AN EXTENDED INTERIOR-POINT METHOD FOR TRANSMIT BEAMFORMING IN MULTI-GROUP MULTICASTING

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ABSTRACT

We address the problem of transmit beamforming to multiple cochannel multicast groups where the total transmitted power is minimized subject to quality-of-service (QoS) constraints at the receivers. The iterative second-order cone programming (SOCP) approach recently proposed in [2] is a powerful tool to solve this problem and has certain advantages over other state-of-the-art techniques like the semidefinite relaxation (SDR) approach. However, we show in this paper that its computational complexity is unnecessarily high. Therefore, we propose to eliminate the superfluous part of its computational effort by combining the concept of the iterative SOCP method with that of interior-point methods. Computer simulations show that by this means, we can substantially reduce the computational complexity of the iterative SOCP method while preserving its good performance.

Index Terms— Multicasting, downlink beamforming, convex optimization, second-order cone programming, interior-point methods

1. INTRODUCTION

Multicasting, e.g., for streaming media, has traditionally been considered as a task performed at the network layer by routing protocols. In emerging wireless networks, however, this task can be performed at the physical layer by exploiting the broadcasting property of the wireless medium. Many wireless communication standards, such as, e.g., Multimedia Broadcast Multicast Service (MBMS) in LTE-A, provision the use of multiple antennas and channel state information (CSI) at the transmitter enabling multicast transmit beamforming. In transmit beamforming, differently weighted replicas of the signals are transmitted over multiple antennas. The aim is to radiate the power in a spatially selective way, so that the main portion of the power is radiated into the desired directions. Exploiting this spatial selectivity, one can increase the spectral efficiency of the system by multiplexing different cochannel multicast groups in space rather than in time or frequency.

The problem of downlink beamforming in a multi-group multicasting (MGM) scenario, where G independent data streams are transmitted towards G cochannel multicast groups of multiple mobile users, was first considered in [1]. The aim of the proposed method in the latter reference is to obtain the complex beamforming weight vectors which minimize the total transmitted power subject to receiver QoS constraints. The problem obtained by this QoS-based design is a non-convex NP-hard optimization problem.

Using SDR, the original non-convex problem is approximated in [1] by a semidefinite programming (SDP) problem which can be solved in polynomial time. However, due to the relaxation involved, the solution to the SDP problem may be infeasible for the original problem. Therefore, the SDR approach needs to be accompanied by randomization (see, e.g., [1] and references therein) and proper power scaling. With help of the latter two methods, a feasible approximate solution can often be generated from the solution to the SDP problem. In the MGM scenario, proper power scaling can only be achieved by solving an additional linear programming (LP) problem for each randomization instance.

In [2] and [4], an alternative convex approximation approach has been proposed for the MGM scenario. Rather than applying the SDR, the original problem is approximated by a SOCP problem whose feasible set is a subset of the original feasible set. Hence, the solution to the SOCP problem, provided it exists, is always feasible for the original problem and the use of randomization and costly power control is avoided. Since the approximated feasible set is only a subset of the original feasible set, an iterative method has been proposed in [2] and [4] where the approximated feasible set is updated in every iteration in order to successively improve the approximate solution.

In each iteration of the algorithm in [2], the solution to a SOCP problem is obtained with high accuracy using interior-point methods [3]. However, since this solution is updated in the next iteration anyway, a high accuracy of these intermediate solutions is not required. In this paper, we therefore propose to obtain intermediate solutions which are only suboptimal to the SOCP problems to reduce the computational complexity of the method in [2]. These suboptimal intermediate solutions can be obtained by performing only one iteration of the interior-point method used to solve the SOCP problem. In other words, we solve only an approximation of the SOCP problem rather than the SOCP problem itself which requires less computational effort. When the updates

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of the feasible set become negligibly small, we successively increase the approximation accuracy similar to the traditional interior-point methods to compute the final optimal weight vector with sufficiently high accuracy. We demonstrate our idea using the barrier method as a simple and illustrative example of an interior-point method. However, our idea can be applied to other interior-point methods as well, e.g., to more efficient primal-dual interior-point methods [3]. Our simulation results show that the proposed method achieves the same performance as the method in [2] at a substantially reduced computational complexity. Hence, there is no price we have to pay for this complexity reduction.

Remark: In this paper, we restricted our study to MGM transmit beamforming for brevity. However, all the presented concepts can straightforwardly be applied to distributed beamforming in MGM relay networks of [4] as well.

Notation: We denote transpose and Hermitian by $(\cdot)^T$ and $(\cdot)^H$, respectively. The operators $\mathrm{E}\{\cdot\}$, $\mathrm{tr}\{\cdot\}$ and $\|\cdot\|_2$ denote the statistical expectation, trace and Euclidean norm, respectively. $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix which contains the elements of a vector $\mathbf{a}$ on its diagonal. $\otimes$ stands for the Kronecker product. $\boldsymbol{I}_G$ denotes the $G \times G$ identity matrix and $\boldsymbol{e}_k$ denotes the kth column of $\boldsymbol{I}_G$. $\mathrm{Re}\{\cdot\}$ and $\angle\{\cdot\}$ denote the real part and the angle of their argument, respectively.

2. SIGNAL MODEL AND PROBLEM STATEMENT

We consider the wireless MGM scenario of [1]. In this scenario, a multi-antenna transmitter with N antenna elements communicates with G multicast groups of multiple destination users, $\{\mathcal{G}_1, \ldots, \mathcal{G}_G\}$, where $\mathcal{G}_k$ is the set of indexes of users participating in multicast group k, $1 \leq G \leq M$ and M denotes the total number of single-antenna receivers. $\boldsymbol{h}_m$ represents the $N \times 1$ vector of complex flat fading channel gains between each transmit antenna and the receive antenna of the mth user where $m \in \{1, \ldots, M\}$. Each receiver belongs to a single group only, i.e., $\mathcal{G}_k \cap \mathcal{G}_l = \emptyset$, for $k \neq l$ and $\cup_k \mathcal{G}_k = \{1, \ldots, M\}$. Let $\boldsymbol{w}_k^*$ denote the $N \times 1$ complex beamforming weight vector applied to the transmitting antenna array for the kth group. In the following, we assume that the transmitted information symbols are zero-mean with unit variance and mutually uncorrelated [1]. Then, the total power transmitted by the antenna array is given by $\sum_{k=1}^{G} ||\boldsymbol{w}_k||_2^2$. The multicast beamforming problem is to minimize the total power of the signal radiated at the transmitter subject to the destination QoS constraints. To measure QoS at each destination, the signal-to-interference-plus-noise ratio (SINR) is used. Thus, the QoS constraints guarantee that the SINR at each user is kept above a predefined threshold. Assuming that the CSI in terms of the instantaneous channel vectors $\{\boldsymbol{h}_m\}_{m=1}^M$ and the receiver noise powers $\{\sigma_m^2\}_{m=1}^M$ are known at the transmitter, the optimization problem can be written as

$$\begin{aligned} \min_{\{\boldsymbol{w}_k\}_{k=1}^G} \quad & \sum_{k=1}^{G} \|\boldsymbol{w}_k\|_2^2 \\ \text{s. t.} \quad & \frac{|\boldsymbol{w}_k^H \boldsymbol{h}_m|^2}{\sum_{l \neq k} |\boldsymbol{w}_l^H \boldsymbol{h}_m|^2 + \sigma_m^2} \geq \gamma_m, \\ & \forall m \in \mathcal{G}_k,\ \forall k, l \in \mathcal{K} \end{aligned} \tag{1}$$

where $\mathcal{K} = \{1, \ldots, G\}$. The problem in (1) is a non-convex optimization problem and it is generally difficult to solve. To obtain a simpler optimization problem, the original non-convex problem can be approximated by a convex SDP problem as in [1] or by a convex SOCP problem as in [2]. In the following, we focus on the SOCP approach which exhibits certain advantages over the SDP approach.

By introducing the new matrix and vector

$$\boldsymbol{H}_{k,m} \triangleq \begin{bmatrix} \sigma_m^2 & \boldsymbol{0}^T \\ \boldsymbol{0} & (\boldsymbol{I}_G - \mathrm{diag}\{\boldsymbol{e}_k\}) \otimes \boldsymbol{h}_m \boldsymbol{h}_m^H \end{bmatrix}^{1/2}, \tag{2}$$
$$\forall m \in \mathcal{G}_k,\ \forall k \in \mathcal{K}, \qquad \boldsymbol{w} \triangleq [1, \boldsymbol{w}_1^T, \ldots, \boldsymbol{w}_G^T]^T$$

the mth constraint in problem (1) can be written as

$$|\boldsymbol{w}_k^H \boldsymbol{h}_m| \geq \sqrt{\gamma_m} \|\boldsymbol{H}_{k,m} \boldsymbol{w}\|_2. \tag{3}$$

Using the following approximation

$$|\boldsymbol{w}_k^H \boldsymbol{h}_m| \geq \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\}, \tag{4}$$

i.e., substituting the magnitude of $\boldsymbol{w}_k^H \boldsymbol{h}_m$ by its real part, the non-convex problem (1) is turned into the following convex one

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \|\boldsymbol{w}\|_2 \\ \text{s. t.} \quad & \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\} \geq \sqrt{\gamma_m} \|\boldsymbol{H}_{k,m} \boldsymbol{w}\|_2, \\ & \forall m \in \mathcal{G}_k,\ \forall k \in \mathcal{K}, \qquad w_1 = 1. \end{aligned} \tag{5}$$

Problem (5) is a SOCP problem and can be solved efficiently using interior-point methods [3]. However, due to the strengthening of the constraints according to (4), the solution to (5) is generally only suboptimal for (1) and may be very inaccurate. Therefore, an iterative method has been proposed in [2] to successively improve the approximate solution. The idea is to adapt the approximation of the feasible set in the $(i+1)$th iteration to the solution $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ of (5) in the ith iteration. This is accomplished by rotating the phase of $\boldsymbol{h}_m^{(i+1)}$ such that the approximation in (4) is tight for $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ which is the case if $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{h}_m^{(i+1)}$ is real and non-negative.

Hence, we define the channel rotation recursion as [2]

$$\boldsymbol{h}_m^{(i+1)} \triangleq \boldsymbol{h}_m^{(i)} \exp\left(-j\alpha_{k,m}^{(i)}\right) \tag{6}$$

where $\alpha_{k,m}^{(i)}$ denotes the rotation angle in the ith iteration and is chosen as $\alpha_{k,m}^{(i)} = \angle\left(\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{h}_m^{(i)}\right)$.

As shown in [2], this makes it possible to find an improved solution $\boldsymbol{w}_{k,\mathrm{opt}}^{(i+1)}$ in the next iteration than $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ of the current iteration. Note that rotating the phase of $\boldsymbol{h}_m$ according to (6) does not affect the original problem (1). It has been shown in [2] that the objective value decreases monotonously with the iterations as long as $\alpha_{k,m}^{(i)} \neq 0$ for all active constraints (i.e., constraints satisfied with equality) in at least one multicast group in the current iteration.

3. PROPOSED METHOD

In the iterative method proposed in [2], a sequence of optimization problems (5) have to be solved and as a consequence, the complexity of this method is unnecessarily high. In each iteration, the SOCP problem (5) is solved using an interior-point method which is an iterative method itself. To make them distinguishable, let us refer to the iterations of successive channel rotations indexed by (i) in (6) as *outer iterations* and to those of the interior-point method as *inner iterations*. In a particular outer iteration, performing inner iterations to convergence yields the optimal solution to the SOCP problem (5). However, as the feasible set is then updated in the outer iterations, it is not necessary to process the inner iterations up to convergence. Therefore, the idea presented in this paper is to avoid this unnecessarily high accuracy of intermediate solutions and obtain solutions which are only suboptimal to problem (5). This can be achieved by performing only a single inner iteration per outer iteration which corresponds to solving an approximation of the SOCP problem (5) rather than problem (5) itself. This reduces the computational complexity of the iterative method in [2].

We explain our idea considering the example of the barrier method [3] as a particular interior-point method which is most straight-forward. We note, however, that our idea can be applied to other interior-point methods as well. Let us first briefly review the concept of the barrier method. A common tool to solve convex problems, such as, e.g., equality-constrained convex problems, is Newton's method [3]. However, convex problems with inequality constraints cannot directly be solved using this method. Therefore, in the barrier method, the solution to an inequality-constrained convex problem is obtained by solving a sequence of equality-constrained convex problems each of which is solved using Newton's method. Towards this aim, the original inequality constrained convex problem, in our case problem (5), is approximated by a convex problem without inequality constraints as shown next. Following the derivation of the barrier method in [3], we incorporate the inequality constraints of problem (5) into the objective function using the indicator function $I_{-}(u)$ as follows

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \|\boldsymbol{w}\|_2 + \sum_{m=1}^{M} I_{-}\left(\sqrt{\gamma_m}\|\boldsymbol{H}_{k,m}\boldsymbol{w}\|_2 - \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\}\right) \\ \text{s. t.} \quad & w_1 = 1. \end{aligned} \tag{7}$$

where the indicator function for non-positive reals $I_{-}(u)$ is defined as

$$I_{-}(u) = \begin{cases} 0 & u \leq 0 \\ \infty & u > 0 \end{cases}$$

The problem in (7) is equivalent to problem (5), i.e., problem (5) is turned into an equivalent equality-constrained problem. However, the objective function of problem (7) is non-differentiable. Accordingly, Newton's method, which requires twice differentiable functions, cannot be applied directly. To overcome this problem, we use the idea of the barrier method to approximate the indicator function with a differentiable one. Here, we use the logarithmic barrier function as an approximation [3]. Using the generalized logarithm for the second-order cone [3], the approximation of the indicator function in problem (7), can be defined as [3]

$$\hat{I}_{-}(\boldsymbol{w}) = -\left(\frac{1}{t}\right)\log\left(\left(\mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\}\right)^2 - \gamma_m\|\boldsymbol{H}_{k,m}\boldsymbol{w}\|_2^2\right)$$

which is twice differentiable and where the parameter t controls the accuracy of the approximation. That is, the larger the value of t, the closer the logarithmic approximation is to the original indicator function I_{-}. Substituting the original with the approximated indicator function in problem (7), one obtains the following approximation[3]

$$\min_{\boldsymbol{w}} \quad t\|\boldsymbol{w}\|_2^2 + \phi(\boldsymbol{w}) \qquad \text{s.t.} \quad w_1 = 1 \tag{8}$$

where the function ϕ denotes the logarithmic barrier for the SOCP problem (5) and is defined as

$$\phi(\boldsymbol{w}) = -\sum_{m=1}^{M}\log\left(\left(\mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\}\right)^2 - \gamma_m\|\boldsymbol{H}_{k,m}\boldsymbol{w}\|_2^2\right). \tag{9}$$

Problem (8) is an approximation of the original SOCP problem (5). The function in (9) is convex (which is not obvious but has been shown in [3] for SOCPs in general) and thus, problem (8) is convex. The domain of problem (8) is equal to the feasible set of problem (5). To achieve an accurate approximate solution, the parameter t has to be chosen large. However, problem (8) is difficult to solve via Newton's method for a large value of t except when the starting point is chosen close to the optimum of (8) (see [3]). Therefore, an iterative approach is used in the barrier method where in each iteration, problem (8) is solved for a particular value of t. In the first iteration, t is chosen small and it is then increased in every iteration where the solution obtained in the current iteration serves as the starting point in the next iteration. This iterative scheme terminates when the solution to problem (5) is approximated with sufficiently high accuracy.

However, as mentioned earlier, this high accuracy is not required for intermediate solutions of the iterative method of [2]. Therefore, we propose to use only the first iteration of the barrier method to obtain an approximate solution to problem (5). This corresponds to combining the barrier method with the iterative scheme of [2] as follows. In each iteration of the proposed method, rather than solving problem (5), we solve problem (8) for a small t which can be done more efficiently using Newton's method. Instead of increasing t in each iteration as in the interior-point method, we perform the rotation of (6) and keep t fixed as long as the rotation is significant. As soon as the rotations become negligible (i.e., as soon as $\max_m\{|\alpha_{k,m}^{(i)}|\} < \epsilon$ for some small ϵ), the feasible set only changes marginally and we can start increasing t in every iteration like in traditional interior-point methods.

This guarantees that, once a good approximation of the feasible set has been found, we obtain the final weight vector with sufficiently high accuracy. As in the traditional barrier method, the solution obtained in a given iteration with Newton's method always serves as the starting point for Newton's method in the following iteration. This assures that Newton's method converges even for increasing t. This procedure is repeated for a fixed number of iterations I. As in the iterative method proposed in [2], in this method, the initial channel vector is assumed to be equal to the original channel vector, i.e., $\boldsymbol{h}_m^{(1)} = \boldsymbol{h}_m$. The proposed iterative method is summarized in Table 1 where $\mu > 1$.

Initialization: $\boldsymbol{h}_m^{(1)} = \boldsymbol{h}_m, \quad t^{(1)} > 0$
for $i = 1 \dots I$
Solve problem (8) with $\boldsymbol{h}_m = \boldsymbol{h}_m^{(i)}$ and $t = t^{(i)}$
Perform the rotation of (6) with $\alpha_{k,m}^{(i)} = \angle \left(\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{h}_m^{(i)} \right)$
if $\max_m \{ \lvert \alpha_{k,m}^{(i)} \rvert \} < \epsilon$ **, then:** $t^{(i+1)} = \mu \cdot t^{(i)}$
otherwise: $t^{(i+1)} = t^{(i)}$
end

Table 1: Proposed Iterative Procedure

As shown in our simulation results, the method in Table 1 achieves the same performance as the iterative method of [2]. However, this is achieved at a significantly reduced computational cost since most of the inner iterations have been eliminated.

In this work, we have focused on the logarithmic barrier method for simplicity. However, the idea presented in this work can also be applied to all other interior-point methods which are based on the concept of successively increasing an accuracy parameter t. The more sophisticated primal-dual interior-point methods [3] belong to this class of interior-point methods.

4. ITERATIVE FEASIBILITY SEARCH FOR THE PROPOSED METHOD

Due to the strengthening of the constraints according to (4), the approximated feasible set used in (5) is restricted compared to the original feasible set of problem (1). Hence, for the original channel vectors $\boldsymbol{h}_m$, problem (5) may be infeasible even if problem (1) is feasible. Obviously, this infeasibility is then inherited by problem (8) as well: If problem (5) is infeasible, the domain of problem (8) is empty. To mitigate this drawback, the same concept of feasibility search that is used in the iterative approach of [2] can be applied to our method. The idea in [2] is to find rotated versions of the original channel vectors for which problem (5) becomes feasible [2]. Towards this aim, problem (5) is reformulated as a feasibility problem whose solution always exists [3]

$$\begin{aligned} \min_{\boldsymbol{z},\,\boldsymbol{w}} \quad & \boldsymbol{1}^T \boldsymbol{z} \\ \text{s. t.} \quad & \sqrt{\gamma_m} \|\boldsymbol{H}_{k,m} \boldsymbol{w}\|_2 - \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\} \le z_m, \\ & \forall m \in \mathcal{G}_k,\ \forall k \in \mathcal{K}, \qquad w_1 = 1 \\ & z_m \ge 0,\ \forall m \in \mathcal{G}_k,\ \forall k \in \mathcal{K}. \end{aligned} \tag{10}$$

where the elements of vector $\boldsymbol{z} = [z_1, \dots, z_M]^T$ are a measure of how far the QoS constraint of each receiver is from being satisfied and where $\boldsymbol{1}$ is the $M \times 1$ vector whose elements are all equal to one. Note that problem (5) is feasible if and only if $\boldsymbol{1}^T \boldsymbol{z} = 0$. In [2], the iterative phase rotation of $\boldsymbol{h}_m$ described above has been applied to problem (10) in order to successively decrease $\boldsymbol{1}^T \boldsymbol{z}$ to zero and thus find a feasible initial approximation. Instead of solving (10) in every iteration, we again reduce the unnecessarily high accuracy of intermediate solutions. We can apply the same idea used for the approximation of problem (5) to problem (10). Therefore, we use the indicator function to incorporate the inequality constraints into the objective function and then approximate the indicator function. This leads to an approximated version of (10) in the following form [3]

$$\min_{\boldsymbol{z},\,\boldsymbol{w}} \quad t \boldsymbol{1}^T \boldsymbol{z} + \psi(\boldsymbol{w}, \boldsymbol{z}) \qquad \text{s. t.} \quad \tilde{w}_1 = 1 \tag{11}$$

where ψ denotes the logarithmic barrier for the SOCP problem (10) and is defined as

$$\begin{aligned} \psi(\boldsymbol{w}, \boldsymbol{z}) = -\sum_{m=1}^{M} \Big[\log \Big(\big(z_m + \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{h}_m\} \big)^2 \\ - \gamma_m \|\boldsymbol{H}_{k,m} \boldsymbol{w}\|_2^2 \Big) + \log(z_m) \Big]. \end{aligned} \tag{12}$$

Replacing problem (10) with problem (11) in the iterative feasibility search of [2], we obtain the computationally more efficient feasibility search of Table 1 where problem (8) is replaced by problem (11). In the feasibility search in [2], the infeasibility $\boldsymbol{1}^T \boldsymbol{z}$ is reduced in every iteration. This applies for the proposed algorithm as well. When $\boldsymbol{1}^T \boldsymbol{z} = 0$, a feasible initial rotation has been found and one can proceed with the algorithm in Table 1 performed on problem (8) using this initial rotation.

5. COMPLEXITY ANALYSIS

The SOCP problem (5) can be solved with a worst-case complexity of $\mathcal{O}(G^3 N^3 M^{1.5})$ using interior-point methods where the terms $\mathcal{O}(G^3 N^3 M)$ and $\mathcal{O}(\sqrt{M})$ correspond to the complexity per iteration and the number of iterations required in the interior-point method, respectively [5]. The complexity per iteration of the proposed method is the same, i.e., $\mathcal{O}(G^3 N^3 M)$, whereas the number of required iterations is slightly larger than $\mathcal{O}(\sqrt{M})$ since in addition to increasing the parameter t, the feasible sets are updated. If we assume I_1 significant updates of the feasible set during which t is kept fixed, the proposed method requires $I_1 + \mathcal{O}(\sqrt{M})$ iterations with a complexity of $\mathcal{O}(G^3 N^3 M)$. In the method of [2], problem (5) is solved approximately I_1 times to achieve the same performance. Hence, it requires $I_1 \cdot \mathcal{O}(\sqrt{M})$ iterations with the per-iteration complexity of the proposed method

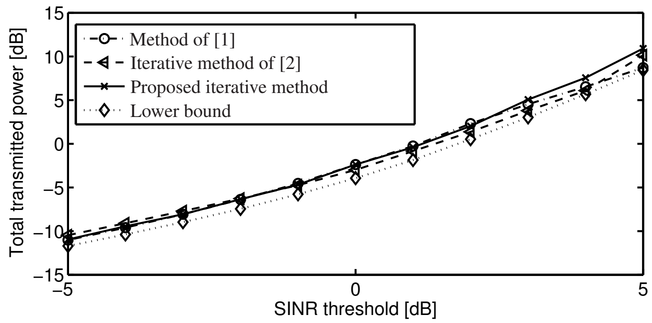

Fig. 1: Total transmitted power versus SINR threshold.

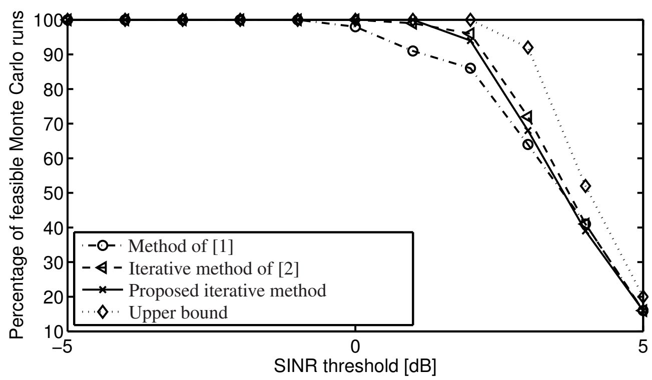

Fig. 2: Percentage of feasible Monte Carlo runs versus SINR threshold.

which results in a higher overall computational complexity. The SDR-based approach of [1] consists of two parts that both add to the complexity. Firstly, a single SDP problem with a worst-case complexity of $\mathcal{O}(M^2(GN + M)^{2.5})$ (see [4] and [6]) is solved. Secondly, a LP problem with a worst-case complexity of $\mathcal{O}(G^{3.5} + MG^{3.5})$ [1] is solved N_{rand} times where N_{rand} stands for the number of generated candidate vectors of which, according to [1], a few hundred are usually required. Thus, as M increases, the computational complexities of the SOCP-based methods decrease relative to that of the SDR-based method.

6. SIMULATION RESULTS

In our simulations, we consider a multicasting scenario in which a transmitter with $N = 6$ antennas serves $G = 3$ multicast groups of similar size where the total number of users is $M = 16$. Without loss of generality, we assume equal noise powers and SINR thresholds at the receivers. Further, we assume flat fading Rayleigh channels with unit variance. For the SDR-based approach, 100 candidate vectors have been generated. For the original iterative SOCP method of [2], we have chosen the number of iterations as $I_1 = 3$ and for the proposed method as $I_2 = 10$. The parameters of the proposed method have been set to $t^{(1)} = 0.1$, $\mu = 8$ and $\epsilon = 30°$. All results (apart from the runtime results) are averaged over 300 Monte Carlo runs.

Fig. 1 displays the total transmitted power versus the minimal required SINR for the SDR-based method, the iterative SOCP method of [2] and the proposed iterative method. A lower bound, which is a byproduct provided by the SDR-based method, is also shown and corresponds to the total transmitted power obtained by solving the relaxation of problem (1). Note that this lower bound is not achievable in general as the solution to the relaxed problem usually lies outside the original feasible set. In Fig. 2, the percentage of feasible Monte Carlo runs versus the SINR threshold is depicted. The relaxation of problem (1) now serves as an upper bound since it may be feasible even if the non-relaxed problem is infeasible.

We can observe from these figures that the performance of the proposed method is similar to the performance of the method of [2]. However, we achieve this performance at a significantly reduced computational complexity. The performance of the method of [1] is similar as well but its complexity is significantly higher in this multi-group multicasting scenarios with a large number of users.

In order to complement our theoretical complexity analysis, we compare the runtimes of the different methods observed during the simulations. In Table 2, the runtime of each of the three competing methods for solving an exemplary instance of problem (1) is provided. The proposed method clearly outperforms the state-of-the-art methods in terms of runtime.

Method of [1]	Method of [2]	Proposed method
1.691	0.447	0.217

Table 2: Comparison of runtime in seconds

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