Fusion of Greedy Pursuits for Compressed Sensing Signal Reconstruction

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***Abstract*—Greedy Pursuits are very popular in Compressed Sensing for sparse signal recovery. Though many of the Greedy Pursuits possess elegant theoretical guarantees for performance, it is well known that their performance depends on the statistical distribution of the non-zero elements in the sparse signal. In practice, the distribution of the sparse signal may not be known *a priori*. It is also observed that performance of Greedy Pursuits degrades as the number of available measurements decreases from a threshold value which is method dependent. To improve the performance in these situations, we introduce a novel fusion framework for Greedy Pursuits and also propose two algorithms for sparse recovery. Through Monte Carlo simulations we show that the proposed schemes improve sparse signal recovery in clean as well as noisy measurement cases.**

***Index Terms*—Compressed sensing, Sparse Recovery, Greedy Pursuits, Fusion**

I. Introduction

Compressed Sensing (CS) [1], [2] uses sparsity of the signal to reduce the number of measurements required to represent the signal. In general, reconstruction of the signal from the compressed measurements is NP-hard. In literature, a variety of suboptimal solutions in polynomial time have been proposed for this purpose. They can be broadly classified as Convex Relaxation methods [3], [4], Bayesian framework [5], [6], and Greedy Pursuits (GP) [7]–[9].

We consider only GP in this paper. GP iteratively estimate the support-set by selecting one or more atoms in each iteration, until a convergence criteria is met. In general, each iteration of GP consists of two steps: atom(s) selection step , and residual updating step. Popular examples of GP include Matching Pursuit (MP) [7], Orthogonal Matching Pursuit (OMP) [8], Subspace Pursuit (SP) [9], and Compressive Sampling Matching Pursuit (CoSaMP) [10].

$Notations$: Bold upper case and bold lower case Roman letters denote matrices and vectors respectively. Calligraphic letters and Upper case Greek alphabets are used to denote sets. $\|.\|_p$ represents the p^{th} -norm. $\mathbf{A}_{\mathcal{T}}$ denote the column sub-matrix of $\mathbf{A}$ formed by the columns of $\mathbf{A}$ listed in the set $\mathcal{T}$. $\mathbf{x}_{\mathcal{T}}$ denote the sub-vector formed by the elements of $\mathbf{x}$ whose indices are listed in the set $\mathcal{T}$. $\mathcal{T}^c$ denotes the complement of the set $\mathcal{T}$ w.r.t. the set $\{1, 2, \ldots, N\}$. For a set $\mathcal{T}$, $|\mathcal{T}|$ denotes its cardinality (size), and for a scalar c, $|c|$ denotes the magnitude of c. $\mathbf{A}^T$, and $\mathbf{A}^\dagger$ denote the transpose, and pseudo inverse of the matrix $\mathbf{A}$ respectively. $\mathbb{E}$ denotes the expectation operator which is approximated by the sample mean taken over a large number of trials.

Empirically, it has been observed that the recovery performance of the GP varies and depends on the nature of the sparse signal [11], [12]. For e.g., OMP may perform better than SP or *vice versa* for some types of signals. If the underlying statistical distribution of the non-zero values of the signal is known *a priori*, we can use the best recovery algorithm suitable for that type of signal. But in many practical situations, we may not have this prior knowledge and hence, we cannot use the best method as the *best* method is signal dependent.

It can be also seen that any greedy pursuit algorithm requires a minimum number of measurements, which is method dependent, for sparse signal recovery. Also, all GP perform poorly in very low dimension measurement regimes. But often many applications provide very less number of measurements and hence lower dimension measurement regimes are particularly important in reality.

To address these issues, we propose a novel fusion framework which fuses the information from two GP and estimates the correct atoms from the union of the support-sets of the two which we refer to as *Fusion of Greedy Pursuits* (FuGP). In this paper, we use OMP and SP as two examples of GP, but they can be replaced with any other GP.

II. CS framework and Greedy Pursuits

Consider the standard signal acquisition model which acquires a signal $\mathbf{x} \in \mathbb{R}^N$ via linear measurements using

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ represents a measurement (sensing) matrix, $\mathbf{b} \in \mathbb{R}^M$ represents the measurement vector, and $\mathbf{w} \in \mathbb{R}^M$ represents the additive measurement noise in the system. In CS framework, we have, $M \ll N$ and (1) is a well known ill-posed problem. But with the additional knowledge that the signal is K-sparse[1] ($K < M$), we can uniquely recover the signal under a few assumptions [1], [2].

OMP [8] selects one prominent atom which gives the maximum correlation value between the columns of $\mathbf{A}$ and

[1]A signal is said to be K-sparse if at most K of its elements are non-zeros.

the regularized measurement vector in every iteration. OMP finds support-set of a K-sparse signal in K iterations.

SP [9] first selects the prominent K columns of $\mathbf{A}$ from a matched filter output. In subsequent steps of the iterations, SP refines the initial solution by performing a test for subsets of K columns in a group, and maintains a list of K columns of $\mathbf{A}$. The refinement of the solution set continues as long as the l_2-norm of the residue decreases.

III. Fusion Framework for GP

We will start with an experiment which shows the motivation of the proposed fusion framework and its significance in sparse recovery.

Consider a CS system where the signal dimension is 500, and the sparsity level is 20. Using our notations, we have, $N = 500$ and $K = 20$. In this example, let us assume that the signal is a Gaussian sparse signal in a clean measurement setup (see Section IV for more details about the simulation setup). We consider two CS sparse recovery algorithms viz. OMP and SP for reconstruction of the signal. Let $\mathcal{T}$ denotes the actual support-set, and $\hat{\mathcal{T}}_{OMP}$ and $\hat{\mathcal{T}}_{SP}$ denote the support-sets estimated by OMP and SP respectively. Let $\hat{\mathcal{T}}_{OMP}^{true} = \mathcal{T} \cap \hat{\mathcal{T}}_{OMP}$ and $\hat{\mathcal{T}}_{SP}^{true} = \mathcal{T} \cap \hat{\mathcal{T}}_{SP}$ represent the sets of true atoms estimated by OMP and SP respectively. Then, we have, $|\mathcal{T}| = |\hat{\mathcal{T}}_{OMP}| = |\hat{\mathcal{T}}_{SP}| = K$, $0 \leq |\hat{\mathcal{T}}_{OMP}^{true}| \leq K$, and $0 \leq |\hat{\mathcal{T}}_{SP}^{true}| \leq K$.

Table I presents the average (over $10,000$ trials) number of true atoms in the estimated support-sets for Gaussian sparse signals for different values of α where α is defined as

$$\alpha = M/N. \tag{2}$$

α denote the fraction of measurements, also called the normalized measure of problem indeterminacy [12]. The details of the simulation are given in Section IV-B. It can be seen from Table I that for $\alpha = 0.12$, the average number of correctly estimated atoms by OMP is 8.1, and by SP is 10.5. Interestingly, the average number of correct atoms in the union of the support-sets estimated by OMP and SP is 12.4, closer to the true value 20. Also, using the property of union operator in set theory, it is guaranteed that the union of the estimated support-sets always contain at least as many true atoms as in the estimated support-set of the best performing algorithm.

$\alpha = M/N$	0.10	0.11	0.12	0.13	0.14
$Avg\lvert\hat{\mathcal{T}}_{OMP}^{true}\rvert$	5.6	6.7	8.1	10.1	12.6
$Avg\lvert\hat{\mathcal{T}}_{SP}^{true}\rvert$	5.8	7.9	10.5	13.2	15.6
$Avg\lvert\hat{\mathcal{T}}_{OMP}^{true} \cup \hat{\mathcal{T}}_{SP}^{true}\rvert$	7.9	9.9	12.4	15	17.1

TABLE I
AVERAGE NUMBER OF CORRECTLY ESTIMATED ATOMS BY OMP AND SP, FOR GAUSSIAN SPARSE SIGNAL, IN CLEAN MEASUREMENT CASE, AVERAGED OVER $10,000$ TRIALS ($N = 500$, $K = 20$).

These observations lead to the possibility of estimating more correct atoms from the union set than that individually estimated by OMP and SP algorithms. The exhaustive search among the atoms in the union set is $\binom{2K}{K}$ ($\binom{40}{20}$ in our experiment) in the worst case, which is significantly small as compared to the original where it is $\binom{N}{K}$ ($\binom{500}{20}$ in our experiment). But for larger values of K, $\binom{2K}{K}$ is still very large. Hence, by employing some efficient scheme to select K atoms from $2K$ atoms, we may be able to improve the number of correctly estimated atoms and improve the quality of the reconstructed sparse signal.

A. Proposed Fusion Framework

To develop the fusion framework using OMP and SP as ingredient algorithms, let us call the union of the estimated support-sets as *joint support-set* and denote by $\Gamma = \hat{\mathcal{T}}_{OMP} \cup \hat{\mathcal{T}}_{SP}$. Also, we call the intersection of the estimated support-sets as *common support-set* and denote by $\Lambda = \hat{\mathcal{T}}_{OMP} \cap \hat{\mathcal{T}}_{SP}$. We have, $|\hat{\mathcal{T}}_{OMP}| = |\hat{\mathcal{T}}_{SP}| = K$, $0 \leq |\Lambda| \leq K$ and $K \leq |\Gamma| \leq 2K$. In the fusion framework, our task is to pick K atoms from the joint support-set with $|\Gamma|$ atoms. Assuming $M \geq 2K$, we propose a least-squares based method in FuGP for this purpose.

Now, let us define two algorithmic functions which will be used in the proposed sparse recovery algorithms.

Definition. *Let $\mathbf{A} \in \mathbb{R}^{M \times N}$, $\mathbf{b} \in \mathbb{R}^{M \times 1}$ and K be the sparsity level. Also let $\mathcal{T}_{init}$ denote the initial support-set with $|\mathcal{T}_{init}| < K$. Then we define the following algorithmic functions.*

$$\hat{\mathcal{T}}_{OMP} = OMP(\mathbf{A}, \mathbf{b}, K, \mathcal{T}_{init})$$
$$\hat{\mathcal{T}}_{SP} = SP(\mathbf{A}, \mathbf{b}, K, \mathcal{T}_{init})$$

where $|\hat{\mathcal{T}}_{OMP}| = |\hat{\mathcal{T}}_{SP}| = K$.

The functions "*OMP*" and "*SP*" execute Algorithms 1 and 2 respectively. Note that by putting $\mathcal{T}_{init} = \varnothing$ in Algorithms 1 and 2, we get classic OMP and SP respectively.

Algorithm 1 OMP with Initial Support

Inputs: $\mathbf{A}_{M \times N}$, $\mathbf{b}_{M \times 1}$, K, and $\mathcal{T}_{init}$
Ensure: $|\mathcal{T}_{init}| \leq K - 1$
1: $k = |\mathcal{T}_{init}|$;
2: $\mathbf{r}_k = \mathbf{b} - \mathbf{A}_{T_{init}} \mathbf{A}_{T_{init}}^{\dagger} \mathbf{b}$;
3: $\hat{\mathcal{T}}_k = \mathcal{T}_{init}$;
4: **repeat**
5: $\quad k = k + 1$;
6: $\quad i_k = \underset{i=1:N, i \notin \hat{\mathcal{T}}_{k-1}}{\arg\max} \ \mathbf{a}_i^T \mathbf{r}_{k-1}$;
7: $\quad \hat{\mathcal{T}}_k = i_k \cup \hat{\mathcal{T}}_{k-1}$;
8: $\quad \mathbf{r}_k = \mathbf{b} - \mathbf{A}_{\hat{\mathcal{T}}_k} \mathbf{A}_{\hat{\mathcal{T}}_k}^{\dagger} \mathbf{b}$;
9: **until** $(k \geq K)$;
10: $\hat{\mathcal{T}} = \hat{\mathcal{T}}_k$;
Outputs: $\hat{\mathcal{T}}$, $\mathbf{x}_{\hat{\mathcal{T}}} = \mathbf{A}_{\hat{\mathcal{T}}}^{\dagger} \mathbf{b}$, and $\mathbf{x}_{\hat{\mathcal{T}}^c} = \mathbf{0}$.

Since both methods (OMP and SP) agree on the atoms selected in Λ, we give more confidence on these atoms as compared to any other atom in Γ. Hence FuGP includes these

Algorithm 2 SP with Initial Support

Inputs: $\mathbf{A}_{M\times N}$, $\mathbf{b}_{M\times 1}$, K, and $\mathcal{T}_{init}$

Ensure: $|\mathcal{T}_{init}| \leq K-1$

$k = 0$;
$\mathbf{r}_k = \mathbf{b} - \mathbf{A}_{T_{init}}\mathbf{A}^{\dagger}_{T_{init}}\mathbf{b}$;
$\hat{\mathcal{T}}_k = \mathcal{T}_{init}$;
repeat
  $k = k+1$;
  J = indices of the K highest amplitude components of $\mathbf{A}^T\mathbf{r}_{k-1}$;
  $\tilde{\mathcal{T}} = J \cup \hat{\mathcal{T}}_{k-1}$;
  $\mathbf{v}_{\tilde{\mathcal{T}}} = \mathbf{A}^{\dagger}_{\tilde{\mathcal{T}}}\mathbf{b}$, $\mathbf{v}_{\tilde{\mathcal{T}}^c} = \mathbf{0}$;
  $\hat{\mathcal{T}}_k$ = indices corresponding to the K largest magnitude entries in $\mathbf{v}$;
  $\mathbf{r}_k = \mathbf{b} - \mathbf{A}_{\hat{\mathcal{T}}_k}\mathbf{A}^{\dagger}_{\hat{\mathcal{T}}_k}\mathbf{b}$;
until $(\|\mathbf{r}_k\|_2 \geq \|\mathbf{r}_{k-1}\|_2)$;
$\hat{\mathcal{T}} = \hat{\mathcal{T}}_{k-1}$;

Outputs: $\hat{\mathcal{T}}$, $\mathbf{x}_{\hat{\mathcal{T}}} = \mathbf{A}^{\dagger}_{\hat{\mathcal{T}}}\mathbf{b}$, and $\mathbf{x}_{\hat{\mathcal{T}}^c} = \mathbf{0}$.

atoms in the solution set. Now, we need to identify only $K-|\Lambda|$ atoms from the remaining $|\Gamma|-|\Lambda|$ atoms. Applying least-squares on the atoms in Γ, we form an intermediary solution for the signal. The remaining $K-|\Lambda|$ support atoms, we denote by $\tilde{\mathcal{T}}$, are estimated as the $K-|\Lambda|$ indices corresponding to the largest magnitudes in $\mathbf{v}$ which are not in Λ. Now, the support-set is estimated as the union of the atoms in the sets $\tilde{\mathcal{T}}$ and Λ. Finally, the sparse signal estimate is found from the estimated support-set using the least-squares. The main steps of the FuGP algorithm for *fusing* estimated support-sets of OMP and SP are summarized in Algorithm 3.

Algorithm 3 *FuGP* Algorithm

Inputs: $\mathbf{A}_{M\times N}$, $\mathbf{b}_{M\times 1}$, and K

$\hat{\mathcal{T}}_{OMP} = OMP(\mathbf{A},\mathbf{b},K,\varnothing)$; ▶ Using Algorithm 1
$\hat{\mathcal{T}}_{SP} = SP(\mathbf{A},\mathbf{b},K,\varnothing)$; ▶ Using Algorithm 2
$\Lambda = \hat{\mathcal{T}}_{OMP} \cap \hat{\mathcal{T}}_{SP}$; ▶ $(0 \leq |\Lambda| \leq K)$
$\Gamma = \hat{\mathcal{T}}_{OMP} \cup \hat{\mathcal{T}}_{SP}$; ▶ $(K \leq |\Gamma| \leq 2K)$
$\mathbf{v}_{\Gamma} = \mathbf{A}^{\dagger}_{\Gamma}\mathbf{b}$, $\mathbf{v}_{\Gamma^c} = \mathbf{0}$; ▶ intermediary signal estimate
$\tilde{\mathcal{T}}$ = indices corresponding to the $(K-|\Lambda|)$ largest magnitude entries in $\mathbf{v}$ which are not in Λ;
$\hat{\mathcal{T}} = \tilde{\mathcal{T}} \cup \Lambda$;

Output: $\hat{\mathcal{T}}$, $\mathbf{x}_{\hat{\mathcal{T}}} = \mathbf{A}^{\dagger}_{\hat{\mathcal{T}}}\mathbf{b}$, and $\mathbf{x}_{\hat{\mathcal{T}}^c} = \mathbf{0}$.

The computational demand of FuGP algorithm is a little more than the added computational cost of the two underlying methods. For e.g., the computational complexity of both OMP and SP are $\mathcal{O}(MNK)$ independently. The computational complexity of FuGP in this case remains $\mathcal{O}(MNK)$. To save the running time, we can run both SP and OMP in parallel and then apply FuGP on the estimated support sets. It may be also observed that the memory requirement of core part of FuGP (steps 3-6 in Algorithm 3) is only $\mathcal{O}(N)$.

Remarks:

- The fusion framework and FuGP algorithm are scalable and can be easily extended to accommodate more than two greedy pursuit algorithms.
- The performance of FuGP directly depends on the number of correct atoms in the joint support-set. Hence, we should choose the underlying algorithms such that join support-set contains maximum number of correct atoms.

Iterative Fusion: Proposed FuGP aims to identify the correct atoms present the joint support-set. Hence it can at most identify all the correct atoms in the joint support-set. But in lower dimensional measurement regimes, the joint support-set may not contain all correct atoms and FuGP will surely miss the correct atoms which are not included in the joint support-set. To address this issue and hence improve the performance, we propose an iterative version of the fusion algorithm called *Iterative Fusion of Greedy Pursuits* (IFuGP).

In k^{th} iteration, we find the common support-set Λ_k and call OMP and SP with Λ_k as the initial support-set to identify the remaining atoms in the support. Λ_0 is initialized as null set. Λ_k is updated as the common atoms in the newly estimated support-sets by OMP and SP. We fuse the support-sets newly estimated by OMP and SP to find the new estimate of the support-set. The iteration continues as long as the l_2-norm of the residue decreases. This procedure opens a possibility to include more correct atoms in the joint support-set Γ_k which are not in the previous iteration $k-1$. It may observed that we internally call FuGP in each iteration.

IFuGP algorithm is explained in Algorithm 4. Observe that Steps 6-13 in Algorithm 4 essentially forms the FuGP algorithm. IFuGP is computationally more demanding than FuGP.

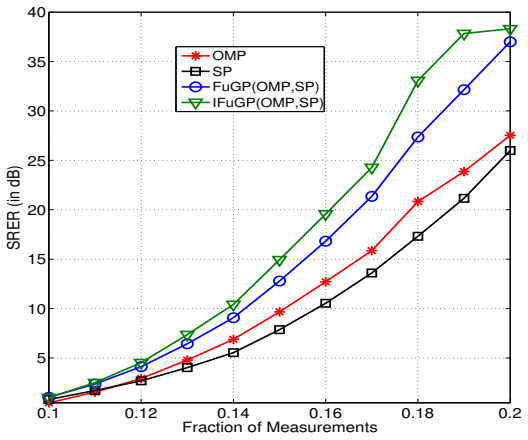
Algorithm 4 *IFuGP* Algorithm

Inputs: $\mathbf{A}_{M\times N}$, $\mathbf{b}_{M\times 1}$, and K

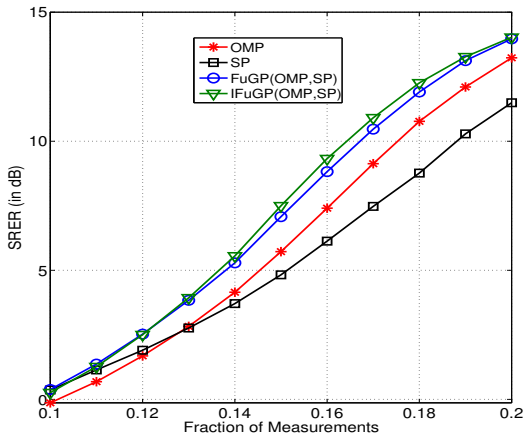
$k = 0$;
$\mathbf{r}_0 = \mathbf{b}$;
$\Lambda_0 = \varnothing$;
repeat
  $k = k+1$;
  $\hat{\mathcal{T}}_{OMP} = OMP(\mathbf{A},\mathbf{b},K,\Lambda_{k-1})$; ▶ Using Algorithm 1
  $\hat{\mathcal{T}}_{SP} = SP(\mathbf{A},\mathbf{b},K,\Lambda_{k-1})$; ▶ Using Algorithm 2
  $\Lambda_k = \mathcal{T}_{OMP} \cap \mathcal{T}_{SP}$; ▶ $0 \leq |\Lambda_k| \leq K$
  $\Gamma_k = \mathcal{T}_{OMP} \cup \mathcal{T}_{SP}$; ▶ $K \leq |\Gamma_k| \leq 2K$
  $\mathbf{v}_{\Gamma_k} = \mathbf{A}^{\dagger}_{\Gamma_k}\mathbf{b}$, $\mathbf{v}_{\Gamma_k^c} = \mathbf{0}$; ▶ intermediary signal estimate
  $\tilde{\mathcal{T}}_k$ = indices corresponding to the $(K-|\Lambda_k|)$ largest magnitude entries in $\mathbf{v}$ which are not in Λ_k;
  $\hat{\mathcal{T}}_k = \tilde{\mathcal{T}}_k \cup \Lambda_k$; ▶ $|\hat{\mathcal{T}}_k| = K$
  $\mathbf{r}_k = \mathbf{b} - \mathbf{A}_{\hat{\mathcal{T}}_k}\mathbf{A}^{\dagger}_{\hat{\mathcal{T}}_k}\mathbf{b}$;
until $(\|\mathbf{r}_k\|_2 \geq \|\mathbf{r}_{k-1}\|_2)$;
$\hat{\mathcal{T}} = \hat{\mathcal{T}}_{k-1}$;

Output: $\hat{\mathcal{T}}$, $\mathbf{x}_{\hat{\mathcal{T}}} = \mathbf{A}^{\dagger}_{\hat{\mathcal{T}}}\mathbf{b}$, and $\mathbf{x}_{\hat{\mathcal{T}}^c} = \mathbf{0}$.

In this paper, for notational brevity, we denote FuGP(OMP,SP) and IFuGP(OMP,SP) by FuGP and IFuGP



(a) Clean Measurement



(b) Noisy Measurement (*SMNR =15 dB*)

Fig. 1. Gaussian sparse signals: Signal-to-Reconstruction-Error Ratio (SRER) vs. Fraction of Measurements ($N = 500$, $K = 20$)

respectively.

IV. Simulation and Results

We did extensive Monte Carlo simulations to evaluate performance of the proposed methods. We explain the simulation setup and define the performance measure used for comparing different methods, in this section.

A. Performance Measure

Signal-to-Reconstruction-Error Ratio (SRER): Let $\mathbf{x}$ and $\hat{\mathbf{x}}$ denote the original and reconstructed sparse signal vector. SRER is a performance measure build on top of Mean Square Error information. SRER (in dB) is defined as

$$\textit{SRER (in dB)} \triangleq 10\log_{10}\frac{\mathbb{E}\|\mathbf{x}\|_2^2}{\mathbb{E}\|\mathbf{x}-\hat{\mathbf{x}}\|_2^2} \tag{3}$$

Signal to Measurement-Noise Ratio (SMNR):: Let σ_s^2 and σ_n^2 denote the power of each element of signal and noise vector respectively. For noisy measurement simulations, we define SMNR in dB as

$$\textit{SMNR (in dB)} = 10\log_{10}\frac{\mathbb{E}\|\mathbf{x}\|_2^2}{\mathbb{E}\|\mathbf{w}\|_2^2} = 10\log_{10}\frac{K\sigma_s^2}{M\sigma_n^2}. \tag{4}$$

B. Experimental Setup

Many of the GP including OMP and SP provide theoretical guarantees for convergence, but the theoretical bounds are by and large "pessimistic" worst case bounds. In general, all CS sparse recovery methods work efficiently near this region. But in many applications, the number of measurements may be very limited due to many practical reasons. Hence we are more interested in the lower dimensional measurement regimes where the sparse recovery methods begin to collapse. To compare the performance of the proposed methods with OMP and SP in this highly under-sampled region, we choose small values of α where α is defined in (2).

The main steps involved in the simulation are the following:

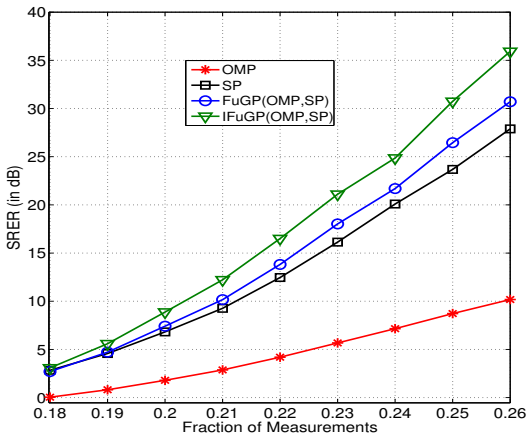
1) Fix K, N and choose an α so that the number of measurements M is an integer.
2) Generate elements of $\mathbf{A}_{M\times N}$ independently from $\mathcal{N}(0,\frac{1}{M})$ and normalize each column norm to unity.
3) Choose K locations uniformly over the set $\{1,2\ldots\mathrm{N}\}$ and fill these locations of $\mathbf{x}$ based on the choice of signal characteristics:
   a) *Gaussian sparse signal:* non-zero values independently from $\mathcal{N}(0,1)$.
   b) *Rademacher sparse signal:* non-zero values are set to +1 or -1 with probability $\frac{1}{2}$. They are also known as "constant amplitude random sign" signals.

   Set remaining $N-K$ locations of $\mathbf{x}$ as zeros.
4) For noisy regime, the additive noise $\mathbf{w}$ is a Gaussian random vector whose elements are independently chosen from $\mathcal{N}(0,\sigma_w^2)$ and for clean case, $\mathbf{w}$ is set to zero.
5) The measurement vector $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w}$.
6) Apply the reconstruction methods independently.
7) Repeat steps 3-6 T times. T indicates the number of times $\mathbf{x}$ independently generated, for a fixed $\mathbf{A}$.
8) Repeat steps 2-7 S times. S indicates the number of times $\mathbf{A}$ is independently generated.
9) Calculate SRER by averaging over $S \times T$ data
10) Repeat steps 2-9 for different values of α.

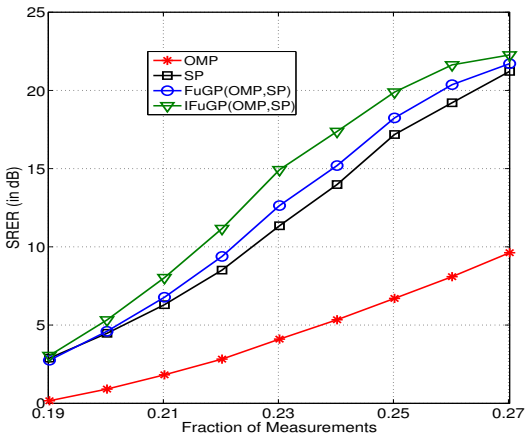
C. Results and Discussions

We performed Monte Carlo simulation with following parameters. $N = 500$, $K = 20$, $S = 100$, $T = 100$. i.e., we generated the measurement matrix $\mathbf{A}$ 100 times and for each realization of $\mathbf{A}$, we generated a sparse signal with ambient dimension 500 and sparsity level $K = 20$, 100 times.

Gaussian Sparse Signal: The performance of FuGP and IFuGP with OMP and SP as ingredient methods for Gaussian sparse signals in clean as well as noisy measurement cases are shown in Fig. 1. FuGP showed a significant improvement as compared to the ingredient methods OMP and SP in both cases. IFuGP was able to give an improvement over FuGP. For example, in Fig. 1(a), for $\alpha = 0.18$, FuGP gave 6.5 dB (31%) and 10 dB (58%) SRER improvement respectively over OMP and SP in clean measurement case. For the same scenario,



(a) Clean Measurement



(b) Noisy Measurement (*SMNR = 15 dB*)

Fig. 2. Rademacher sparse signals: Signal-to-Reconstruction-Error Ratio (SRER) vs. Fraction of Measurements ($N = 500$, $K = 20$)

IFuGP(OMP, SP) was able to improve the performance further and showed 12 dB (59%), and 16 dB (91%) improvement respectively over OMP and SP. Compared to FuGP, IFuGP gave 5.72 dB (21%) additional performance improvement in SRER.

In the noisy measurement case (refer Fig. 1(b)), for $\alpha = 0.18$, FuGP improved the performance by 1.1 dB (11%) and 3.1 dB (36%) over OMP and SP respectively. For the same situation, IFuGP gave 1.5 dB (13%), and 3.5 dB (40%) additional SRER as compared to OMP and SP respectively, and also showed 0.35 dB (3%) SRER improvement over FuGP.

Rademacher Sparse Signal: The results of simulation for Rademacher sparse signal is shown in Fig. 2. Here also FuGP and IFuGP showed performance improvement over OMP and SP. In the clean measurement case (refer Fig. 2(a)), for $\alpha = 0.25$, FuGP gave 18 dB (203%) and 2.8 dB (12%) SRER improvement that OMP and SP respectively. IFuGP further improved the performance and gave SRER improvement by 22dB (252%) and 7 dB (30%) respectively over OMP and SP. In this case IFuGP showed 4.3 dB (16%) SRER improvement over FuGP.

In the noisy measurement simulation (refer Fig. 2(b)), for $\alpha = 0.25$, by employing FuGP, an additional SRER improvement of 11.5 dB (172%) and 1.1 dB (6%) was achieved as compared to OMP and SP respectively. In this case also, IFuGP continued to improve the performance over FuGP (1.7 dB (9%) SRER improvement than FuGP) resulting in 13.2 dB (196%) and 2.7 dB (16%) SRER improvement over OMP and SP respectively.

From the simulation results, it can be seen that FuGP and IFuGP improved the sparse signal recovery consistently in all the cases as compared to the ingredient methods. The robustness against noise was shown in noisy measurement simulations for an SMNR = 15 dB, which closely matches many application scenarios.

Reproducible Results: In the spirit of reproducible research, we provide necessary Matlab codes downloadable in the following website: `http://www.ece.iisc.ernet.in/~ssplab/Public/FuGP.tar.gz`. The code reproduces the simulation results shown in Fig. 1, and Fig. 2.

V. Conclusions

We proposed a novel fusion framework for Greedy Pursuits and also proposed two algorithms to recover the sparse signals. Using simulations we showed that the proposed scheme can improve the sparse signal recovery performance of Greedy Pursuits in clean as well as noisy measurement cases.

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