A REALIZATION OF FIR FILTERS WITH SIMULTANEOUSLY VARIABLE BANDWIDTH AND FRACTIONAL DELAY

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ABSTRACT

This paper introduces a realization of finite-length impulse response (FIR) filters with simultaneously variable bandwidth and fractional delay (FD). The realization makes use of impulse responses which are two-dimensional polynomials in the bandwidth and FD parameters. Unlike previous polynomial-based realizations, it utilizes the fact that a variable FD filter is typically much less complex than a variable-bandwidth filter. By separating the corresponding subfilters in the overall realization, significant savings are thereby achieved. A design example, included in the paper, shows about 65 percent multiplication and addition savings compared to the previous polynomial-based realizations. Moreover, compared to a recently introduced alternative fast filter bank approach, the proposed method offers significantly smaller group delays and group delay errors.

1. INTRODUCTION

Variable digital filters are gaining interest due to the increasing demand for reconfigurable systems required in, for example, emerging communication systems to support several different standards and operation modes [1–3]. This paper deals with digital finite-length impulse response (FIR) filters which have both a variable bandwidth (VBW) and a variable fractional delay (VFD) [4,5]. Specifically, impulse responses that are two-dimensional polynomials in the bandwidth and fractional delay (FD) are considered. Filters based on polynomial impulse responses offer offline design and simple online update when switching from one mode to another. This is done by utilizing the Farrow structure and its generalizations which make use of a number of fixed FIR subfilters whose outputs are fed to variable multipliers. However, the overall computational complexity of such filters may be prohibitive.

To reduce the complexity, this paper introduces a new realization as an alternative to those in [4, 5]. The proposed realization utilizes the fact that a Farrow-structure-based VFD filter, in many cases, is much less complex than a VBW filter. By separating the corresponding subfilters in the overall realization, significant savings are therefore feasible. A design example, included in the paper, shows some 65% savings compared to the previous polynomial-based realizations. Another design example shows that the proposed approach offers significantly smaller group delays and group delay errors as compared to a recently introduced alternative [6].

Following this introduction, Section II states the problem under consideration and reviews the previous realizations. Section III introduces the proposed realization. Section IV discusses the design whereas Section V provides a design example. Finally, Section VI concludes the paper.

2. PROBLEM FORMULATION AND REVIEW OF PREVIOUS REALIZATIONS

The desired frequency function $D_H(j\omega T, b, d)$ is

$$D_H(j\omega T, b, d) = \begin{cases} e^{-j\omega T(N_H/2+d)}, & \omega T \in [0,\, b-\Delta] \\ 0, & \omega T \in [b+\Delta,\, \pi] \end{cases} \tag{1}$$

for $b \in [b_l,\, b_u]$ and $d \in [-1/2,\, 1/2]$. For each pair of values, d and b, $D_H(j\omega T, b, d)$ corresponds to a lowpass filter with passband and stopband edges at $b-\Delta$ and $b+\Delta$, respectively. The parameter b thus represents the center of the transition band. In the passband, an FD of $N_H/2 + d$ is desired.

The problem is now to form an FIR filter $H(z, b, d)$ for which $H(e^{j\omega T}, b, d)$ approximates $D_H(j\omega T, b, d)$. A simple way to achieve this is to realize $H(z, b, d)$ of order $N_H = N_F + N_G$ in a cascade form as

$$H(z, b, d) = F(z, b)G(z, d) \tag{2}$$

where

$$F(z, b) = \sum_{p=0}^{P} b^p F_p(z) \tag{3}$$

and

$$G(z, d) = \sum_{k=0}^{K} d^k G_k(z). \tag{4}$$

Here, the filter $F(z, b)$ is a VBW filter with $F(e^{j\omega T}, b)$ approximating $D_F(j\omega T, b)$ given by

$$D_F(j\omega T, b) = \begin{cases} e^{-j\omega T N_F/2}, & \omega T \in [0,\, b-\Delta] \\ 0, & \omega T \in [b+\Delta,\, \pi] \end{cases} \tag{5}$$

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for all $b \in [b_l, b_u]$ whereas $G(z, d)$ is a VFD filter with $G(e^{j\omega T}, d)$ approximating $D_G(j\omega T, d)$ given by

$$D_G(j\omega T, d) = e^{-j\omega T(N_G/2+d)}, \ \omega T \in [0, \ b_u - \Delta] \qquad (6)$$

for all $d \in [-1/2, \ 1/2]$. The subfilters $F_p(z)$ and $G_k(z)$ are fixed, and to reduce the complexity, they are here chosen to be linear-phase FIR filters. The subfilters $F_p(z)$ are then Type I linear-phase filters, thus of even order N_F and with symmetric impulse responses $f_p(n) = f_p(N_F - n)$. As to the subfilters $G_k(z)$, there are two options. When $G_k(z)$ are of even order N_G, they are of Type I (Type III) for even (odd) values of k. In the odd-order case, they are instead of Type II (Type IV) for even (odd) values of k. In both cases, the impulse responses are symmetric (anti-symmetric) for even (odd) values of k, thus $g_k(n) = (-1)^k g_k(N_G - n)$. In this short paper, only the even-order case is considered.

A drawback of using this cascade form is that transients will distort the signal whenever b or d (or both) is altered, depending on the order in which the filtering operations take place. To avoid transients, all variable parameters must be located after the delay elements, thus never at the input of a subfilter. Structures that avoid transients can be derived by expanding the product $F(z, b)G(z, d)$ to place all variable parameters at the output of the resulting subfilters.

A straightforward expansion of $F(z, b)G(z, d)$ gives

$$H(z, b, d) = \sum_{p=0}^{P} \sum_{k=0}^{K} E_{pk}(z) b^p d^k \qquad (7)$$

with

$$E_{pk}(z) = F_p(z)G_k(z). \qquad (8)$$

The corresponding realization is a linear combination of the $(P+1)(K+1)$ subfilters $E_{pk}(z)$ weighted with the variable multipliers $b^p d^k$. Besides $(P+1)(K+1) - 1$ actual variable multipliers $b^p d^k$, we need $PK + P + K - 2$ multiplications for computing b^p and d^k as well as all their product combinations. The computation of these values can be avoided by first rewriting $H(z, b, d)$ as [5]

$$H(z, b, d) = \sum_{p=0}^{P} b^p C_p(z, d) \qquad (9)$$

with

$$C_p(z, d) = \sum_{k=0}^{K} d^k E_{pk}(z), \qquad (10)$$

and then utilizing Farrow structures with interconnected variable multiplier cascades [similar to the proposed realization as seen later in Figs. 1 and 2].

In both of the cases above, considered in [4, 5], respectively, the subfilters $E_{pk}(z)$ are not realized as $E_{pk}(z) = F_p(z)G_k(z)$. Instead, $E_{pk}(z)$ are free linear-phase filters of different appropriate types, which can be determined by the type of $F_p(z)G_k(z)$. An advantage of using free subfilters $E_{pk}(z)$ is that the design of the overall filter $H(z, b, d)$ then amounts to solving a convex problem. A drawback, however, is that all subfilters $E_{pk}(z)$ then have the same order (at least approximately), namely $N_H = N_F + N_G$, where N_F and N_G are the orders required for $F(e^{j\omega T}, b)$ and $G(e^{j\omega T}, d)$ to approximate $D_F(e^{j\omega T}, b)$ and $D_G(e^{j\omega T}, d)$ in (5) and (6), respectively, with roughly the same approximation error as the overall approximation error. This leads to an unnecessarily high complexity as $(P+1)(K+1)$ subfilters of order N_H are then required. To reduce the complexity, one should therefore realize the overall filter in terms of subfilter cascades $F_p(z)G_k(z)$ because the order N_G of $G_k(z)$ is typically much lower than the order N_F of $F_p(z)$, at least for narrow- to mid-band lowpass filters. Most of the subfilters in the overall realization can then have a low order, N_G, giving a substantial reduction of the overall complexity. This is the rationale behind our proposed structure.

3. PROPOSED STRUCTURE

The proposed structure is seen in Fig. 1 with $G(z, d)$ being realized as in Fig. 2. The overall transfer function is

$$H(z, b, d) = \sum_{p=0}^{P} F_p(z)G(z, d)(b - b_0)^p \qquad (11)$$

where $G(z, d)$ is given by (4). There are mainly two features that distinguish the proposed structure from the previous structures discussed earlier. Firstly, all fixed subfilters are realized in cascade form $F_p(z)G_k(z)$. Secondly, instead of b^p, the terms $(b - b_0)^p$ are used where $b_0 = (b_l + b_u)/2$. This corresponds to a VBW filter $F(z, b)$ according to

$$F(z, b) = \sum_{p=0}^{P} (b - b_0)^p F_p(z). \qquad (12)$$

The rationale behind the use of $b - b_0$, instead of b, was discussed in [7]. One advantage is that it enables us to find the minimum order required for $F_p(z)$ in a simple manner, as discussed later in Section 4.

3.1. Implementation Complexity

For the fixed subfilters $F_p(z)$ and $G_k(z)$, the overall numbers of required multiplications and additions are

$$C_m = (P+1)\left([N_F/2 + 1 + K[N_G/2 + 1] - \lceil K/2 \rceil\right) \qquad (13)$$

and

$$C_a = (P+1)\left(N_F + KN_G - \lceil K/2 \rceil\right), \qquad (14)$$

respectively. Further, the number of delay elements is

$$C_d = N_F + (P+1)N_G, \qquad (15)$$

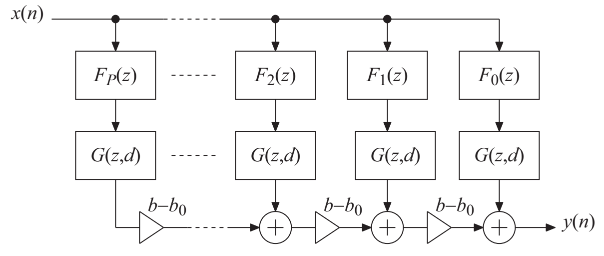

Fig. 1. Proposed structure with $G(z, d)$ realized as shown in Fig. 2.

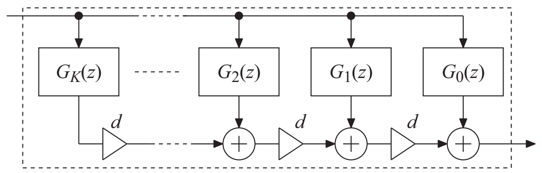

Fig. 2. Realization of $G(z, d)$ used in Fig. 1.

provided they are appropriately shared between the subfilters. Moreover, the variable parts need $P + (P + 1)K = (P + 1)(K+1)-1$ additional general multiplications and additions.

4. FILTER DESIGN

In this section, we outline the design for the case where the maximum of the modulus of a complex error function is minimized. Other types of errors and design criteria like least-squares [8], can naturally be considered as well after some minor appropriate modifications.

Specifically, the maximum of the modulus of the error function $E(j\omega T, b, d)$, given by

$$E(j\omega T, b, d) = H(e^{j\omega T}, b, d) - D_H(j\omega T, b, d), \quad (16)$$

is minimized over $\omega T \in \Omega(b)$, $b \in [b_l, b_u]$, and $d \in [-1/2, 1/2]$, where $H(e^{j\omega T}, b, d)$ is given by (11) with $z = e^{j\omega T}$. Further, $D_H(j\omega T, b, d)$ is the desired function given by (1), whereas $\Omega(b)$ is the union of the passband and stopband regions according to

$$\Omega(b) = [0,\ b - \Delta] \cup [b + \Delta,\ \pi]. \quad (17)$$

For given values of the subfilter orders, say N_F and N_G, as well as the number of subfilters, $P+1$ and $K+1$, the overall filter $H(z, b, d)$ is designed by solving the following approximation problem:

Approximation Problem: Find the unknowns $f_p(n) = f_p(N_F - n)$ for $n = 0, 1, \ldots, N_F/2$ and $p = 0, 1, \ldots, P$, $g_k(n) = (-1)^k g_k(N_G - n)$ for $n = 0, 1, \ldots, N_G/2$ and $k = 1, 2, \ldots, K$, as well as δ, to minimize δ subject to

$$|E(j\omega T, b, d)| \le \delta. \quad (18)$$

In this short paper, the order N_G is assumed to be even and thus the subfilters $G_k(z)$ are of Type I and Type III for even and odd values of k, respectively. This means that $G_0(z) = z^{-N_G/2}$, i.e., a pure delay, and that the center taps $g_k(N_G/2)$ are zero for odd values of k. The overall number of filter parameters, to be optimized, is consequently $(P+1)(N_F/2+1) + K(N_G/2+1) - \lceil K/2 \rceil$.

This optimization problem is nonlinear because of the use of cascaded subfilters. It is therefore not possible to guarantee a globally optimum solution. In addition, without a reasonably good starting point, the optimization tends to be slow and may end up in a poor local optimum. It is therefore beneficial to find a good initial solution to the optimization. Furthermore, we need to determine the values required for P and K, as well as the values of N_F and N_G, so that the overall complexity is minimized. Taking the above aspects into account, given also a targeted approximation error δ_e, the overall filter $H(z, b, d)$ is designed in four steps as follows:

1. Estimate the required filter orders N_F and N_G, say $\widehat{N}_F$ and $\widehat{N}_G$.

2. Determine the values of P and K.

3. For each combination of the filter orders N_F and N_G around the estimated orders $\widehat{N}_F$ and $\widehat{N}_G$:

(a) Design a VBW filter $F(z, b)$ in the form of (12) by minimizing δ_F subject to $|E_F(j\omega T, b)| \le \delta_F$ where $E_F(j\omega T, b) = F(e^{j\omega T}, b) - D_F(j\omega T, b)$ over all $b \in [b_l, b_u]$ and with $D_F(j\omega T, b)$ as is (5). This gives the impulse response values $f_p(n) = f_p(N_F - n)$, $n = 0, 1, \ldots, N_F/2$, $p = 0, 1, \ldots, P$, of $F_p(z)$ in (12).

(b) Design a VFD filter $G(z, b)$ in the form of (4) by minimizing δ_G subject to $|E_G(j\omega T, d)| \le \delta_G$ where $E_G(j\omega T, d) = G(e^{j\omega T}, d) - D_G(j\omega T, d)$ over all $d \in [-1/2, 1/2]$ and with $D_G(j\omega T, d)$ as in (6). This gives the impulse response values $g_k(n) = (-1)^k g_k(N_G - n)$, $n = 0, 1, \ldots, N_G/2$, $k = 1, 2, \ldots, K$, of $G_k(z)$ in (4). (Recall that $G_0(z) = z^{-N_G/2}$ is adopted here and thus needs not be determined in the optimization.)

(c) Use $f_p(n)$ and $g_k(n)$ obtained above as the initial impulse response values in a further nonlinear optimization routine that solves the Approximation Problem stated above in this section. If $\delta \le \delta_e$, store the result.

4. Among all solutions stored in Step 3(c), select the one with lowest complexity. If several solutions have the same complexity, select the one with lowest delay.

In Step 1, the subfilter orders N_F and N_G are estimated as

$$\widehat{N}_F = -\frac{2\pi \log_{10}(10\delta_e^2)}{3\Delta}, \quad \widehat{N}_G = -\frac{4\pi \log_{10}(10\delta_e^2)}{3(\pi - b_u + \Delta)} \quad (19)$$

rounded to the nearest even integer. These are based on the order estimation formula in [9] using passband and stopband ripples equal to δ_e together with appropriate transition bandwidths as explained below.

Firstly, δ_e emanates from a worst-case analysis (ignoring higher-order terms), and the fact that joint optimization typically suppresses the overall approximation error by a factor of two. Specifically, if $\left|F(e^{j\omega T}, b) - D_F(j\omega T, b)\right| \leq \delta_F$, $\left|G(e^{j\omega T}, d) - D_G(j\omega T, b)\right| \leq \delta_G$, and $N_H = N_F + N_G$, it follows that $\left|F(e^{j\omega T}, b)G(e^{j\omega T}, d) - D_H(j\omega T, b, d)\right| \lessapprox \delta_F + \delta_G$ where $D_H(j\omega T, b, d) = D_F(j\omega T, b)D_G(j\omega T, d)$ according to (1), (5), and (6). This follows by using the triangle inequality and the fact that $F(e^{j\omega T}, b)$ is a linear-phase filter and thus can be expressed as $F(e^{j\omega T}, b) = [1 \pm \delta_F(\omega T, b)]e^{-j\omega T N_F/2}$. With $\delta_F = \delta_G = \delta_e$, the overall approximation error will then be smaller than the error $2\delta_e$ which subsequently will be reduced in the joint optimization. Secondly, for the VBW filter $F(z, b)$ in (12), we have $F(z, b_0) = F_0(z)$. This means that $F(z, b_0)$ is a regular lowpass filter with passband and stopband edges at $b_0 - \Delta$ and $b_0 + \Delta$, respectively. Thus, its transition bandwidth is 2Δ, which then results in $\widehat{N}_F$ in (19). Thirdly, the widest band the VFD filter $G(z, d)$ should handle occurs for $b = b_u - \Delta$, implying a transition bandwidth of $\pi - (b_u - \Delta)$, from which $\widehat{N}_G$ in (19) then follows.

In Step 2, P and K are determined by separately designing the VBW filter $F(z, b)$ and the VFD filter $G(z, d)$ to approximate $D_F(j\omega T, b)$ and $D_G(j\omega T, d)$ in (5) and (6), respectively, with the approximation error δ_e. This is based on the facts that, for $d = 0$, we have $H(z, b, 0) = F(z, b)$, whereas the most stringent requirement for $G(z, d)$ occurs for $b = b_u$ in which case $G(e^{j\omega T}, d)$ should approximate a VFD filter for all $d \in [-1/2,\ 1/2]$. The values of P and K are then found using available design methods for VBW and VFD filters [7, 10].

In Step 3, the problems in parts (a) and (b) are convex, each of which has a unique global optimum. They can be solved using any regular solver for such problems. The problem in part (c) is nonlinear because of the cascaded subfilters. In this paper, the nonlinear optimization problem is solved using the general-purpose routine *fminimax* in MATLAB together with the real-rotation theorem [11], which states that minimizing $|f|$ is equivalent to minimizing $\Re\{fe^{j\Theta}\}$, $\forall\Theta \in [0,\ 2\pi]$. The optimization problem is then solved with ωT and Θ discretized to dense enough grids. Here, the passband (stopband) region contains 101 (201) points and Θ consists of 12 points. Further, we have used 11 values of b evenly distributed between b_l and b_u, and with 6 values of d evenly distributed between 0 and $1/2$.

5. DESIGN EXAMPLES

Example 1: The following specification is considered: $b_l = 0.22\pi$, $b_u = 0.54\pi$, $\Delta = 0.12\pi$, and $\delta_e = 0.01$. With the structure in Fig. 1, designed as outlined in Section 4, this specification is met with $P = 4$, $K = 3$, $N_F = 26$, and $N_G = 6$. Figure 3 plots the modulus of the frequency response and error for the filter.

The overall realization requires 120 multiplications and 210 additions for the fixed subfilters. Using the structure in [5], the corresponding figures are, for the subfilter order $N_H = N_F + N_G = 32$, 330 and 630, respectively. Thus, multiplication and addition savings of 64% and 67% are achieved. The price to pay is an increased number of delay elements, from 32 to 56. The delay is the same for both structures though, namely $N_H/2 = 16$. The additional number of general multiplications and additions required for the variable parts are also the same, namely 19.

Example 2: This example compares the proposed technique with the recently introduced alternative fast filter bank approach in [6]. We compare the results for the proposed realization with those given in Example A in [6]. The bandwidth is the same as in Example 1 above. Here, the magnitude error δ_{me}, phase error δ_{pe}, and group delay error δ_{ge} are minimized. In the design, their weights are chosen according to their respective maximum values given in [6]. Using the proposed structure with $P = 5$, $K = 3$, $N_F = 36$, and $N_G = 6$, we obtain $\delta_{me} = 0.03$ and $\delta_{pe} = 0.0005$, which are slightly larger than the values reported in [6]. However, the group delay error is here $\delta_{ge} = 0.006$ which is merely 20% of the value reported in [6]. The reason for this small value is that the group delay error in the proposed realization emanates solely from $G(z, d)$ which is of low order. Furthermore, the overall group delay of the proposed realization is also significantly smaller, namely 21 as compared to 169, i.e., only 12.5% of that value. Hence, the main features of the proposed realization, compared to that in [6], appear to be considerably smaller group delays and group delay errors. Figure 4 plots the group delay response.

As to the implementation cost, the proposed overall realization requires 174 fixed multiplications and 23 general multiplications whereas that of [6] requires 134 fixed multiplications and 41 general multiplications. In an implementation which can benefit from fixed arithmetic, the complexities of the two realizations will therefore be similar as variable (general) multipliers are more expensive to implement than fixed multipliers. As a rough estimate (as different fixed multipliers can have different costs), the multiplication complexities become equal, here, when the cost of a general multiplication is 2.2 times that of a fixed multiplication. On average, a general multiplication is even more costly than that.

6. CONCLUSION

This paper introduced a realization of FIR filters with simultaneously variable bandwidth and FD. Unlike previous similar realizations, it utilizes the fact that a VFD filter is typically much less complex than a VBW filter. By separating the corresponding subfilters in the overall realization, significant complexity savings can be achieved. This was demonstrated through a design example that showed 65% savings. Furthermore, as demonstrated in another example, the proposed approach offers significantly smaller group delays and group delay errors, as compared to a recently introduced alternative fast filter bank approach.

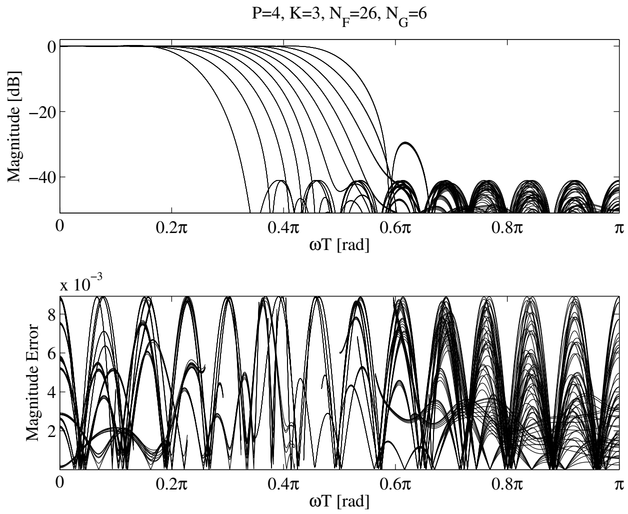

Fig. 3. Modulus of the frequency response (upper) and error $E(j\omega T, b, d)$ (lower), for the Example 1 filter. Note that the modulus is the same for d and $-d$ due to conjugate symmetry. Note also that the bumps seen in some of the magnitude responses occur in the transition region which is acceptable.

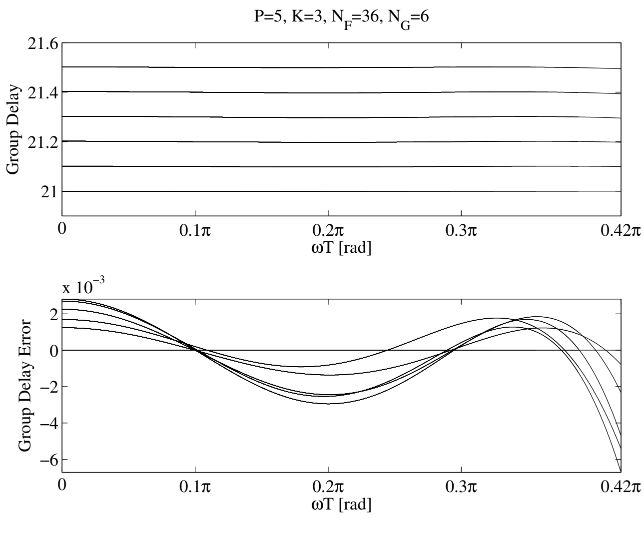

Fig. 4. Group delay (upper) and group delay error (lower), in the passbands, for the Example 2 filter.

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