# A Consensus Approach for Cooperative Communications in Cognitive Radio Networks

Ivan Stupia[1], Riccardo Andreotti[2], Vincenzo Lottici[2] and Luc Vandendorpe[1]
[1]ICTEAM, Université Catholique de Louvain, Louvain la Neuve, Belgium
[2]Department of Information Engineering, University of Pisa, Pisa, Italy
Email:{ *ivan.stupia,luc.vandendorpe*}@uclouvain.be, { *riccardo.andreotti,vincenzo.lottici*}@iet.unipi.it

***Abstract*—In this paper, we investigate a decentralized power allocation (PA) algorithm, i.e., without the need of a fusion center, for a coalition of bit-interleaved coded (BIC) OFDM-based cognitive radio (CR) devices. Taking advantages of a game theoretical description for our problem, it is shown that the reliability of the secondary cooperative link can be remarkably improved through a simple average consensus algorithm keeping the interference produced to the primary users (PUs) under a prescribed threshold. Finally, the performance gain of the proposed PA policy is highlighted comparing it with conventional power allocation algorithms.** [1]

## I. Introduction

Several solutions have been proposed in the last years to deal with the growing demand of the limited radio spectrum resource, like link adaptation techniques [1] and cognitive radio (CR) paradigms [2]. However, the latter does not allow unlicensed users, called secondary users (SUs), to reach a good coverage or network connectivity, due to the emission limits imposed to the SUs. In fact, SUs can transmit over the same bands assigned to licensed users, called primary users (PUs), under the constraint that the interference caused to them is kept below a predetermined threshold. For this reason, cooperative protocols can offer suitable solutions to improve SUs performance, [3]. In particular, we consider a specific scenario where a set of cooperative users, i.e. a coalition, of BIC OFDM CR terminals transmits in packet-oriented fashion. Moreover, we assume that the cooperation happens at the channel coding level, according to an Hybrid Distributed Forward Error Correction (H-DFEC) protocol as proposed in [4] and [5].

Our work focuses on deriving a novel distributed PA algorithm that improves the reliability of the cooperative link. First of all, a proper figure of merit, for packet oriented transmission systems, is identified in the packet error rate (PER). In order to derive a simple yet accurate expression of the PER metric, the well-known link performance evaluation methodology called effective SNR (ESNR) mapping is here extended to account for the H-DFEC protocol. In detail, we take advantage of the $\kappa$ESM methodology, originally proposed in [6] for hybrid automatic repeat request (HARQ) BIC OFDM systems, which relies on the cumulant moment generating functions of the decoder log-likelihood ratios (LLRs). As shown in [6], the $\kappa$ESM maps into a single scalar value the envisaged performance of a BIC OFDM system that transmits several copies of the same packet, each of them coded with a different coding rate. In the following, the main contributions are pointed out. *i)* The *cooperative* $\kappa$ESM methodology, that maps into a scalar value the predicted performance of the cooperative link, accounting for the contribution of all the coalition members, is first introduced. *ii)* Each member of the coalition only requires the knowledge of a scalar value to evaluate the cooperative $\kappa$ESM, instead of all the channel state information (CSI) and transmission parameters (TPs) of all the other members. *iii)* The PA problem is formalized as a potential game, named max-ESNR game. Capitalizing on the potential games' features, a decentralized algorithm reaching the Nash equilibrium is then proposed by exploiting the class of the consensus algorithms.

[1]The authors wish to acknowledge the activity of the ARC project SCOOP that motivated this work.

## II. System Model

We consider a cooperative system composed of $Q$ SUs transmitting over a BIC OFDM channel made of $N$ subcarriers. In particular, packet-oriented transmissions are taken into account, i.e., each packet coming from the upper layers is mapped into a radio link control protocol data unit (RLC-PDU) of $N_s$ bits, which include the header, the payload and the CRC field.

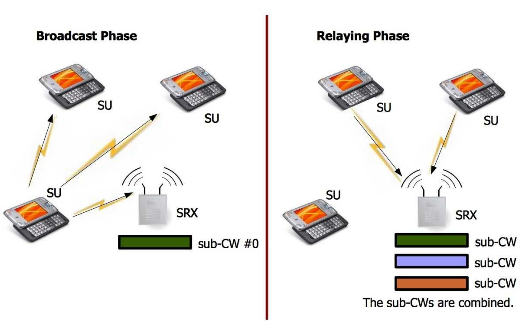


Fig. 1. H-DFEC Protocol.

### A. Cooperative Protocol

In this section, the H-DFEC protocol adopted by the $Q$ cooperative SUs is outlined. As apparent from Fig. 1, the H-DFEC protocol consists of two phases: the broadcast phase and the relaying phase. In the former, the generic $\bar{q}$-th SU encodes the RLC-PDU with a mother code $r$, then the rate matching mechanism punctures the obtained sequence $\{b_k\}$ of encoded binary symbols (EBs), with $1 \leq k \leq N_s/r$, and outputs $N_s/r_{\bar{q}}$ EBs, being $r_{\bar{q}}$ the desired coding rate. Then, according to the BICM paradigm, these EBs are randomly interleaved and mapped into the available resources, obtaining the sequence $\{a_{n,h}\}$, where $n$ $(1 \leq n \leq N)$

identifies the subcarrier index and $h$ $(1 \leq h \leq m_{\bar{q},n})$ the position of the EB in the label of the modulation symbol $s_{\bar{q},n}$ belonging to a $2^{m_{\bar{q},n}}$-QAM constellation. Finally, after the IFFT processing and the cyclic prefix (CP) insertion, the signal is broadcasted to the receiver and to all the members of the coalition, which are assumed to be able to decode without errors the broadcasted signal. In the relaying phase, each member $q$, with $q \neq \bar{q}$, performs the same packet processing of the source. Thus, the RLC-PDU is re-encoded with mother code rate $r$, randomly punctured to obtain the chosen coding rate $r_q$, and sent to the receiver. Thus, assuming a block-fading channel, the received signal on the generic link $q$, $1 \leq q \leq Q$, over the $n$th subcarrier results

$$y_{q,n} = \sqrt{p_{q,n}\gamma_{q,n}} s_{q,n} + w_{q,n} \tag{1}$$

where $\boldsymbol{w}_q \triangleq [w_{q,1}, \cdots, w_{q,N}]$ is the noise vector whose entries are complex-valued circularly symmetric Gaussian RVs with zero mean and unit variance, $\mathbf{p}_q \triangleq [p_{q,1}, \cdots, p_{q,N}] \in \mathcal{P}_q$ is the power allocation vector, $\mathcal{P}_q \triangleq \{\mathbf{p}_q \in \Re^N : \sum_{n=1}^{N} p_{q,n} \leq P_q\}$ is the set of the allowed power allocation vectors and $\gamma_{q,n}$ is the per-carrier signal-to-noise ratio. Thus, the destination evaluates the sequence of LLRs $\Lambda_{k,q}$ relevant to each received signal and then combines the LLRs associated to the same packet so that

$$\mathcal{L}_k = \mathbf{q}_k \boldsymbol{\Lambda}_k, \tag{2}$$

with $\boldsymbol{\Lambda}_k = [\Lambda_{k,1}, \Lambda_{k,2}, \cdots, \Lambda_{k,Q}]^T$ and $\mathbf{q}_k = [q_{k,1}, q_{k,2}, \cdots, q_{k,Q}]^T$, where $q_{k,q} \in \{0, 1\}$ is the puncturing bit relevant to the EB $b_k$ forwarded by the $q$th SU. The LLR flow $\mathcal{L}_k$ is finally fed to the decoder, which performs the decoding and checks the CRC. It is worth noting that the H-DFEC protocol is analogous to a distributed HARQ mechanism as proposed in [4]. In the latter, time diversity is exploited, since the transmitter sends several copies of the same packet, while here, the spatial diversity offered by the members of the coalition is exploited.

### *B. Cognitive Environment*

In the considered cognitive scenario, the primary system accounts for two different kinds of PUs, according to their positions with respect to the SUs. The first class is formed by *underlay* PUs (UPUs), which are set geographically at a certain distance with respect to the SUs. In this way, the SUs can transmit over the same frequencies used by the UPUs, if the interference caused to them is below the threshold $T_j$, $j = 1, \cdots, J$, being $J$ the number of UPUs. More formally,

$$\mathbf{p} \in \mathcal{P}_U \triangleq \{\mathbf{p} \in \Re^{NQ} : \sum_{q=1}^{Q} \sum_{\phi(n)=j} p_{q,n} \leq T_j, \;\; 1 \leq j \leq J\}, \tag{3}$$

where $\mathbf{p}$ is the collection of the PA vectors of all the SUs and $\phi(n) = j$ a mapping function denoting that UPU $j$ transmits over the $n$th subcarrier. The second class consists of *interweave* PUs (IPUs), which are located near to the SUs. Here, the SUs transmit over the frequencies left free by the IPUs and the interference is caused by the out-of-band emissions of the SUs' signals, so that

$$\mathbf{p} \in \mathcal{P}_I \triangleq \{\mathbf{p} \in \Re^{NQ} : \sum_{q=1}^{Q} \sum_{n=1}^{N} K_{q,n}^{(l)} p_{q,n} \leq I_l, \;\; 1 \leq l \leq L\}, \tag{4}$$

where $\mathcal{P}_I$ is the set of feasible PA vectors satisfying the interweave constraints $I_l$, $l = 1, \cdots, L$, being $L$ the number of IPUs, and $K_{q,n}^{(l)}$ a positive coefficient depending on the spectral shape of the transmitted signal and on the distance between the $q$th secondary transmitter and the $l$th IPU.

## III. Power Allocation Game

In this section, a novel metric is first derived, which enables a simple yet accurate description of the cooperative link performance. Exploiting the proposed metric, a power allocation game, aimed at improving the coalition's performance under constraints on the interference caused to the PUs, is then introduced.

First of all, let us recall the effective SNR (ESNR) mapping concept, usually adopted in point-to-point communications. The ESNR method estimates the instantaneous performance of a link, mapping the per-subcarrier SNRs values along with the employed TPs (i.e. power allocation vector, bit loading, etc.) into a single scalar value, named ESNR. This value univocally corresponds to a PER value, according to the adopted coding rate, that represents the estimate of the link performance. In particular, we rely on the $\kappa$ESM methodology proposed in [7] that predicts the performance of a BIC OFDM link by mapping it into an equivalent BPSK system over AWGN channel, whose SNR would be equal to the ESNR evaluated as follows:

$$\Gamma(\mathbf{p}_q) \triangleq -\log\left[\frac{\sum_{n=1}^{N} \alpha_{q,n} e^{-p_{q,n}/\rho_{q,n}}}{\sum_{n=1}^{N} m_{q,n}}\right]. \tag{5}$$

In (5), $\rho_{q,n}$ is a constant value depending on the SNR and the modulation adopted on the $n$th subcarrier and $\alpha_{q,n}$ is a constant depending only on the modulation adopted on the $n$th subcarrier. Recalling the above-mentioned analogy between the H-DFEC protocol and the HARQ mechanism, we introduce the concept of *cooperative* ESNR, relying on the aggregated $\kappa$ESM methodology derived in [6] for BIC OFDM system employing HARQ protocols.

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**Proposition 1**. For $1 \leq q \leq Q$, the cooperative effective SNR $\tilde{\Gamma}$ relevant to the $q$th SU can be approximated as

$$\tilde{\Gamma}\left(\mathbf{p}_q, \mathbf{p}_{-q}\right) \simeq g(\tilde{\Gamma}(\mathbf{p}_{-q}), \eta_q) + f(\Gamma(\mathbf{p}_q), \theta_q), \tag{6}$$

where $\mathbf{p}_{-q}$ denotes the collection of the power allocation vectors of all the SUs but the $q$th, $\eta_q \triangleq \frac{r_q}{R_{-q}}$, $\theta_q \triangleq \frac{R_{-q}}{r_q}$ and

$$g(\Gamma, a) \triangleq \begin{cases} -\log\left[1 + a(\mathrm{e}^{-\Gamma} - 1)\right], & r_q \leq R_{-q} \\ \Gamma, & r_q > R_{-q} \end{cases}, \tag{7}$$

$$f(\Gamma, a) \triangleq \begin{cases} \Gamma, & r_q \leq R_{-q} \\ -\log\left[1 + a(\mathrm{e}^{-\Gamma} - 1)\right], & r_q > R_{-q} \end{cases}, \tag{8}$$

being $r_q$ the code rate of the user $q$ and $R_{-q} = \min_{j \neq q} \{r_j\}$.

---

The reader can refer to [6] for the demonstration, here omitted for the sake of brevity. Now, some observation are in order. *i)* The cooperative ESNR is the SNR of an equivalent BPSK system over AWGN channel that has the same performance of the cooperative link composed of $Q$ SUs. *ii)* The contribution of the other members of the coalition, in (6), is represented by the scalar value $\tilde{\Gamma}(\mathbf{p}_{-q})$.

Now, the PA game can be introduced. Formally, the game, called max-ESNR game, in strategic form, is identified by the triplet $<\mathcal{Q},\mathcal{P},\mathcal{U}>$, being $\mathcal{Q}$ the set of players, corresponding to the $Q$ SUs, $\mathcal{P}$ the strategy set, corresponding to the feasible power allocation vectors and $\mathcal{U}=\{u_q\}_{q=1}^{Q}$ the set of utilities, where

$$u_q(\mathbf{p}_q,\mathbf{p}_{-q}) = g(\tilde{\Gamma}(\mathbf{p}_{-q}),\eta_q) + f(\Gamma(\mathbf{p}_q),\theta_q). \tag{9}$$

Resorting to the potential games theory [9], the set of pure Nash equilibria is found investigating the local optima of a global potential function, that represents the incentive of all the players to change their strategies. From (5), it can be demonstrated that

$$\phi(\mathbf{p}) \triangleq \sum_{q=1}^{Q}\sum_{n=1}^{N}\alpha_{q,n}e^{-p_{q,n}/\rho_{q,n}} \tag{10}$$

is a potential function for the proposed game. Thus, solving the following optimization problem

$$(\mathrm{P1}): \begin{array}{ll} \underset{\mathbf{p}\succeq\mathbf{0}}{\text{minimize}} & \sum_{q=1}^{Q}\sum_{n=1}^{N}\alpha_{q,n}\mathrm{e}^{-p_{q,n}/\rho_{q,n}} \\ \text{subject to} & \mathbf{p}\in\mathcal{P} \end{array},$$

where $\mathcal{P}=\mathcal{P}_1\times\cdots\times\mathcal{P}_Q\times\mathcal{P}_U\times\mathcal{P}_I$, the set of Nash equilibria is found. Being (P1) a convex optimization problem, the existence and uniqueness of the Nash equilibrium is guaranteed.

## IV. Distributed Power Allocation Strategy

Within the context outlined above, in this section, we derive a decentralized PA strategy solving the NE problem. First of all, the power allocation vector corresponding to the NE is derived by maximizing the potential function and then, based on the knowledge of the equilibrium point expression, a decentralized strategy relying on a simple consensus algorithm is proposed.

### *A. Successive Set Reduction Algorithm*

In this section, we take advantage of the SSR algorithm, originally proposed in [10], to develop the structure of the PA procedure which maximizes the potential function while guarantees the given coexistence constraints. The rationale of our approach lies in the iterative optimization of the power allocation vector employed by the coalition of SUs, wherein, at each step of the algorithm, an optimal power increment $\delta\mathbf{p}^{(j)}$ is evaluated over a suitably defined reduced set of feasible power values $\mathcal{K}_j$. Formally, let us express $\mathbf{p}^*$ as a sum of $J$ contributions, such that

$$\mathbf{p}^* \triangleq \sum_{j=1}^{J}\delta\mathbf{p}^{(j)}, \tag{11}$$

where the $j$th increment $\delta\mathbf{p}^{(j)}$ is carried out by optimizing the function $\phi(\mathbf{p})$ over the reduced set of feasible power values

$$\mathcal{K}_j \triangleq \left\{\delta\mathbf{p}\in\Re^{NQ}: \sum_{q=1}^{Q}\sum_{n=1}^{N}\frac{\delta p_{q,n}}{\Theta_{q,n}^{(j)}}\le 1\right\}, \tag{12}$$

with $\{\Theta_{q,n}^{(j)}\}_{n=1}^{N}$ representing the *extreme* points of the set, i.e., the maximum increments of power per subcarrier that do not violate any of the constraint of the problem, assuming that subcarrier is the only active one. The extreme points are expressed by

$$\Theta_{q,n}^{(j)} \triangleq \min\left\{\bar{P}_q^{(j)},\bar{T}_n^{(j)},\left\{\bar{I}_{l,q,n}^{(j)}\right\}_{l=1}^{L}\right\}, \tag{13}$$

with

$$\bar{P}_q^{(j)} \triangleq P_q - \textstyle\sum_{\nu=1}^{N}p_{q,\nu}^{(j-1)},\ \bar{T}_n^{(j)} \triangleq T_{\phi(n)} - \displaystyle\sum_{q=1}^{Q}\sum_{\phi(\nu)=\phi(n)}p_{q,\nu}^{(j-1)},$$

$$\bar{I}_{l,q,n}^{(j)} \triangleq \frac{I_l-\left(\sum_{k=1}^{Q}\sum_{\nu=1}^{N}p_{k,\nu}^{(j-1)}K_{k,\nu}^{(l)}\right)}{K_{q,n}^{(l)}},\ \forall 1\le l\le L,$$

and $p_{q,n}^{(j-1)}\triangleq\sum_{\nu=1}^{j-1}\delta p_{q,n}^{(\nu)}$, wherein the reduced set $\mathcal{K}_j$ is obtained as the intersection of the halfspace of the positive power increments with the halfspace lying below the hyperplane passing by the extreme points. Therefore, defining

$$\phi^{(j)}(\delta\mathbf{p}) \triangleq \phi\left(\mathbf{p}^{(j-1)}+\delta\mathbf{p}\right) = \sum_{q=1}^{Q}\sum_{n=1}^{N}\tilde{\alpha}_{q,n}^{(j-1)}\mathrm{e}^{-\frac{\delta p_{q,n}}{\rho_{q,n}}}, \tag{14}$$

where $\tilde{\alpha}_{q,n}^{(j-1)} \triangleq \alpha_{q,n}\mathrm{e}^{-p_{q,n}^{(j-1)}/\rho_{q,n}}$, the reduced-set optimization problem can be rewritten as

$$(\mathrm{P2}): \begin{array}{ll} \underset{\delta\mathbf{p}\succeq\mathbf{0}}{\text{minimize}} & \phi^{(j)}(\delta\mathbf{p}) \\ \text{subject to} & \sum_{q=1}^{Q}\sum_{n=1}^{N}\frac{\delta p_{q,n}}{\Theta_{q,n}^{(j)}}\le 1. \end{array}$$

Let us note that the problem (P2) is equivalent to that proposed in [7] for non-cognitive point-to-point communications, so that

$$\delta p_{q,n}^{(j)} = \rho_{q,n}\left[\mu^{(j)}-\log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}^{(j-1)}\Theta_{q,n}^{(j)}}\right)\right]^{+} \tag{15}$$

with,

$$\mu^{(j)} \triangleq \frac{1+\sum_{q=1}^{Q}\sum_{n=1}^{N}\frac{\rho_{q,n}}{\Theta_{q,n}^{(j)}}\log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}^{(j-1)}\Theta_{q,n}^{(j)}}\right)}{\sum_{q=1}^{Q}\sum_{n=1}^{N}\frac{\rho_{q,n}}{\Theta_{q,n}^{(j)}}}. \tag{16}$$

Table I summarizes the SSR algorithm. Finally, let us remark some features of the SSR algorithm. *i)* It iteratively reduces the set of feasible power increments until, after $J$ steps, any increment would violate one of the constraints ($\Theta_{q,n}^{(J)}=0,\ \forall q,n$). *ii)*. The solution is reached in a greedy-fashion, i.e., at each step the best local choice is evaluated through the closed-form solution (15).

| |
|---|
| **Initialize**: $j=1$, $\mathbf{p}^{(0)}=\mathbf{0}$ |
| **Do** |
| Evaluate $\mathbf{p}^{(j)}$ and $\{\tilde{\alpha}_{q,n}^{(j-1)},\Theta_{q,n}^{(j)}\}\ \forall q,n$ |
| Evaluate $\delta\mathbf{p}^{(j)}$ according to (15) and (16) |
| Set $j\leftarrow j+1$ |
| **While** $\left(\|\boldsymbol{\Theta}^{(j)}\|>0\right)$ |
| **Set** $J=j$ |
| **Output**: $\mathbf{p}^*=\sum_{j=1}^{J}\delta\mathbf{p}^{(j)}$ |

TABLE I
Pseudo-Code of the SSR Algorithm

### B. Distributed Consensus Algorithm

Now, let us recall that the network-wide interference constraints do not allow the SU to perform an individual choice of the power distribution. Thus, in order to keep the PA strategy as decentralized as possible while imposing global interference constraints, the proposed idea is to solve each iteration of the SSR algorithm in a distributed manner, once that an agreement on the amount of per user allowed interference is reached. Stated in mathematical terms, the generic $q$th SU transmit with a power vector $\mathbf{p}_q = \sum_{j=1}^{J} \delta\mathbf{p}_q^{(j)}$, where the increments $\{\delta\mathbf{p}_q^{(j)}\}_{j=1}^{J}$ are obtained by solving the following minimization problem

$$
(\mathrm{P3}): \quad \begin{array}{ll} \underset{\delta\mathbf{p}_q \succeq \mathbf{0}}{\text{minimize}} & \sum_{n=1}^{N} \tilde{\alpha}_{q,n}^{(j-1)} \mathrm{e}^{-\delta p_{q,n}^{(j)}/\rho_{q,n}} \\ \text{subject to} & \sum_{n=1}^{N} \frac{\delta p_{q,n}}{\Theta_{q,n}^{(j)}} \leq \vartheta_q^{(j)}, \end{array}
$$

$0 \leq \vartheta_q^{(j)} \leq 1$ representing the share of allowed interference negotiated by the $q$th SU. This problem is identical to that proposed in [7] for non-cognitive scenarios whose solution is

$$
\frac{\delta p_{q,n}}{\rho_{q,n}} = \left[\mu(\vartheta_q) - \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right)\right]^{+}, \tag{17}
$$

where, for the sake of readability, the dependence on the iteration index $j$ has been omitted. Now, let us remark that the solution (17) can be interpreted as a waterfilling-like representation where

$$
\mu(\vartheta_q) \triangleq \frac{\vartheta_q + \sum_{n=1}^{N} \frac{\rho_{q,n}}{\Theta_{q,n}} \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right)}{\sum_{n=1}^{N} \frac{\rho_{q,n}}{\Theta_{q,n}}} \tag{18}
$$

plays the role of the water level. Some observations are now in order. *i)* We assume a non-hierarchical coalition. Hence, since each SU must be able to act as source, it shall have sufficient power to transmit its own packet to the other SUs (fairness requirement). *ii)* In order to avoid excessive waste of power during the signaling phase, communication between SUs shall be provided through a low-power channel. As a consequence, each SU can exchange information only with its neighborhood (robustness requirement). *iii)* The objective function monotonically decreases with the power and, looking at (15), it can be noted that the power increment $\delta\mathbf{p}_q^{(j)}$ is directly proportional to the water level $\mu_q$.

For the above reasons, we propose the following criterion to endogenously compute the parameter $\boldsymbol{\vartheta}$.

$$
(\mathrm{P4}): \quad \begin{array}{ll} \underset{\boldsymbol{\vartheta} \succeq \mathbf{0}}{\text{maximize}} & \prod_{q=1}^{Q} \mu(\vartheta_q) \\ \text{subject to} & \sum_{q=1}^{Q} \vartheta_q \leq 1. \end{array}
$$

Solving the well-known optimization problem (P4), we obtain

$$
\vartheta_q^* = \left[\lambda + \frac{1}{Q} - \sum_{n=1}^{N} \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right)\right]^{+} \tag{19}
$$

with

$$
\lambda = \frac{\sum_{q=1}^{Q} \sum_{n=1}^{N} \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right)}{Q}. \tag{20}
$$

Thus, the NE problem turns into the problem of finding the optimal share of interference whose expression is given by (19). This problem is solved taking advantage of the framework for consensus algorithms provided by [8], where the states of all the agents of a self-organizing networked system must converge to a prescribed function (typically the average) of the initial states. Therefore, the interaction topology of the secondary network can be represented by an indirected graph $G = (V, E)$, with vertices $V = \{1, 2, ..., Q\}$ and edges $E = \{(q, q') \in V \times V : a_{qq'} \neq 0\}$, $a_{qq'}$, being the generic element of the graph adjacency matrix $\mathbf{A}$. In the following, we refer to the vertices as nodes, and the edges as links. Each node $q \in \{1, 2, \cdots, Q\}$ is associated to a value, $\lambda_q$, which must be averaged over the network, and a dynamical variable $x_q(k)$, also called state of the node, representing the estimate of the average values at the $k$th step. Thus, the discrete time average consensus is evaluated as

$$
\mathbf{x}(k+1) = \mathbf{x}(k) + \epsilon\mathbf{L}\mathbf{x}(k), \tag{21}
$$

where $\mathbf{x}(k) \triangleq [x_1(k), x_2(k), \cdots, x_Q(k)]^T$, $\epsilon$ represents the step size, and $\mathbf{L}$ is the Laplacian matrix associated to the graph whose generic element $L_{q,q'}$ is defined as

$$
L_{q,q'} \triangleq \begin{cases} \Delta_q & \text{if } q = q' \\ -1, & \text{if } q' \in \{\nu \in V : a_{q,\nu} \neq 0\} \\ 0 & \text{otherwise.} \end{cases} \tag{22}
$$

It is worth noting that the algorithm (21) represents an iterative gradient method solving the quadratic programming problem

$$
(\mathrm{P5}): \quad \begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2}\mathbf{x}^T\mathbf{L}\mathbf{x} \\ \text{subject to} & \sum_{q=1}^{Q} x_q = \sum_{q=1}^{Q} \lambda_q. \end{array}
$$

Thanks to this equivalent formulation, it can be demonstrated that the average consensus is reached under the condition $0 < \epsilon < \frac{1}{2\Delta_m}$, $\Delta_m$ being the maximum degree among all nodes. Thus, under the assumptions that the graph is strongly connected, the algorithm converges to the average of the initial state of all nodes

$$
\lambda = \sum_{q=1}^{Q} \lambda_q / Q. \tag{23}
$$

Eventually, from equations (19), (20) and (23), we yield

$$
\vartheta_q(k) = \left[x_q(k) + \frac{1}{Q} - \sum_{n=1}^{N} \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right)\right]^{+}, \tag{24}
$$

with

$$
x_q(0) = \lambda_q \triangleq \sum_{n=1}^{N} \log\left(\frac{\rho_{q,n}}{\tilde{\alpha}_{q,n}\Theta_{q,n}}\right). \tag{25}
$$

In [8], it is also demonstrated that, for the discrete-time algorithm described by eqn. (21), the consensus is globally exponentially reached with a minimum speed that is proportional to the algebraic connectivity of the graph, defined as the second eigenvalue of the Laplacian matrix. Finally, let us remark that the proposed distributed consensus algorithm tacitly assumes that each node is able of directly communicating only with its neighbors, so that it is more robust to imperfect information on the strategic environment or failures of some of the secondary nodes.

| Parameter | Value |
|---|---|
| Payload/CRC length | 1024/32 bits |
| Active subcarriers/FFT size | 1320/2048 |
| CP length | 160 samples |
| Bandwidth | 20 MHz |
| Modulation and Coding scheme | convolutional code, 4-QAM |
| Path-loss model | NLOS urban scenario@2GHz |
| Noise power level | -100 dBm |
| Short-term fading model | ITU Pedestrian B |

TABLE II
PARAMETERS AND FEATURES OF THE BIC OFDM SYSTEM.

## V. SIMULATION RESULTS AND CONCLUSION

Numerical simulations have been carried out to assess the performance of the proposed cooperative protocol over a typical wireless channel environment, with system parameters listed in Tab. II. The primary network is composed of 5 PUs (2 underlay, 3 interweave) whose geographical positions have been randomly decided. The secondary network is composed of 4 SUs uniformly distributed over a circle whose center is represented by the secondary receiver. The underlay PUs must have a minimum distance of 200m from the edge of the secondary network and the interference threshold is set to $-110$ dBm. Fig. 2 shows the PER performance improvement due to the proposed ESNR-based PA (EPA) strategy with respect to a uniform PA with step-ladder (SL) policy [11], for a TDMA case. The same figure also compares the PER curves obtained through the proposed cooperative strategy (C-EPA), for a secondary network organized in coalitions made of $Q = 2$ and 4 SUs, with the PER curve obtained in the TDMA case.

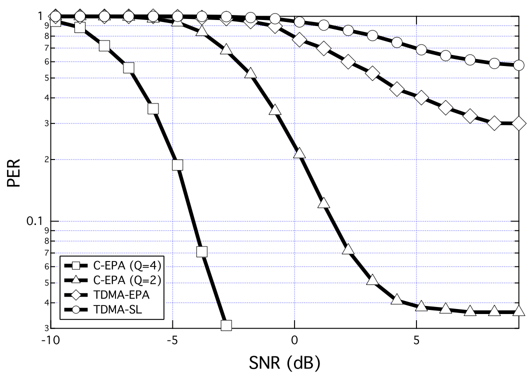


Fig. 2. PER comparison.

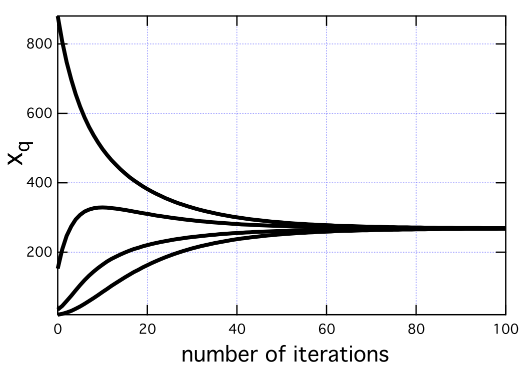


Fig. 3. Sample trajectories from an experiment with a 4-nodes networks.

The PER curves are obtained averaging over 1000 independent channel realizations. As apparent, since the proposed algorithm is capable of effectively exploiting the spatial diversity offered by the members of the coalition, the larger the coalition size the greater the performance improvement. Fig. 3 illustrates the behavior of the consensus algorithm through a states trajectory obtained for a single channel realization with a coalition of 4 SUs and $\epsilon = 0.1$, where, after the topology-formation phase is completed, the nodes begin the iterative computation with their neighbors. Some features of the PA scheme are listed below.

- The max-ESNR PA strategy takes into account the cooperative gain through a scalar value, i.e. the cooperative ESNR.
- Simulation results demonstrated that the proposed scheme remarkably boosts link performance over conventional TDMA.
- The proposed decentralized algorithm requires only local signaling between SUs.
- The proposed PA strategy improves the PER performance with respect to conventional PA strategies, thus allowing a better coverage for the secondary network.

*Future Works*. Since the consensus algorithm proposed here is useful only for those channels whose coherence time is sufficiently greater than the convergence time, a dynamic consensus algorithm is currently under investigation. In this case, the agents shall aim to track the average of the individual time-varying conditions in order to handle the dynamic nature of the CR scenario.

## REFERENCES


[1] S.T. Chung, and A.J. Goldsmith, *Degree of Freedom in Adaptive Modulation: A Unified View*, IEEE Tr. on Comm., vol.49, no.9, pp.1561-1571, Sept.2001.
[2] S. Haykin, *Cognitive Radio: Brain Empowered Wireless Communications*, IEEE JSAC, vol.23, no.2, pp.201-219, Feb.2005.
[3] K. B. Letaief and W. Zhang, *Cooperative communications for cognitive radio networks,* Proc. of the IEEE, vol.97, no.5, pp.878893, 2009.
[4] M. Varga, M.A. Badiu, V. Bota, *Combined Hybrid ARQ and Link Adaptation for Coded Cooperation in Block-Fading Channels*, JNCW 2011 - NEWCOM++/COST 2100 joint Workshop, pp.1-7, March 2011.
[5] M. Levorato, F. Librino, and M. Zorzi, *Integrated Cooperative Opportunistic Packet Forwarding and Distributed Error Control in MIMO Ad Hoc Networks*, IEEE Tr. on Comm., vol.59, no.8, pp.2215-2227, August 2011.
[6] I. Stupia, R. Andreotti, F. Giannetti, V. Lottici, L. Vandendorpe, *Adaptive Hybrid ARQ for Goodput Optimization in BIC-OFDM Systems*, Multi-Carrier Systems and Solutions 2011 (MC- SS 2011), pp.1-5, 2011.
[7] I. Stupia, V. Lottici, F. Giannetti, and L. Vandendorpe, *Link Resource Adaptation for Multiantenna Bit-Interleaved Coded Multicarrier Systems*, accepted to IEEE Tr. on Sig. Proc., 2012.
[8] R. Olfati-Saber, J.A. Fax, R.M. Murray, *Consensus and Cooperation in Networked Multi-Agent Systems*, Proc. of the IEEE, Vol.95, no.1, pp.215-233, Jan.2007.
[9] M.J. Osborne, A. Rubinstein, *A Course in Game Theory*, The MIT press,1994.
[10] I. Stupia, R. Andreotti, V. Lottici and L. Vandendorpe, *A Game Theoretical Approach for Coded Cooperation in Cognitive Radio Networks*, ISCCSP2012 (invited paper), pp.1-5, 2012.
[11] G. Bansal, M.J. Hossain, and V.K. Bhargava, *Optimal and Suboptimal Power Allocation Schemes for OFDM-based Cognitive Radio Systems*, IEEE Tr. on Wir. Comm., vol.7, no.11, pp.4710-4718, Nov. 2008.