes." Cambridge, MA: MIT Press, 1963.

[2] D. J. C. MacKay, "Good error-correcting codes based on very sparse matrices," IEEE Trans. Information Theory, vol. 45, no. 2, pp. 399-431, March 1999.

[3] J.Ha, D. Klinc, J. Kwon, and Steven W. McLaughlin, "Layered BP Decoding for Rate-Compatible Punctured LDPC Codes", IEEE Communication Letter, Vol. 11, pp. 440442, May. 2007

[4] Sangjoon Park, Sunyoung Lee , Keumchan Whang, "Shuffled BP decoding for punctured LDPC codes." 14th Asia-Pacific Conference on Communications, 2008.

[5] S. Chen, T. Zhang, and Y. Xin, "Relaxed K-best MIMO signal detector design and VLSI implementation," IEEE Transactions on very large scale integration VLSI systems, vol. 15, no. 3, pp. 328-337, 2007.

[6] A. Chakrabarti, A. de Baynast, A. Sabharwal, and B. Aazhang, "Low-Density Parity-Check Codes for the Relay Channel," IEEE Journals on Selected Areas in Communications, vol. 25, no. 2, pp. 280-291, Feb. 2007.

[7] P. Razaghi and W. Yu, "Bilayer Low-Density Parity-Check Codes for Decode-and-Forward in Relay Channels," IEEE Trans. on Inform. Theory, vol. 53, no. 10, pp. 3723-3739, Oct. 2007.

[8] M. Wu, P. Weitkemper, D. Wubben, and K. D. Kammeyer, "Comparison of distributed LDPC coding schemes for decode-and-forward relay channels," in Proceedings of the International ITG Workshop on Smart Antennas (WSA '10), pp. 127-134, February 2010.

[9] Marwan H. Azmi, Jun Li, Jinhong Yuan and Robert Malaney, "Soft Decode-and-Forward using LDPC coding in Half-Duplex Relay Channels," IEEE International Symposium on Information Theory (ISIT), 2011.

[10] Ma S, Yang Y, Sharif H, "Distributed MIMO technologies in cooperative wireless networks," IEEE Commun Mag 2011, 49(5):78-82.