A NOVEL SHOCK FILTER FOR IMAGE RESTORATION AND ENHANCEMENT

Romulus Terebes, Monica Borda*, Christian Germain**, Olivier Lavialle***

*Technical University of Cluj-Napoca, 28 Memorandumului Street, 400114, Cluj-Napoca, Romania
**Univ. Bordeaux, IMS Laboratory, UMR 5218, Crs. de la Liberation, 33405 Talence, France
Email:{romulus.terebes,monica.borda}@com.utcluj.ro,{christian.germain,olivier.lavialle}@ims-bordeaux.fr

ABSTRACT

The paper proposes a novel shock filter for image restoration and enhancement tasks. The method is put in terms of a system of partial differential equations that describes both the evolution of the processed image and of its smoothed second order derivative. The method employs selective smoothing terms acting on robust diffusion directions and its efficiency is proven in the experimental part of the paper on both real and synthetic images.

Index Terms—partial differential equations, image restoration, diffusion equations.

1. INTRODUCTION

Partial differential equations (PDE)-based filters are modeling an image restoration or enhancement process through a partial differential equation that regards a degraded image *I(x,y)* as the initial state of a diffusion process and relates the image's spatial derivatives with its time derivative. A classical method that devoted a lot of interest is the anisotropic diffusion equation which is essentially driven by a non-linear diffusivity function *g(·)* taking as argument the gradient vector norms of the evolving image *U(·,t)* [1].

Using the notation $U(x,y,0) = I(x,y)$, the original formulation of the anisotropic diffusion consists in:

$$\frac{\partial U}{\partial t} = div(g(|\nabla U|)\nabla U) . \tag{1}$$

The solution of the equation at some time instant (or observation scale) t is approximated on the numerical domain by an iterative filter which computes recursively solutions from fine to coarser scales.

A common formalism used in the literature to describe the action of a PDE-based filter in each pixel of the processed image is based on an orthonormal basis. Let $\boldsymbol{\eta} = \nabla \mathbf{U}/|\nabla \mathbf{U}|$ denote the vector collinear with the edge directions passing through a pixel and $\boldsymbol{\xi} \perp \boldsymbol{\eta}$ a vector oriented along the structure directions. Equation (1) can be put then in the following terms:

$$\frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} + c_\eta U_{\eta\eta} , \tag{2}$$

with $c_\xi = g(|\nabla U|)$ and $c_\eta = [|\nabla U| g(|\nabla U|)]'$, representing the diffusion coefficients along the two axes.

Equation (2) allows a better understanding of the filter's behavior. For constant diffusivity functions, (2) is equivalent to the classical heat equation, inducing low pass-filtering actions in each pixel of the processed image. However, for one of the choices indicated in [1]:

$$g(s) = [1 + (s/K)^2]^{-1} , \tag{3}$$

it can be shown that the equation has always positive diffusion coefficients along $\boldsymbol{\xi}$ and it can have negative (for $|\nabla U| > K$) or positive coefficients (for $|\nabla U| < K$) along the $\boldsymbol{\eta}$ diffusion axis.

The choice of the diffusion function that can lead to negative diffusion coefficients in the direction orthogonal to edges is deliberate in the original Perona-Malik model for allowing edge enhancement to take place. This deliberate inversion of a forward diffusion process that is smoothing out noise, is characteristic for another family of PDE based filters – shock filters – that was specially designed to deal with blur-like image degradations. In its simple form, such a filter can be put in terms of the following equation:

$$\frac{\partial U}{\partial t} = -sign(U_{\eta\eta}) \cdot |\nabla U| . \tag{4}$$

Equation (4) was proposed by Osher and Rudin [2] and it can be made stable only in the numerical domain by appropriate numerical schemes relying on slope limiter functions for limiting the solution near discontinuities.

Both filters given by equations (3) and (4) were generalized since their introduction by several authors. The filter (3) was made more robust with respect to noise in [3]; the modification is related to a simple Gaussian pre-smoothing of the input image, prior to the estimation of the diffusivity function:

$$g(|\nabla U|) \to g(|\nabla U_\sigma|) = g[\nabla(|G_\sigma * U|)]. \tag{5}$$

Using (5), noise amplification is avoided and edge enhancement can still take place for relatively small standard deviations (σ) of the Gaussian filter [3].

For shock filters major improvements were introduced in [4], [5] and [6]. The approach in [4] also uses a Gaussian pre-smoothing, making the filter more robust with respect to noise; the modified shock filter equation is:

$$\frac{\partial U}{\partial t} = -sign(G_\sigma * U)_{\eta\eta}|\nabla U| + c_\xi U_{\xi\xi}\,. \quad (6)$$

The second term in (6) is a directional diffusion term that induces a smoothing action along the structure's directions.

The method proposed in [5] employs robust diffusion directions estimated via a structure tensor based approach. The corresponding equation:

$$\frac{\partial U}{\partial t} = -sign(G_\sigma * U)_{vv}|\nabla U| \quad (7)$$

induces shock filtering along the direction of the eigenvector $\boldsymbol{v}$ that corresponds to the largest eigenvalue of the structure tensor computed at a semi-local, integration scale ρ [5]:

$$J_\rho(\nabla U_\sigma) = \begin{pmatrix} G_\rho * (\frac{\partial U_\sigma}{\partial x})^2 & G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} \\ G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} & G_\rho * (\frac{\partial U_\sigma}{\partial y})^2 \end{pmatrix}. \quad (8)$$

The approach in [6] was proposed directly in terms of a directional interpretation:

$$\begin{aligned}\frac{\partial U}{\partial t} &= \alpha_f(U - U_0) + \alpha_r[h_\tau(|G_\sigma * \nabla U|)U_{\eta\eta} + U_{\xi\xi}] \\ &\quad - \alpha_e[1 - h_\tau(|G_\sigma * \nabla U|)]sign(G_\sigma * I)_{\eta\eta}|\nabla U|\end{aligned} \quad (9)$$

and it governs the image restoration process by adaptively inducing directional or isotropic smoothing and edge enhancement processes. This is accomplished by using the fuzzy edge detector function *h(.)*. For low gradients the filter has an isotropic smoothing action ($h_\tau(|G_\sigma * \nabla U|) \cong 1$) whereas, for high smoothed gradient norms ($h_\tau(|G_\sigma * \nabla U|) \cong 0$), the equation can be put in the following terms:

$$\frac{\partial U}{\partial t} = \alpha_f(U - U_0) + \alpha_r U_{\xi\xi} - sign(G_\sigma * I)_{\eta\eta}|\nabla U|\,, \quad (10)$$

inducing unidirectional smoothing along the structure directions and edge enhancement on the orthogonal axis. We refer to the original publication [6] for a description of the other parameters.

All the methods presented above are scalar equations operating on the values of the luminance function that describes the image content. Gilboa et al. proposed in [7] a different solution for image restoration problems. The method combines the PDE formalism with the framework of complex functions of real variables for handling additive Gaussian noise and Gaussian blur image degradation scenarios. The associated PDE is:

$$U_t = \frac{\partial U}{\partial t} = -\frac{2}{\pi}\arctan(a\frac{U_I}{\theta})|\nabla U| + \lambda U_{\eta\eta} + \tilde{\lambda} U_{\xi\xi}\,, \quad (11)$$

with :

$$\lambda = re^{j\theta} = \lambda_R + j\lambda_I \quad (12)$$

a complex variable. The evolving image is defined as a complex function :

$$U(x,y) = U_R(x,y) + jU_I(x,y) \quad (13)$$

and a real scalar $\tilde{\lambda}$ is used by the authors to induce smoothing along the structure directions. The second order derivative along the gradient vector direction is approached in (11) by dividing the imaginary part of the evolving image with the argument θ of the complex number acting as parameter of the method; the authors prove that this approximation holds for sufficiently small values for θ. The use of the inverse arctangent function favors faster sharpening for larger second order derivatives values and the real part of the solution defines the restored image [7].
As the authors show, the method eliminates the need of performing Gaussian pre-smoothing as in (6), (7) or (9) and is more efficient in handling Gaussian blur and Gaussian noise degradation scenarios.

The complex diffusion term was used also in [8] in a hybrid scalar/complex formulation, that insures relative independence with respect to the stopping time (*t*).

2. PROPOSED METHOD

2.1 Continuous model

We derive our method from a theoretical analysis of the PDEs describing the evolution of the real and imaginary parts of an image processed with a complex shock filter. Equation (11) develops to:

$$\begin{cases} U_{R_t} = -\frac{2}{\pi}\arctan(a\frac{U_I}{\theta})|\nabla U| + \lambda_R U_{R_{\eta\eta}} - \lambda_I U_{I_{\eta\eta}} + \tilde{\lambda} U_{R_{\xi\xi}} \\ U_{I_t} = \lambda_I U_{R_{\eta\eta}} + \lambda_R U_{I_{\eta\eta}} + \tilde{\lambda} U_{I_{\xi\xi}} \end{cases} \quad (14)$$

with $U_{R(I)_t}$ denoting, respectively, the time derivatives of the real and imaginary parts.

The last term in (11) is essential for handling noise–like degradations and, by already classical results in PDE-based image processing, the term can be expressed as follows:

$$U_{\xi\xi} = |\nabla U| div(\frac{\nabla U}{|\nabla U|}) = k|\nabla U| \quad (15)$$

i.e. it represents a mean-curvature motion term that propagates isophotes in inner normal direction with a curvature (*k*) dependent speed. The properties of a PDE governed by such a term have been extensively studied; for example in [9] and the references therein, the authors show that, despite being efficient for denoising purposes, the usage of a mean-curvature denoising term leads to results in which each non-convex object will evolve into a convex one that will eventually disappear in finite time. In the case of complex shock filters, as shown in (14), the term affects both the real and the imaginary terms and it can lead to geometrical distortions.

The terms involving the second order derivatives along the gradient directions $U_{R_{\eta\eta}}$ and $U_{I_{\eta\eta}}$, both factored in (14) by a positive weight (λ_R), are also smoothing terms acting non-selectively in each pixel of the processed image. For high λ_R values the usage of such terms can lead to parasite blurring and displacement of edges. On another hand, too low values for the parameter will induce actions on which region-like areas will be less efficiently filtered.

The method we are proposing addresses both the discussed issues and it is developed to act on robust diffusion directions computed using the structure tensor approach:

$$\begin{cases} \mathbf{u} = [\cos(\theta_s), \sin(\theta_s)]^T \\ \mathbf{v} = [-\sin(\theta_s), \cos(\theta_s)]^T \end{cases}, \quad (16)$$

with ***u*** representing the eigenvector corresponding to the smallest eigenvalue of the structure tensor (8).

We model the image restoration process using two functions *U(x,y,t)*- the restored image at scale *t* – and *V(x,y,t)*- a smoothed version of the second order derivative of the restored image, taken along the direction orthogonal to structures ***v***. The two variables are linked through the system of PDEs:

$$\begin{cases} U_t = -\frac{2}{\pi}\arctan(aV)|\nabla U| + \tilde{\lambda}\cdot\{\frac{\partial}{\partial \mathbf{v}}[g(U_v)(U_v)] \\ \qquad + \frac{\partial}{\partial \mathbf{u}}[g(U_u)(U_u)]\} \\ V_t = U_{vv} + \tilde{\lambda}\cdot\{\frac{\partial}{\partial \mathbf{v}}[g(V_v)(V_v) + \frac{\partial}{\partial \mathbf{u}}[g(V_u)(V_u)]\} \end{cases} \quad (17)$$

with:

$$\begin{cases} U_{\mathbf{u}} = \frac{\partial U}{\partial \mathbf{u}} \\ U_{\mathbf{v}} = \frac{\partial U}{\partial \mathbf{v}} \end{cases} \quad (18)$$

and

$$\begin{cases} V_{\mathbf{u}} = \frac{\partial V}{\partial \mathbf{u}} \\ V_{\mathbf{v}} = \frac{\partial V}{\partial \mathbf{v}} \end{cases} \quad (19)$$

denoting the directional derivatives taken along the eigenvectors of the moving orthonormal basis (***u,v***). For modulating the intensity of the selective smoothing/enhancement processes taking place on both directions, we employ Perona-Malik like functions as in [10-11].

The initial condition is identical to the one used in the original formulation of the complex shock filter:

$$\begin{cases} U(x,y,0) = I(x,y) \\ V(x,y,0) = 0 \end{cases}. \quad (20)$$

The first equation in (17) relies on a smoothed approximation of the second order derivative given by the second equation to perform restoration of the input image. From this point of view, our method and the complex shock filter are similar but, in contrast to the formulation of the shock filter, on the direction orthogonal to edges (***v***), the function modeling the evolution of the input image *U* is enhanced only for sharp transitions (i.e. edges) and is smoothed otherwise. This also holds for the function modeling the evolution of the second order derivative *V*.

As pointed out in [10-11] directional derivatives along the structure directions (***u***) can be regarded as confidence factors in the estimated orientation. High values correspond to corners and junctions and these values are expected to fall above the diffusion threshold. The use of Perona-Malik functions limits the mean-curvature motion effects by an inversion of the forward diffusion process. On oriented patterns and on the background of the processed image the directional derivatives have small values and the method induces smoothing for both functions.

2.2. Numerical aspects

We derive the approximation for the continuous equation for its real and imaginary parts. For the time derivative, we use a forward time explicit numerical scheme with a time step of dt=0.1, insuring that the values of *U* and *V* at the scale *t+1* are computed from the known values at scale *t*.

For approximating the gradient value we use the classical *minmod* slope limiter function and forward/backward difference approximations [9]:

$$|\nabla U| \cong \sqrt{m^2[D_x^+(U), D_x^-(U)] + m^2[D_y^+(U), D_y^-(U)]} \quad (21)$$

In order to approximate directional derivatives along the **u** and **v** diffusion axis we employ:

$$\begin{cases} \frac{\partial}{\partial \mathbf{u}}[g(U_u)(U_u)] = g[D_u^+(U)]D_u^+(U) - g[D_u^-(U)]D_u^-(U) \\ \frac{\partial}{\partial \mathbf{v}}[g(U_v)(U_v)] = g[D_v^+(U)]D_v^+(U) - g[D_v^-(U)]D_v^-(U) \end{cases} \quad (22)$$

and similar relations for the *V* function.

We handle the needed subpixel resolution through biquadratic interpolations as indicated in a previous work [11] and we compute the second order derivative involved in the second equation using the Hessian matrix (**H**) [12]:

$$U_{\mathbf{vv}} = trace(\mathbf{Hvv}^T) \quad (23)$$

2.3. Parameters

Our method takes the following parameters:

- ρ - the standard deviation of the structure tensor's Gaussian kernel. In all the experiments we relate this parameter to the size of the support window ($4\rho+1$);
- the Perona –Malik diffusion thresholds *K*. We set these thresholds to be time dependent, being equal to a predefined percentage of the integral value associated to the histograms of the directional derivatives taken in the ***u*** and ***v*** directions;
- a - controling the steepness of the soft sign function;
- $\tilde{\lambda}$ - weights the contribution of the selective smoothing components

3. EXPERIMENTAL RESULTS

This section focuses on presenting a comparative analysis between the proposed method and the PDEs discussed in the introductory section.

In a first experimental setup we consider a simple synthetic image composed of geometrical features that need to be preserved (Fig.1.a). We simulate a mixed degradation scenario by first convolving the image with a Gaussian kernel (σ=1.0) and then by adding Gaussian noise (σ_B=10).

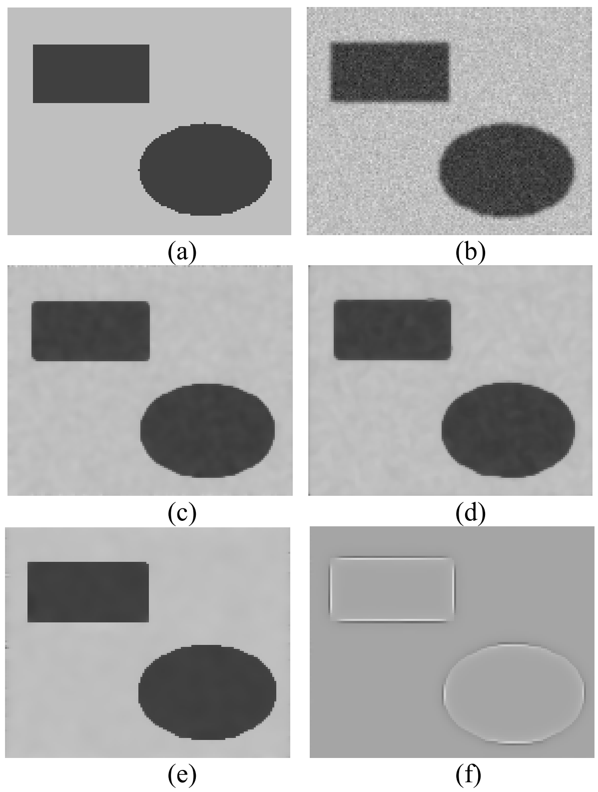

Fig.1. Restoration of a synthetic image degraded by Gaussian blur and noise; a) original image (150x120 pixels); b) degraded image; c) optimal result using (9); d) optimal result using (11); e) restored image using our method (U); f) smoothed second order derivative approximation (V).

For each method we allowed the parameters to vary in search of an optimal result corresponding to the best mean structural similarity index measure (SSIM); the obtained mean SSIMs and the associated PSNR values are shown in Table 1.

Table 1 – Quantitative measures corresponding to the results shown in Fig 1.

Result	SSIM	PSNR [dB]
Degraded image	0.427	25.50
Kornprobst et al. (Fig. 1.c)	0.963	34.15
Complex shock filter (Fig. 1.d)	0.958	33.90
Proposed method (Fig. 1.e)	0.990	36.57

The two existing filters specially designed to handle blur and noise – the Kornprobst et al. and the complex shock filter – are producing close results both in terms of SSIM and PSNR values (Fig.2.c, Fig.2.d). The methods both use mean curvature terms and the associated PDEs had to be stopped rather early (12 iterations for the Kornprobst et al. method and 20 iterations for the complex shock filter). Our method outperforms both considered shock filters; the gains of more than 2dB in PSNR and 0.03 in terms of SSIM value are easily observable on the results in Fig. 1. Our result was obtained after 40 iterations, with Perona-Malik diffusion thresholds set to 95% (***u*** axis) and 35% (***v*** axis) of the integral values of directional derivatives, $\tilde{\lambda}$=1.0 and a=0.2.

In the second experiment we show results (Fig.2) obtained on a real image degraded by blur (σ=1.0) and Gaussian noise (σ_B=10).

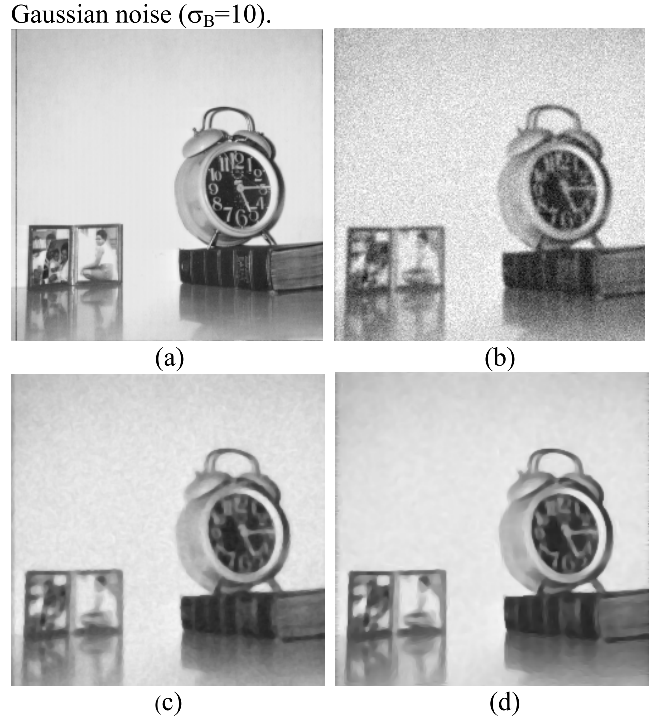

Fig. 2. Restoration of a real image degraded by Gaussian blur and noise; a) original image (256x256 pixels); b) degraded image (PSNR=23.39dB, SSIM=0.379; c) optimal result using eq. (11) (PSNR=28.08dB, SSIM=0.830); d) restored image using our method (PSNR=28.98dB, SSIM=0.883).

The same effects as in the case of the synthetic image are observable; the observation scale for the complex shock filter has to be kept fine in order to limit geometrical distortions. This corresponds to the divergent behavior of the complex shock filters as it was addressed in [8]. Our method limits geometrical distortions and induces efficient smoothing of region like areas.

The proposed method can be applied also to image enhancement tasks and we show in Fig. 3 comparative results obtained in enhancing linear, flow-like structures.

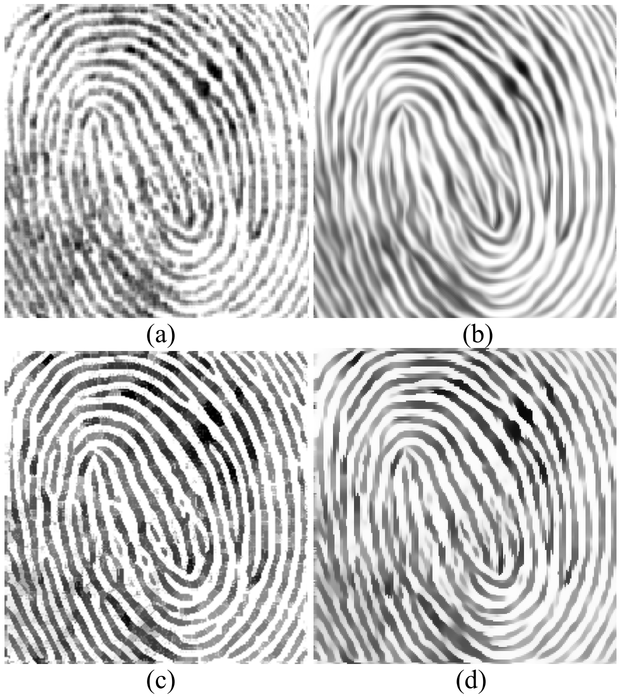

Fig. 3. Enhancement of a fingerprint image; a) original image (226x230 pixels); b) coherence enhancing filter result c) coherence enhancing shock filter result; d) image enhanced using our method.

The original image (Fig. 3.a) was smoothed with the coherence enhancing diffusion filter [9] (Fig. 3.b), processed with the coherence enhancing shock filter (eq. (7)- Fig. 3.c) and with our method (Fig. 3.d).

All the results were produced using a structure tensor with ρ=3 at an observation scale corresponding to 50 iterations of explicit time approximation schemes. The coherence enhancing filter propagates information along the isophotes of the image and successfully enhances the continuity of the gray levels along these directions; the result, however, lacks contrast. This not the case for the coherence enhancing shock filter; information propagation takes place using a series of morphological dilations and erosions and the output image is almost binary. Our method increases the coherence along the isophotes, closes small gaps and adds simultaneously a shock filtering action that increases the contrast of the output image.

4. CONCLUSIONS AND FUTURE WORK

We propose a novel shock filter for image restoration and enhancement tasks. The method is put in terms of a system of partial differential equations describing the evolution of the processed image and of its smoothed second order derivative along the direction orthogonal to the structures composing the image. The results presented in the experimental section show that the method compares favorably with other existing approaches, the price paid being an increased computational complexity.

Future work will be devoted for extending the model to handle the 3D and color image cases.

5. REFERENCES

[1] P. Perona and J. Malik, "Scale space and edge detection using anisotropic diffusion", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol.12, no.7, pp.629-639, 1990.

[2] S. Osher and L. Rudin, "Feature-oriented image enhancement with shock filters", *SIAM J. Numerical Analysis*, vol. *27(*4), pp. 919-940, 1990.

[3] F. Catte, P.L. Lions, J.M. Morel and T. Coll, "*Image selective smoothing and edge detection by nonlinear diffusion"*, *SIAM J. Numer. Anal*., vol.29, *1*, pp. 182-193,1992

[4] L. Alvarez and L. Mazorra, "Signal and image restoration using shock filters and anisotropic diffusion", *SIAM Journal on Numerical Analysis*, vol 31(2), pp, 590-605, 1994.

[5] J.Weickert, "Coherence enhancing shock filters", *Lecture notes on computer science*, vol. 2781, pp.1-8, 2003.

[6] P. Kornprobst, R. Deriche and G.Aubert, "Image coupling, restoration and enhancement via PDE's", *In Proc. IEEE International Conference in Image Processing- ICIP'97,* Santa–Barbara, CA, pp. 458-461, 1997.

[7] G. Gilboa, N.Sochen and Y.Y. Zeevi "Image enhancement and denoising by complex diffusion processes", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol.26 (8), pp.1020-1036, 2004.

[8] C. Ludusan, O. Lavialle, R. Terebes and M. Borda, "Morphological Sharpening and Denoising Using a Novel Shock Filter Model", *In Proc. 4th International Conference on Image and Signal Processing, ICISP 2010*, pp.19-27, Trois-Rivieres, QC, Canada, 2010.

[9] G. Aubert and P. Kornprobst, *Mathematical problems in image processing: partial differential equations and the calculus of variations (Second edition)*, volume 147 of Applied Mathematical Sciences. Springer-Verlag, 2006.

[10] R. Terebes, M. Borda, S. Pop, O. Lavialle , C. Germain and C. Ludusan, "A novel diffusion filter for image restoration and enhancement", *Revue Roumaine des Sciences Techniques- Serie électrotechnique et énergétique*, vol. 55, pag 310-319, 2010.

[11] R. Terebes, M. Borda, Y. Baozong, O. Lavialle, P. Baylou – "A new PDE based approach for image restoration and enhancement using robust diffusion directions and directional derivatives based diffusivities", *In Proc. International Conference on Signal Processing, ICSP'04*, vol. 1, pp.707-712, Beijing, China, 2004.

[12] D Tschumperle and R. Deriche, "Vector-Valued Image Regularization with PDE's : A Common Framework for Different Applications", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 27, No. 4, pp. 506--517, 2005.