

# PERCEPTION-BASED NONLINEAR LOUDSPEAKER COMPENSATION THROUGH EMBEDDED CONVEX OPTIMIZATION

*Bruno Defraene, Toon van Waterschoot, Moritz Diehl and Marc Moonen*

Dept. E.E./ESAT, SCD-SISTA, KU Leuven  
Kasteelpark Arenberg 10, B-3001 Leuven, Belgium

## ABSTRACT

In this paper a novel nonlinear loudspeaker compensation technique is presented which is based on embedded convex optimization. The aim is to compensate for the linear as well as for the nonlinear perceptible distortions incurred in the loudspeaker. To this end, a psychoacoustic model is adopted and a convex optimization based problem formulation is set up. In order to solve the resulting convex optimization problems in a fast and reliable way, a projected gradient optimization method is proposed. From comparative objective evaluation experiments, it is concluded that the proposed nonlinear loudspeaker compensation technique indeed improves the average audio quality scores.

**Index Terms**— Sound perception, loudspeaker compensation, nonlinear model, Hammerstein model, convex optimization.

## 1. INTRODUCTION

Achieving a high perceived audio quality is undoubtedly a main concern in the development of any audio reproduction system. In general, the loudspeakers in such a system have a non-ideal response introducing both linear and nonlinear distortions in the reproduced audio signal. This may result in a significant degradation of the perceived audio quality [1].

Loudspeaker compensation techniques aim at reducing the effects caused by the non-ideal loudspeaker characteristics. The idea is to apply a digital compensation operation in cascade with the audio reproduction channel to counteract the response errors and nonlinearities introduced by the loudspeakers. Traditionally, loudspeakers have been modeled by *linear* systems such as FIR filters, IIR filters, warped

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filters or Kautz filters. The aim of linear loudspeaker compensation (also known as equalization) techniques is then to identify/approximate and apply the inverse digital filter to the audio signal prior to playback in order to reduce the linear distortions [2].

However, the small and low-cost loudspeakers that are ubiquitous in mobile devices are also causing a high level of *nonlinear* distortion, especially at high playback levels. This nonlinear behaviour can be taken into account by using nonlinear loudspeaker models such as Hammerstein models, Wiener-Hammerstein models and Volterra models. The aim of nonlinear loudspeaker compensation techniques is then to invert the nonlinear system under consideration [3]. This is in general a computationally expensive approach.

In this paper, a novel nonlinear loudspeaker compensation technique is presented, which aims to compensate for the linear as well as for the nonlinear distortions incurred in the loudspeaker. The novelty compared to existing loudspeaker compensation techniques is twofold. Firstly, a convex optimization procedure is embedded into the algorithm to carry out the nonlinear loudspeaker compensation. Approaches based on *embedded convex optimization* have been successful in related signal processing applications, e.g. [4] [5]. Secondly, the proposed compensation technique is *perception-based*: a psychoacoustic model which captures knowledge about the human perception of sounds is employed, and allows to minimize the resulting perceptible distortion.

This paper is organized as follows. In Section 2, the nonlinear loudspeaker compensation technique is introduced in a framework of embedded convex optimization and the inclusion of a psychoacoustic model is discussed. In Section 3, a projected gradient optimization method is presented for solving the convex optimization problems at hand in a fast and reliable way. In Section 4, simulation results are given for a comparative audio evaluation experiment. Finally, in Section 5, some concluding remarks are presented.

## 2. NONLINEAR LOUDSPEAKER COMPENSATION

The aim of nonlinear loudspeaker compensation is to precompensate for the linear as well as for the nonlinear distortions incurred in the loudspeaker. Figure 1 shows the operation of

the proposed compensation technique. Frame-by-frame processing of the digital input audio signal  $x[n]$  is applied, using non-overlapping input frames  $x_m \in \mathbb{R}^N, m = 0, 1, \dots, M$ . The loudspeaker is modeled by a Hammerstein model, i.e. a memoryless nonlinearity with a linear region  $[-U, U]$ , followed by a linear finite impulse response (FIR) filter with impulse response  $h[n], n = 0 \dots L-1$ . Before it is fed into the loudspeaker, the input frame  $x_m$  passes through the nonlinear loudspeaker compensation block. For a given input frame  $x_m$ , the nonlinear loudspeaker compensation consists of the following steps:

1. Calculate the instantaneous global masking threshold  $t_m \in \mathbb{R}^{\frac{N}{2}+1}$  of the input frame  $x_m$  using a psychoacoustic model (see Subsection 2.2).
2. Calculate a compensated input frame  $v_m^* \in \mathbb{R}^N$  as the solution of a convex optimization problem, such that the resulting output frame  $y_m^*$  is perceptually as close as possible to  $x_m$  (see Subsection 2.1).

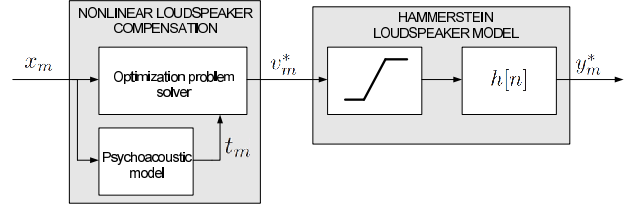
## 2.1. Optimization problem formulation

The core of the proposed nonlinear loudspeaker compensation technique consists in calculating the solution of a convex optimization problem for each input frame. The optimal compensated input frame  $v_m^*$  is calculated from the knowledge of the input frame  $x_m$  and its instantaneous masking threshold  $t_m$ . A necessary constraint on  $v_m^*$  is that the amplitude of the samples be contained within the linear region  $[-U, U]$  of the memoryless nonlinearity. The objective function reflects the amount of perceptible distortion added between  $y_m$  and  $x_m$ . The optimization problem is then formulated as an inequality constrained frequency domain weighted L2-distance minimization, i.e.<sup>1</sup>

$$\begin{aligned} v_m^* &= \arg \min_{v_m \in \mathbb{R}^N} f(v_m) \quad \text{s.t.} \quad -u \leq v_m \leq u \\ &= \arg \min_{v_m \in \mathbb{R}^N} \frac{1}{2N} \sum_{i=0}^{N-1} w_m(i) |Y_m(e^{j\omega_i}) - X_m(e^{j\omega_i})|^2 \\ &\quad \text{s.t.} \quad -u \leq v_m \leq u \end{aligned} \quad (1)$$

where  $\omega_i = (2\pi i)/N$  represents the discrete frequency variable,  $X_m(e^{j\omega_i})$  and  $Y_m(e^{j\omega_i})$  are the discrete frequency components of  $x_m$  and  $y_m$  respectively, the vector  $u = U \cdot 1_N$  contains the upper amplitude level of the linear region (with  $1_N \in \mathbb{R}^N$  a vector of ones), and  $w_m(i)$  are the weights of a perceptual weighting function to be defined in subsection 2.2.

<sup>1</sup>Superscripts  $T$  and  $H$  denote the transpose and the Hermitian transpose, respectively.



**Fig. 1.** Perception-based nonlinear loudspeaker compensation: schematic overview

The optimization problem (1) can be rewritten as follows,

$$\begin{aligned} v_m^* &= \arg \min_{v_m \in \mathbb{R}^N} \frac{1}{2} (y_m - x_m)^H \underbrace{D^H W_m D}_{\triangleq Q_m} (y_m - x_m) \\ &\quad \text{s.t.} \quad -u \leq v_m \leq u \\ &= \arg \min_{v_m \in \mathbb{R}^N} \frac{1}{2} (H v_m + \tilde{H} v_{m-1}^* - x_m)^T Q_m (H v_m + \tilde{H} v_{m-1}^* - x_m) \\ &\quad \text{s.t.} \quad -u \leq v_m \leq u \\ &= \arg \min_{v_m \in \mathbb{R}^N} \frac{1}{2} v_m^T \underbrace{H^T Q_m H}_{\text{Hessian } A_m} v_m + \underbrace{(H^T Q_m^T (\tilde{H} v_{m-1}^* - x_m))^T}_{\text{Gradient } b_m} v_m \\ &\quad \text{s.t.} \quad -u \leq v_m \leq u \end{aligned} \quad (2)$$

where  $D \in \mathbb{C}^{N \times N}$  is the unitary Discrete Fourier Transform (DFT) matrix,  $W_m \in \mathbb{R}^{N \times N}$  is a diagonal weighting matrix with positive diagonal elements  $w_m(i)$ , obeying the symmetry property  $w_m(i) = w_m(N-i)$  for  $i = 1, 2, \dots, \frac{N}{2}-1$ , and the matrices  $H \in \mathbb{R}^{N \times N}$  and  $\tilde{H} \in \mathbb{R}^{N \times N}$  are defined as

$$\begin{aligned} H &= \begin{bmatrix} h[0] & 0 & \dots & \dots & \dots & \dots & 0 \\ h[1] & h[0] & 0 & \dots & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h[L-1] & & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & h[L-1] & \dots & h[1] & h[0] \end{bmatrix} \quad (3) \\ \tilde{H} &= \begin{bmatrix} 0 & \dots & 0 & h[L-1] & \dots & h[2] & h[1] \\ 0 & \dots & 0 & 0 & h[L-1] & \dots & h[2] \\ \vdots & & & & \ddots & \ddots & \vdots \\ \vdots & & & & & \ddots & h[L-1] \\ \vdots & & & & & & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix} \quad (4) \end{aligned}$$

It is remarked that the objective function in (2) is a quadratic function and that the constraint functions are affine, hence optimization problem (2) constitutes a *quadratic program* (QP). As it can be shown that the Hessian matrix  $A_m = H^T Q_m H$  in (2) is guaranteed to be real and positive definite, the optimization problem is a strictly convex quadratic program.

## 2.2. Perceptual weighting function

In order for the objective function in optimization problem (1) to reflect the amount of perceptible distortion added between input frame  $x_m$  and output frame  $y_m$ , the perceptual weighting function  $w_m$  must be constructed judiciously. The rationale behind applying signal-dependent weights in the summation (1) is the psychoacoustic fact that distortion at certain frequencies is more perceptible than distortion at other frequencies, and that the relative perceptibility is mostly signal-dependent. Two phenomena of human auditory perception are responsible for this.

A first phenomenon is the *absolute threshold of hearing*, which is defined as the required intensity (dB) of a pure tone such that an average listener will just hear the tone in a noiseless environment. The absolute threshold of hearing is a function of the tone frequency and has been measured experimentally. A second phenomenon is *simultaneous masking*, where the presence of certain spectral energy (the masker) masks the simultaneous presence of weaker spectral energy (the maskee), or in other words, renders it imperceptible. Combining both phenomena, the instantaneous global masking threshold of a signal gives the amount of distortion energy (dB) at each frequency bin that can be masked by the signal.

In this framework, consider the input frame  $x_m$  to act as the masker, and  $y_m - x_m$  as the maskee. By selecting the weights  $w_m(i)$  in (1) to be exponentially decreasing with the value of the global masking threshold of  $x_m$  at frequency bin  $i$ , the objective function effectively reflects the amount of perceptible distortion introduced. This can be specified as

$$w_m(i) = \begin{cases} 10^{-\alpha t_m(i)} & \text{if } 0 \leq i \leq \frac{N}{2} \\ 10^{-\alpha t_m(N-i)} & \text{if } \frac{N}{2} < i \leq N-1 \end{cases} \quad (5)$$

where  $t_m$  is the global masking threshold of  $x_m$  (in dB). Appropriate values of the compression parameter  $\alpha$  have been determined to lie in the range 0.04-0.06.

The instantaneous global masking threshold  $t_m$  of an input frame  $x_m$  is calculated by using part of the ISO/IEC 11172-3 MPEG-1 Layer 1 psychoacoustic model 1 [6]. The major steps in its computation are the following:

1. *Spectral analysis and SPL normalization*: A high-resolution spectral estimate of the input frame is calculated. After a normalization operation and a Hann windowing operation on the input signal frame, the PSD estimate is obtained through a 512-point Fast Fourier Transform (FFT).
2. *Identification of tonal and non-tonal maskers*: The output of the FFT is used to determine relevant tonal and non-tonal maskers in the spectrum of the audio signal.
3. *Calculation of individual masking thresholds*: an individual masking threshold is calculated for each masker in a decimated set of tonal and non-tonal maskers, using

fixed psychoacoustic rules. Essentially, the individual masking threshold depends on the frequency, loudness level and tonality of the masker.

4. *Calculation of global masking threshold*: Finally, the global masking threshold is calculated by a power-additive combination of the tonal and non-tonal individual masking thresholds, and the absolute threshold of hearing.

## 3. FAST PROJECTED GRADIENT OPTIMIZATION

The core of the nonlinear loudspeaker compensation technique described in Section 2 consists in the solution of successive instances of the convex quadratic optimization problem (2). The focus of this section is on the class of *projected gradient methods* for solving the convex QPs. In Subsection 3.1, the standard projected gradient method is introduced. In Subsection 3.2, an improved projected gradient method is presented, which reaches an optimal convergence.

### 3.1. Standard projected gradient method

In each iteration of the standard projected gradient method, first a step is taken along the negative gradient direction of the objective function, after which the result is orthogonally projected onto the convex feasible set, thereby maintaining feasibility of the iterates. A low computational complexity per iteration is the main asset of projected gradient methods, provided that the orthogonal projection onto the convex feasible set and the gradient of the objective function can easily be computed.

Introducing the notation  $v_m^k$  for the  $k$ th iterate of the  $m$ th frame, the main steps in the  $(k+1)$ th iteration of the projected gradient method can be written as follows:

- Take a step of stepsize  $s_m^k$  along the negative gradient direction :

$$\tilde{v}_m^{k+1} = v_m^k - s_m^k \nabla f(v_m^k) \quad (6)$$

where the gradient is computed as

$$\nabla f(v_m^k) = H^T Q_m^T (H v_m + \tilde{H} v_{m-1}^* - x_m) \quad (7)$$

- Project  $\tilde{v}_m^{k+1}$  orthogonally onto the convex feasible set  $\Omega$  of (2), which is defined as

$$\Omega = \{v_m \in \mathbb{R}^N \mid -u \leq v_m \leq u\} \quad (8)$$

An orthogonal projection  $\Pi_\Omega(\tilde{v}_m^{k+1})$  onto  $\Omega$  can be shown to come down to performing a simple componentwise hard clipping operation (with lower bound  $-U$  and upper bound  $U$ ), i.e.

$$v_m^{k+1} = \Pi_\Omega(\tilde{v}_m^{k+1}) = \arg \min_{v_p \in \Omega} \frac{1}{2} \|v_p - \tilde{v}_m^{k+1}\|_2^2 \quad (9)$$

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**Algorithm 1** Optimal projected gradient method
 

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**Input**  $x_m \in \mathbb{R}^N$ ,  $v_{m-1}^* \in \mathbb{R}^N$ ,  $v_m^0 = c_m^0 \in \Omega$ ,  $\gamma_m^0 \in (0, 1)$ ,  
 $U$ ,  $h$ ,  $W_m$

**Output**  $v_m^* \in \mathbb{R}^N$

- 1: Calculate Lipschitz constant  $C_m$  [using (13)]
  - 2: Calculate convexity parameter  $\mu_m$  [using (14)]
  - 3:  $\kappa_m = \frac{C_m}{\mu_m}$
  - 4:  $k = 0$
  - 5: **while** convergence is not reached **do**
  - 6:  $\tilde{v}_m^{k+1} = c_m^k - \frac{1}{C_m} \nabla f(c_m^k)$  [using (7)]
  - 7:  $v_m^{k+1} = \Pi_{\Omega}(\tilde{v}_m^{k+1})$  [using (10)]
  - 8: Calculate  $\gamma_m^{k+1}$  from  $(\gamma_m^{k+1})^2 = (1 - \gamma_m^{k+1})(\gamma_m^k)^2 + \kappa_m \gamma_m^{k+1}$
  - 9:  $\delta_m^k = \frac{\gamma_m^k(1 - \gamma_m^k)}{(\gamma_m^k)^2 + \gamma_m^{k+1}}$
  - 10:  $c_m^{k+1} = v_m^{k+1} + \delta_m^k(v_m^{k+1} - v_m^k)$
  - 11:  $k = k + 1$
  - 12: **end while**
  - 13:  $v_m^* = v_m^k$
- 

where

$$v_m^{k+1}(i) = \begin{cases} -U & \text{if } \tilde{v}_m^{k+1}(i) < -U \\ \tilde{v}_m^{k+1}(i) & \text{if } -U \leq \tilde{v}_m^{k+1}(i) \leq U \\ U & \text{if } \tilde{v}_m^{k+1}(i) > U. \end{cases}, i = 0 \dots N-1 \quad (10)$$

### 3.2. Optimal projected gradient method

In this subsection, a projected gradient optimization method is presented that reaches an optimal convergence for the class of convex optimization problems with *strongly convex* objective functions. This method was first proposed in [7] and variants of the method have been applied in diverse applications, e.g. real-time clipping of audio signals [8].

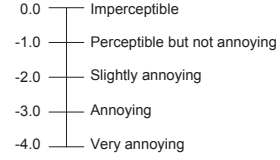
Algorithm 1 summarizes the optimal projected gradient optimization method. In each iteration, a standard projected gradient step is performed on a potentially infeasible weighted sum of two previous feasible iterates. Knowledge of the Lipschitz constant  $C_m$  of the gradient  $\nabla f$  and the convexity parameter  $\mu_m$  of  $f$  on the set  $\Omega$  is assumed. In order to establish  $C_m$  and  $\mu_m$  for optimization problem (2), the next two lemmas are proposed.

**Lemma 3.1.** (cfr: [7]) *Let function  $f$  be twice continuously differentiable on set  $\Omega$ . The gradient  $\nabla f$  is Lipschitz continuous on set  $\Omega$  with Lipschitz constant  $C$  if and only if*

$$\|\nabla^2 f(z)\| \leq C, \forall z \in \Omega \quad (11)$$

**Lemma 3.2.** (cfr: [7]) *Let function  $f$  be twice continuously differentiable on set  $\Omega$ . The function  $f$  is strongly convex on set  $\Omega$  with convexity parameter  $\mu$  if and only if there exists  $\mu > 0$  such that*

$$\nabla^2 f(z) \geq \mu I, \forall z \in \Omega \quad (12)$$



**Fig. 2.** The ITU-R five-grade impairment scale

Using Lemma 3.1, it is proved that the Lipschitz constant  $C_m$  can be computed as

$$\begin{aligned} C_m &= \|A_m\| = \max_{1 \leq i \leq N} \lambda_i(A_m) \\ &= \max_{1 \leq i \leq N} \lambda_i(H^H D^H W_m D H) \end{aligned} \quad (13)$$

where  $\lambda_i(A_m)$ ,  $i = 1 \dots N$ , denote the eigenvalues of  $A_m$ .

Using Lemma 3.2, it is proved that the convexity parameter  $\mu_m$  can be computed as

$$\begin{aligned} \mu_m &= \min_{1 \leq i \leq N} \lambda_i(A_m) \\ &= \min_{1 \leq i \leq N} \lambda_i(H^H D^H W_m D H). \end{aligned} \quad (14)$$

## 4. SIMULATION RESULTS

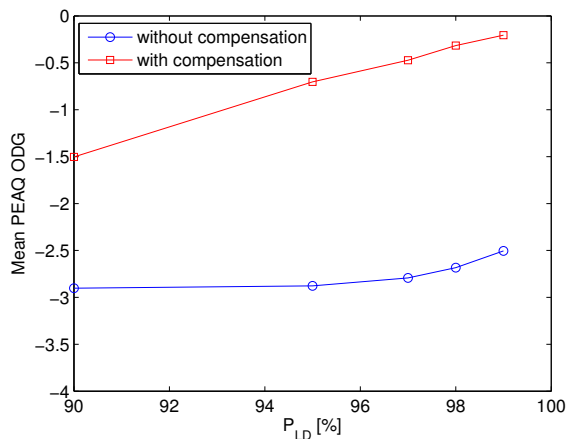
For audio quality evaluation purposes, a test database consisting of 8 audio excerpts was compiled (see Table 1 for details). A first set of excerpts (numbers 1-5) was extracted from different commercial audio CDs. A second set of excerpts (numbers 6-8) was extracted from an ITU CD-ROM associated to Recommendation BS.1387-1 [14].

The loudspeaker was modeled with a Hammerstein model, using a hard clipping nonlinearity with linear region  $[-U, U]$  and a 5th-order lowpass filter with impulse response  $h = [1 \ 0.7 \ 0.5 \ 0.3 \ 0.1]^T$ . Each audio signal in the test database was passed through the Hammerstein loudspeaker model, with and without performing a preceding perception-based nonlinear loudspeaker compensation step. The following parameters were used for nonlinear loudspeaker compensation:  $N = 512$ ,  $\alpha = 0.04$ , and using  $\kappa_m$  iterations of Algorithm 1 for all instances of optimization problem (2), such that the solution accuracy  $\epsilon = f(v_m^{\kappa_m}) - f(v_m^*) = 10^{-12}$ . Simulations were performed for five different degrees of nonlinear loudspeaker distortion  $P_{LD}$ . The parameter  $U$  was set such that the amplitude of the input audio signal  $x[n]$  exceeded the linear region  $[-U, U]$  for  $P_{LD} = \{90, 95, 97, 98, 99\}\%$  of the samples.

For each of a resulting total of  $8 \times 5 \times 2 = 80$  processed audio signals, the PEAQ (*Perceptual Evaluation of Audio Quality*) [14] measure was calculated. The PEAQ ODG (Objective

**Table 1.** Audio excerpts database used for comparative audio quality evaluation (16 bit mono at 44.1 kHz)

Nr.	Name	Texture	Composition	Style	Duration [s]	Sample <sub>start</sub>	Sample <sub>end</sub>	Origin
1	poulenc.wav	polyphonic	instrumental	classical	17.8	400000	1183000	[9]
2	rhcp.wav	polyphonic	instrumental	rock	9.8	468996	900000	[10]
3	pierle.wav	polyphonic	instrumental+vocal	pop	11.7	2234000	2750000	[11]
4	chopin.wav	monophonic	instrumental	classical	17.8	50000	836200	[12]
5	kraftwerk.wav	polyphonic	instrumental	electronic	17.2	7480000	8240000	[13]
6	breftri.wav	monophonic	instrumental	classical	19.7	1	869675	[14]
7	crefsax.wav	monophonic	instrumental	classical	10.9	1	479026	[14]
8	grefcla.wav	monophonic	instrumental	classical	6.9	1	302534	[14]

**Fig. 3.** Mean PEAQ ODG for audio signals processed with and without perception-based nonlinear loudspeaker compensation, as a function of the degree of nonlinear loudspeaker distortion  $P_{LD}$ .

Difference Grade) has a range between 0 and -4, corresponding to the ITU-R impairment scale depicted in Figure 2.

In Figure 3, the average PEAQ ODG score over all 8 audio signals is plotted as a function of the degree of nonlinear loudspeaker distortion  $P_{LD}$ . A monotonically increasing average audio quality score is observed for increasing  $P_{LD}$ . Clearly, including the perception-based nonlinear loudspeaker compensation is seen to improve the average objective audio quality score significantly, and this for all considered degrees of nonlinear loudspeaker distortion<sup>2</sup>.

## 5. CONCLUSION

In this paper a novel nonlinear loudspeaker compensation technique was presented. By including a psychoacoustic model and embedding convex optimization into the algorithm, it was possible to minimize the perceptible distortion.

<sup>2</sup>Processed audio signals from this simulation are available online on <http://homes.esat.kuleuven.be/~bdefraen/>.

A fast projected gradient optimization method was proposed for solving the resulting convex optimization problems. Comparative objective evaluation experiments have shown that the proposed nonlinear loudspeaker compensation improves the average objective audio quality score, and this for different degrees of nonlinear loudspeaker distortion.

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