JOINT USER SELECTION AND BEAMFORMING IN INTERFERENCE LIMITED COGNITIVE RADIO NETWORKS

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ABSTRACT

We consider optimal beamforming and user admission in cellular secondary networks where radio access to the licensed spectrum is granted under the premise that the interference level created in the primary network falls below a predefined threshold and Quality of Service (QoS) demands of the secondary users (SUs) need to be fulfilled. In scenarios where the interference constraints are restrictive, or the number of users in the primary network is large, costly user admission generally becomes unavoidable. We propose an iterative transmit beamforming and power allocation technique for interference limited cognitive networks in which a computationally efficient user selection procedure naturally integrates.Our iterative scheme is based on the uplink-downlink duality property of the network. Further, a simple infeasibility detection procedure is proposed from which a computationally attractive heuristic deflation technique for user selection is derived. Simulation results demonstrate that our user selection scheme represents a well-balanced compromise between user selection performance and computational complexity.

***Index Terms*—** small cell networks, cognitive radio, femtocells, user admission, beamforming

1. INTRODUCTION

Downlink beamforming and power allocation techniques can be employed at the cognitive radio base station (CBS) in order to efficiently control the amount of interference leaked to the primary system while maintaining a required signal-to interference-plus-noise-ratio (SINR) for the secondary users (SUs). The optimal beamforming and power allocation problem in a cognitive radio context has been recently addressesd in [1]-[3]. A common approach, is to relax this problem to a convex form [1], [2] and apply interior point methods to solve it. A technique that is computationally more attractive has been proposed in [2], [3] and uses the uplink-downlink duality theory in order to decouple the beamformer optimization problem from the power allocation. These algorithms exploit the structure of the problem more efficiently, therefore achieve a lower complexity as compared to the interior point methods.

In scenarios where the interference requirements imposed by the primary network are strong and the number of primary users (PUs) is large, admission control at the cognitive base station becomes inevitable. Due to the combinatorial nature of the user admission problem, optimal user selection is generally impractical [4]. A suboptimal user selection and beamformer design has been proposed in [5], where SUs are admitted for transmission based on an orthogonality measure between the SU and PU channel vectors. The elegant method of [4], originally proposed in the context of conventional multiuser downlink networks has been extended in [6] to the cognitive case. This method involves however the use of computationally complex interior point solvers.

Our main contribution in this paper is to devise and naturally integrate an infeasibility detection stage into the iterative optimal beamforming and power allocation algorithm which is based on the uplink-downlink duality theory and has been proposed in [2] and [3]. For this iterative algorithm we further propose a powerful deflation technique that is based on efficient heuristics for user selection.

Throughout this paper we use $(\cdot)^T$, $E\{\cdot\}$, $\|\cdot\|$ to denote the transpose, expectation and vector norm 2 of $(\cdot)$ respectively. $\rho(\cdot)$ stands for the spectral radius of $(\cdot)$, $\boldsymbol{I}$ denotes the identity matrix. Furthermore $\cup$, $\cap$ and $\subseteq$ denote the reunion, intersection and inclusion operators of two sets while $|(\cdot)|$ represents the cardinality of the set $(\cdot)$. All vectors are defined as column vectors unless stated otherwise and the vector inequalities are considered element-wise.

2. SYSTEM MODEL

Consider a cognitive radio basestation (CBS) with N_t transmit antennas in the presence of K single-antenna SUs and L single-antenna PUs. Without loss of generality, we assume that the users are indexed so that users $1,\ldots,K$ are SUs, and users $K+1,\ldots,K+L$ are PUs. Additionally we introduce the ordered set of indexes $S \subseteq \{1,\ldots,K\}$ to indicate the candidate SUs, which are considered to be served by the CBS at a particular time instant.

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The signal transmitted at the CBS is given by $\boldsymbol{x}(n) = \sum_{k\in S} \sqrt{p_k}\boldsymbol{u}_k s_k(n)$, where p_k, $\boldsymbol{u}_k$, $s_k(n)$ denote the power, the unit norm beamforming weight vector and the zero mean unit norm variance information symbol transmitted to the kth user at time instant n, respectively.

The signal at the kth receiver can be written as

$$y_k(n) = \begin{cases} \boldsymbol{h}_k^T \boldsymbol{x}(n) + z_k(n), & k = 1, \ldots, K \\ \boldsymbol{h}_k^T \boldsymbol{x}(n), & k = K+1, \ldots, K+L \end{cases} \quad (1)$$

where $\boldsymbol{h}_k \triangleq [h_{k,1}, \ldots, h_{k,N_t}]^T$ denotes the downlink channel vector of the kth user, with $h_{k,m}$ being the complex channel coefficient between the mth transmit antenna and the kth receiver and $z_k(n)$ is the additive noise of zero mean and variance σ_k^2, assumed to be independent of the information signals. The latter term may also include the interference from the primary system, which is treated as noise by the secondary system. In this paper we assume that the transmitter perfectly knows the instantaneous SU channels, the channel covariance matrices of the PUs and the noise variances at the receivers. Let $\boldsymbol{R}_k \triangleq \mathrm{E}\{\boldsymbol{h}_k \boldsymbol{h}_k^H\}$ denote the downlink channel covariance matrix of the kth user.

The downlink SINR for the kth SU can be defined as

$$\mathrm{SINR}_k^{\mathrm{DL}} \triangleq \frac{p_k \boldsymbol{u}_k^H \boldsymbol{R}_k \boldsymbol{u}_k}{\sum_{\substack{j\neq k\\ j\in S}} p_j \boldsymbol{u}_j^H \boldsymbol{R}_k \boldsymbol{u}_j + \sigma_k^2}, \quad (2)$$

and the interference towards the lth PU is

$$\mathrm{T}_l \triangleq \sum_{k\in S} p_k \boldsymbol{u}_k^H \boldsymbol{R}_{K+l} \boldsymbol{u}_k. \quad (3)$$

The downlink beamforming problem consists in optimally allocating beamformers and powers such that the total transmitted power is minimized and the SINR requirements of the SUs are met. Additionally the interference leaked to the PUs and the total transmit power must be kept below the limits imposed by regulations and hardware restrictions. For the set of users S this problem can be formulated as:

$$\min_{\{\boldsymbol{u}_k, p_k\}} \sum_{k\in S} p_k \quad (4a)$$

$$\text{s.t} \quad \mathrm{SINR}_k^{\mathrm{DL}} \geq \gamma_k;\ \mathrm{T}_l \leq \frac{1}{\gamma_{K+l}};\ l = 1, \ldots, L+1; \quad (4b)$$

$$p_k \geq 0;\ \|\boldsymbol{u}_k\| = 1;\ k \in S, \quad (4c)$$

where γ_k, $1/\gamma_{K+l}$ and $1/\gamma_{K+L+1}$ denote the SINR target of the kth SU, the interference threshold of the lth PU and the maximum transmit power budget, respectively. For compactness, we have expressed in (4b) the total power as the $(L+1)$th interference term, i.e., $\mathrm{T}_{L+1} \triangleq \sum_{k\in S} p_k \boldsymbol{u}_k^H \boldsymbol{R}_{K+L+1} \boldsymbol{u}_k$, for $\boldsymbol{R}_{K+L+1} = \boldsymbol{I}$.

We will refer to problem (4) as $\mathcal{P}_S$ to emphasise the dependency on the set of users considered in this problem. The objective of the CBS is to select the maximum set of users S_o:

$$S_o = \mathrm{argmax}_{S\subseteq\{1,\ldots,K\}} |S| \ \text{s.t.}\ \mathcal{P}_S \text{ is feasible}, \quad (5)$$

and assign optimal beamformers and powers to the users in this set.

3. BEAMFORMER DESIGN AND POWER ALLOCATION

In this section we revise the duality results for problem $\mathcal{P}_S$, that have been previously derived in [2]. Based on these results we present an iterative beamforming scheme with improved convergence behaviour.

It can be proven using similar arguments as in [7] that, strong Lagrange duality holds for the original problem (4). Hence, the Lagrange dual

$$\begin{aligned} \min_{\{q_k\}} \quad & \sum_{k\in S} \gamma_k \sigma_k^2 q_k - \sum_{l=1}^{L+1} q_{K+l} \quad (6) \\ \text{s.t.} \quad & \boldsymbol{I} - q_k \boldsymbol{R}_k + \sum_{\substack{i\neq k\\ i\in S}} \gamma_i q_i \boldsymbol{R}_i + \sum_{l=K+1}^{K+L+1} \gamma_l q_l \boldsymbol{R}_l \succcurlyeq 0 \\ & q_k \geq 0;\ q_{K+l} \geq 0;\ k \in S;\ l = 1, \ldots, L+1 \end{aligned}$$

achieves the same optimum as (4). Relaxing the PU and total power constraints of the original problem (4), a partial dual function of $\mathcal{P}_S$ can be constructed for any non-negative vector of Lagrange multipliers $\boldsymbol{q}_2 = [q_{K+1}, \ldots, q_{K+L+1}]^T$:

$$f_p(\boldsymbol{q}_2) = \min_{\{\boldsymbol{u}_k, p_k\}} \sum_{k\in S} p_k + \sum_{l=1}^{L+1} q_{K+l} \gamma_{K+l} \left(\sum_{k\in S} p_k \boldsymbol{u}_k^H \boldsymbol{R}_{K+l} \boldsymbol{u}_k - 1 \right) \quad (7a)$$

$$\text{s.t. } \gamma_k \sigma_k^2 - p_k \boldsymbol{u}_k^H \boldsymbol{R}_k \boldsymbol{u}_k + \gamma_k \sum_{\substack{j\neq k\\ j\in S}} p_j \boldsymbol{u}_j^H \boldsymbol{R}_k \boldsymbol{u}_j = 0, \quad (7b)$$

$$p_k \geq 0;\ k \in S. \quad (7c)$$

Note that the equalities in (7b) stem from the observation that at the optimum of (4) the SINR constraints are satisfied with equality [2]. Maximizing the partial dual function $f_p(\boldsymbol{q}_2)$ in (7) with respect to $\boldsymbol{q}_2$ yields the partial dual problem of (4):

$$\max_{\boldsymbol{q}_2 \geq \boldsymbol{0}} f_p(\boldsymbol{q}_2), \quad (8)$$

which achieves the same optimum objective value as $\mathcal{P}_S$ and the optimal beamformers in both problems are identical. Making use of the uplink-downlink duality theory in [2] and [7], problem (7) can be equivalently expressed as the virtual uplink beamforming problem:

$$f_d(\boldsymbol{q}_2) = \min_{\{\boldsymbol{u}_k, q_k\}} \sum_{k\in S} \gamma_k \sigma_k^2 q_k - \sum_{l=1}^{L+1} q_{K+l} \quad (9a)$$

$$\text{s.t. } \mathrm{SINR}_k^{\mathrm{UL}} \triangleq \frac{q_k \boldsymbol{u}_k^H \boldsymbol{R}_k \boldsymbol{u}_k}{\boldsymbol{u}_k^H \left(\sum_{\substack{j\neq k\\ j\in S}} q_j \gamma_j \boldsymbol{R}_j + \sum_{l=K+1}^{K+L+1} q_l \gamma_l \boldsymbol{R}_l + \boldsymbol{I} \right) \boldsymbol{u}_k} = 1 \quad (9b)$$

$$q_k \geq 0;\ \|\boldsymbol{u}_k\| = 1;\ k \in S. \quad (9c)$$

such that for any non-negative vector $\boldsymbol{q}_2$, the same optimum objective value and beamformers are obtained by (7) and (9). We remark that the Lagrange multipliers $q_1, \ldots, q_K$ and

$q_{K+1}, \dots q_{K+L+1}$ in (9) can also be interpreted as virtual uplink powers of the SUs and PUs respectively. Maximizing $f_d(\boldsymbol{q}_2)$ with respect to $\boldsymbol{q}_2$ yields

$$\max_{\boldsymbol{q}_2 \geq \boldsymbol{0}} f_d(\boldsymbol{q}_2), \tag{10}$$

which is equivalent to (4) and (8), in terms of optimum objective value and beamformers. The advantage of (10) is that the powers and the beamformers are essentially decoupled.

Optimal beamforming and power minimization

In the following we propose an iterative algorithm to solve $\mathcal{P}_S$ in (4) based on the equivalent reformulations (8) and (10) presented in the previous section. In this section we assume that $\mathcal{P}_S$ is feasible, i.e., there exists a set of beamformers and powers $\{p_k, \boldsymbol{u}_k\} \in \mathcal{F}$, with

$$\mathcal{F} \triangleq \left\{ p_k, \boldsymbol{u}_k \mid \text{SINR}_k^{\text{DL}} \geq \gamma_k, \text{T}_l \leq \frac{1}{\gamma_{K+l}},\ l=1,\dots,L+1, k \in S \right\} \tag{11}$$

denoting the feasible set of (4). Additionally, we assume a feasible set of powers and beamformers to be available as starting point of our iterative algorithm. Obtaining such initial values and treating the cases when $\mathcal{P}_S$ is not feasible are considered in the following section.

Due to the maximin structure of (10) our iterative beamforming and power allocation algorithm consists of two sequential updates 1) the update of the beamformers and SU virtual uplink powers $q_1, \dots, q_K$ for fixed PU virtual uplink powers $q_{K+1}, \dots, q_{K+L+1}$, and 2) the update of the PU virtual uplink powers for fixed beamformers and SU virtual uplink powers. Denoting by t the iteration number, and introducing $[\boldsymbol{q}_1]_{i_k} = q_k$, with $k \in S$ and i_k the index of k in the ordered set S, the algorithm can be summarized as follows:

Algorithm 1:Beamforming and power minimization (BP)

Until the convergence of $\boldsymbol{q}_2$

1. Solve (9) for fixed $\boldsymbol{q}_2$ by iterating the steps:

1.1 Update the SU uplink powers $\boldsymbol{q}_1$ with the solution of the linear system of equations in (9b) for fixed beamformers.

1.2 Update the beamformers $\boldsymbol{u}_k$ as generalized eigenvectors of the expressions defined in (9b) for fixed $\boldsymbol{q}_1$.

2.1 Compute the downlink powers from (7b) and the PU interference levels T_l with (3)

2.2 Perform the PU virtual uplink power updates with

$$q_{K+l}(t+1) = q_{K+l}(t)\gamma_{K+l}T_l(t+1),\ l = 1,\dots,L+1. \tag{12}$$

We remark that, for $\boldsymbol{q}_2 = \boldsymbol{0}$, problem (9) corresponds to the conventional multiuser uplink beamforming problem, for which several algorithms have been proposed [7], [10]. These algorithms can straightforwardly be applied in the case of non-negative $\boldsymbol{q}_2$ due to the following proposition.

Proposition 1: *If (9) is feasible for $\boldsymbol{q}_2 = \boldsymbol{0}$ and a unique set of positive SU virtual uplink powers achieves the optimum, then (9) is feasible for any non-negative virtual uplink powers $\boldsymbol{q}_2$ and the corresponding SU virtual uplink powers are unique and positive.*

We omit the proof due to space limitations.

In Step 1 we use the algorithm proposed in [10], which is proven to exhibit superlinear convergence rate. The procedure in Step 2.1 is motivated by the observation that problems (7) and (9) yield the same optimum value and beamforming weight vectors. The PU virtual uplink power updates proposed in Step 2.2 follow the intuition that in the virtual uplink system described by problem (9) any particular PU transmits with a power depending on the level of interference it experiences. Moreover, from the complementary slackness condition, which is satisfied due to the strong duality of problem (4) we obtain that at the optimum $q_{K+l}(\gamma_{K+l}T_l - 1) = 0$, for all $l = 1, \dots, L+1$. It follows that if an optimal solution of (4) exists, then the algorithm we propose converges to the global optimum.

4. INFEASIBILITY DETECTION AND USER SELECTION

In this section we propose a simple infeasibility certificate that can be used in scenarios in which infeasibility is caused by the constraints imposed by the primary network. We further demonstrate how this detection scheme naturally integrates in the algorithm presented in the previous section and propose a simple user selection method.

We refer to a problem as *SU-feasible* if in the absence of the PU system the SINR constraints can be satisfied with a transmit power below the imposed levels, i.e, the optimum of:

$$\max_{\{\boldsymbol{u}_k, p_k\}} \min_{k \in S} \text{SINR}_k^{\text{DL}}/\gamma_k \ \text{ s.t} \sum_{k \in S} p_k \leq 1/\gamma_{K+L+1} \tag{13}$$

achieves a value greater than 1. Similarly, we define a problem as *strictly SU-feasible*, if the objective of (13) is strictly larger than 1. By further introducing the sets:

$$\mathcal{D}_1 \triangleq \left\{ \boldsymbol{q} \left| -q_j \boldsymbol{R}_j + \sum_{\substack{k \neq j \\ k \in S}} q_k \gamma_k \boldsymbol{R}_k + \sum_{l=K+1}^{K+L+1} q_l \gamma_l \boldsymbol{R}_l \succcurlyeq 0,\ j \in S \right. \right\} \tag{14}$$

$$\mathcal{D}_2 \triangleq \left\{ \boldsymbol{q} \mid \boldsymbol{q}_1^T \boldsymbol{\eta} - \boldsymbol{q}_2^T \boldsymbol{1} > 0 \right\} \tag{15}$$

with $\boldsymbol{q} = [\boldsymbol{q}_1^T,\ \boldsymbol{q}_2^T]^T$ we can state the following:

Proposition 2: *If the original problem (4) is strictly SU-feasible then exactly one of the sets $\mathcal{D} \triangleq \mathcal{D}_1 \bigcap \mathcal{D}_2$ and $\mathcal{F}$ is non-empty.*

Sketch of proof Formulating the feasibility problem associated to (4) and applying the theorem of alternatives [8, Chapter 5.8] it immediately follows that at most one of the sets $\mathcal{F}$ and $\mathcal{D}$ is non-empty. In order to prove that at least one of the sets $\mathcal{F}$ and $\mathcal{D}$ is non-empty, (4) must be reformulated as a conic problem as in e.g., [3]. Subsequently, the strict SU-feasibility assumption of Proposition 2 can be used to prove that a constraint qualification condition holds, with which the proof can be concluded. ■

In order to integrate an infeasibility detection scheme based on the result of Proposition 2 into Algorithm 1, we make the following two remarks. Firstly, we note that the SU-feasibility of (4) implies that (9) is feasible for $\boldsymbol{q}_2 = \mathbf{0}$. Thus, from Proposition 1, it follows that for all $\boldsymbol{q}_2$ updates in (12), problem (9) is feasible. Hence all updates of Algorithm 1 remain meaningful even when the feasibility assumption of $\mathcal{P}_S$ in (4) has been relaxed to strict SU-feasibility. Secondly, it can be proven that the $\boldsymbol{q}_2$ updates (12) are forcing the new virtual uplink powers to be in the set $\mathcal{D}_2$, which is a necessary condition for the infeasibility of (4). This results directly from introducing $\boldsymbol{q}_2(t+1)$ derived with (12) in the expressions of the optimum SU virtual uplink powers obtained from (9b) for fixed $\boldsymbol{q}_2(t)$ and further using Proposition 1.

Thus in order to introduce infeasibility control (IC) into Algorithm 1, it is sufficient to test whether the virtual uplink powers, after each complete round of updates, i.e, after Step 2 in Algorithm 1, are in the set $\mathcal{D}_1$ defined in (14). If this condition is satisfied the user removal procedure presented in the next subsection is initiated. Furthermore, the feasibility testing procedure can be terminated once a point in $\mathcal{F}$ is found. Note that the verification whether $\{p_k, \boldsymbol{u}_k\} \in \mathcal{F}$ does not add to the overall computational complexity of the algorithm as it consists in testing whether the PU interferences T_l and hence the update terms of the PU virtual uplink powers in (12) are smaller than 1. Even though we cannot give a formal upper bound on the number of iterations required to have an update in $\mathcal{D}$, simulations show that in the vast majority of cases an infeasibility certificate is obtained in less than 5 iterations.

4.1. User Selection

The goal of the user selection procedure is to admit as many SUs as possible such that their SINR targets are satisfied and the interference levels are kept below the imposed limits. Therefore we start with the largest set of users S, for which $\mathcal{P}_S$ is strictly SU-feasible and apply Algorithm 1, enhanced with the infeasibility testing procedure described in the beginning of this section. Once infeasibility has been detected, a deflation procedure is initiated and an appropriate user is eliminated. In order to derive a heuristic for the user removal procedure, we recall that infeasibility is detected for the set S of users when the virtual uplink powers are in the set $\mathcal{D}$. Thus, by eliminating the user k with the largest weighted virtual uplink power $q_k\gamma_k$, we observe the largest decrease of the linear expression on the left hand side of the inequality in (15). This, in a sense moves the remaining virtual uplink powers furthest away from the set $\mathcal{D}_2$, and hence problem $\mathcal{P}_S$ for the resulting set S, far away from infeasibility. The advantages of this technique are its simplicity and general good performance as shown from the simulations in the next section. We name the deflation procedure based on this heuristic "Fast Removal" and show in Algorithm 2 the complete outline of Algorithm 1 enhanced with the infeasibility control and the described user selection procedure.

Algorithm 2: BP with IC and Fast Removal:

Initialization: Find the largest set of users S for which (4) is strictly SU-feasible. Initialize $\boldsymbol{q}_2$, $\{\boldsymbol{u}_k\}$
Until convergence of $\boldsymbol{q}_2$:
1-2. Perform Steps 1-2 in Algorithm 1.
3. Test if $\boldsymbol{q} \in \mathcal{D}_1$,
if yes, remove user with largest weighted SU virtual uplink power and go to step 1
if no, test convergence and go to step 1 if necessary

The performance of this heuristic depends however on the existence of an SU with a weighted virtual uplink power, significantly larger than that of the other users. Problems may occur, e.g., when two SUs interfere with each other or create large interference to the same PU. Then, their corresponding virtual uplink powers are relatively close to each other and eliminating the user with the largest power may no longer be the best strategy.

To overcome this problem we propose a more sophisticated user removal procedure referred to as 'Look-Ahead Removal'. This technique consists in an efficient "depth first" branch-search with maximum depth of 1. When there is no SU with a distinctive value of the weighted virtual uplink power $q_k\gamma_k$, the two SUs with the two largest weighted virtual uplink powers are considered as candidates for removal. Each of the alternatives is temporarily eliminated one at a time, the iterative updating procedure is performed, and the evolution of the candidates among the remaining set of users is analysed. The user with the potentially best 'behaviour' in future, i.e., at the next branch level will be kept. In order to describe the 'behaviour' of the users, different metrics can be applied: weighted uplink power, interference level created, etc. In Algorithm 3 we illustrate one metric that we found to perform best. For simplicity and without loss of generality it is assumed that at the stage of the iterative algorithm where infeasibility is detected, the candidate set for removal is formed by the first two users. S^j denotes the set S without the jth user.

Algorithm 3: Look Ahead

1. for j=1,2
1.1 Solve $\mathcal{P}_{S^j}$ until $\boldsymbol{q}$ converges or infeasibility is detected.
1.2 With the obtained virtual uplink powers $\boldsymbol{q}$ compute:

$$m_j = \begin{cases} \sum_{k\in S} \gamma_k\sigma_k^2 q_k - \sum_{l=1}^{L+1} q_{K+l} & \text{if } \mathcal{P}_{S^j} \text{feasible;} \\ |T| \text{ where } T = \{k \in S^j \mid \gamma_k q_k > \gamma_j q_j\} & \text{otherwise.} \end{cases} \tag{16}$$

Return (m_j, infeasibility status)
2. If both $\mathcal{P}_{S^j}$ $j = 1, 2$ have the same infeasibility status eliminate user j with the smallest m_j
Else eliminate user j for which $\mathcal{P}_{S^j}$ is feasible

4.2. SU-feasibility and the Initialization Procedure

We have proposed an infeasibility certficate for problem (4) under the assumption of strict SU-feasibility. We complete the analysis by integrating a method to verify whether problem (4) is strictly SU-feasible, and when this is not satisfied, to propose a new set of users S, such that $\mathcal{P}_S$ in (4) becomes strictly SU-feasible. This stage will serve as an initialization phase for Algorithm 2.

Detecting strict SU-feasibility is straightforwardly obtained from [9], where it was proven that (13) yields the same optimum value and beamformers as the equivalent uplink reformulation:

$$\max_{\{\boldsymbol{u}_k, q_k\}} \min_{k \in S} \ \text{SINR}_k^{\text{UL}}/\gamma_k \ \ \text{s.t} \sum_{k \in S} q_k \leq 1/\gamma_{K+L+1}. \quad (17)$$

Since (17) is essentially a conditioned eigenvalue problem it can be solved with low computation complexity and fast convergence [9]. When the optimum of (17) is strictly larger than 1 for the set S then $\mathcal{P}_S$ is strictly SU-feasible. As initial virtual uplink powers $\boldsymbol{q}_2$ any positive values can be used, while the inital beamformers must be chosen as the optimum beamformers of (17). When the optimum of (17) is smaller than 1, the original problem (4) is SU-infeasible and a user must be removed. Any of the metrics suggested in section 4.1 can be used. This procedure is repeated until a set of users rendering a problem strictly SU-feasible is found.

5. SIMULATIONS

We consider a CBS with 7 antennas, transmitting over Rayleigh fading channels. The interference threshold at the PUs is considered −6dB and the initial number of SUs is fixed to 6. The optimal user set is obtained by exhaustive search.For better comparison we also consider a deflation technique in which one user is randomly removed after each infeasibility decision.

In the first scenario, the number of PUs is fixed to 6, and the SINR targets are increased from 5 to 9 dBs. We show in Table 1 the percentage of correct decisions in term of served number of users, that are taken with the two user selection schemes we proposed. We note that the 'Look Ahead' yields the correct number of users in more than 95% of the cases. The 'Fast Removal' procedure is still performing well, with the correct decision in almost 90% of the cases, hence much more than in the random removal scheme.

Table 1. Correct decisions for increasing SINR targets

SINR / Method	5	6	7	8	9
Random	50%	41%	41.2%	39%	52%
Fast Removal	94%	93%	90%	90%	90%
Look ahead	99%	97%	96%	96%	97%

Simulations have additionally showed that, when the correct number of users is served, the minimum transmit power of the selected set exceeds the power of the optimal combination by no more than 0.4 dBs in the case of the 'Fast-Removal' scheme, and 0.8 dBs when 'Look-Ahead' is employed.

We further show in Table 2 the percentage of infeasibility decisions that were taken in a certain number of runs. It can be noted that in almost 97% of the cases the decision that the particular user configuration is infeasible is taken in less than 5 iterations. In the second scenario the SINR targets of the SUs are set to 8dB and the number of PUs is increased from 5 to 9. We show in Table 3 the percentage of correct decisions in term of served number of users that are taken with the two user selection schemes we proposed.

Table 2. Number of tests to decide infeasibility

No of Runs	1	2	3	4	5	6	7
Percentage	34.29%	50.06%	8.62%	2.46%	1.54%	0.51%	0.47%

Table 3. Correct decisions for increasing number of PUs

PUs / Method	5	6	7	8	9
Random	42%	41%	41%	39%	42%
Fast Removal	90%	93%	90%	88%	85%
Look ahead	94%	94%	99%	96%	93%

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