HILBERT-HUANG TRANSFORM BASED HIERARCHICAL CLUSTERING FOR EEG DENOISING

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ABSTRACT

Empirical mode decomposition (EMD) is a recently introduced decomposition method for non-stationary time series. The sum of the decomposed intrinsic mode functions (IMF) can be used to reconstruct the original signal. However, if the signal is corrupted by wideband additive noise, several IMFs may contain mostly noise components. Hence, it is a challenging study to determine which IMFs have informative oscillations or information free noise components. In this study, hierarchical clustering based on instantaneous frequencies (IF) of the IMFs obtained by the Hilbert-Huang Transform (HHT) is used to denoise the signal. Mean value of Euclidean distance similarity matrix is used as the threshold to determine the noisy components. The proposed method is tested on EEG signals corrupted by white Gaussian noise to show the denoising performance of the proposed method.

***Index Terms*—** Hilbert-Huang Transform, hierarchical clustering, EEG denoising.

1. INTRODUCTION

Linear and non-linear filtering methods are widely studied and applied to eliminate irrelevant components such as power line interference and other corrupting noise before processing biological signals. However, white noise interference canceling is a challenging task [1].

Linear filtering methods are commonly applied where the signal and noise are stationary and their spectra are known and non-overlapping [2]. A well known linear technique, the Wiener filter can be applied for smoothing additive high frequency artifacts under the assumption of wide sense stationarity [3]. In order to process non-stationary signals, time-frequency based approaches such as wavelet transform (WT) and Gabor expansion are considered to be more appropriate. As such, a threshold is applied to the resulting coefficients to reconstruct the filtered signal [4] [5]. Even though it gives satisfactory time-frequency resolution, the trial and error approach for selecting the optimum wavelet function makes it ineffective for denosing signals with white noise and impulses of short duration [6]. Huang at al. has introduced the empirical mode decomposition (EMD) as an alternative method to analyze non-stationary signals [7]. EMD does not require any basis function, i.e., it is a data driven, adaptive method. It decomposes the signal into a few oscillations called intrinsic mode functions (IMF) which are derived from the signal. They are semi-orthogonal functions having fluctuant frequency spectrum [8]. Hilbert-Huang transform (HHT) obtains the instantaneous frequency (IF) of each IMF which is an effective method to analyze IMFs in the frequency domain [9]. However, it is still a challenging study to determine relevant or irrelevant IMFs, which can be considers as the first step for EMD based denoising. Wu and Huang [10] applied a hypothesis test based method to determine the relevant information level of the IMFs. However, it is reported to perform poorly for low frequency components. Information theoretical based approaches such as relative entropy and mutual information are investigated to find irrelevant noisy oscillations [11] [12].

Electroencephalography (EEG) signals are inclined to be corrupted by environmental random noise due to recording mechanism and interference from other sources. This may be modeled as additive white Gaussian noise (AWGN) on EEG signals. In this study, AWGN corrupted epileptic and normal EEG signals are denoised by using the proposed HHT and clustering based method. After computing instantaneous frequencies (IFs) of IMFs, hierarchical clustering is applied to determine a threshold based on the distance metric. IMFs below the threshold are used to reconstruct a filtered version of the given signal.

2. HILBERT-HUANG TRANSFORM

The combination of EMD [7] and Hilbert spectral analysis [9] is known as HHT which represents a non-stationary signal in the time-IF domain. Empirically, HHT is a superior tool for

*This work was partially supported by The Research Fund of The University of Istanbul. Project numbers:14381 and 31474.

non-linear and non-stationary signal processing in the time-frequency domain.

EMD algorithm decomposes a signal into IMFs without the requirement of apriori basis function so that the sum of IMFs equals to the original signal. Each IMF should satisfy two criteria: First, the number of extrema and the number of zero crossings must be equal or must differ by one at most. Second, the mean of the envelopes determined by the local maxima and minima should be zero. Consequently, fluctuations of IMFs can be reduced [13]. The most important stage of the EMD algorithm to find IMFs is called *Sifting*, which involves the following steps [13]:

i) Find local maxima, $M_i, i = 1, 2, \cdots$ and minima, $m_k, k = 1, 2, \cdots$ in $x(n)$.

ii) Compute the corresponding interpolated signals $M(n) = f_M(M_i, n)$, and $m(n) = f_m(m_k, n)$. These are the upper and lower envelopes of the signal.

iii) Let $e(n) = (M(n) + m(n))/2$.

iv) Substract $e(n)$ from the signal: $x(n) = x(n) - e(n)$.

v) Return to step (i) and stop when $x(n)$ remains nearly unchanged.

vi) After obtaining an IMF, $c_i(n)$, remove it from the signal $x(n) = x(n) - c_i(n)$ and return to step (i) if $x(n)$ is not constant or trend, $r(n)$.

Thus, signals can be decomposed into number of IMFs with different band limited spectra. The signal may be reconstructed by the sum of IMFs described as follows;

$$s(n) = \sum_{i=1}^{M} c_i(n) + r(n) \tag{1}$$

where M is the total number of extracted IMFs and $r(n)$ is the final residue. Hilbert-Huang spectrum is defined as the IFs of the decomposed IMFs. First, analytic versions of the IMFs are computed using Hilbert Transform (HT) described as follows [14]:

- Compute N-point one-sided discrete-time Fourier transform (DTFT) $C_i(m)$ of the N real data samples.

- Form the N-point one-sided discrete-time analytic signal

$$C_i^A(m) = \begin{cases} C_i(0) & m = 0 \\ 2C_i(m) & 1 \leq m \leq \frac{N}{2} - 1 \\ C_i(\frac{N}{2}) & m = \frac{N}{2} \\ 0 & \frac{N}{2} + 1 \leq m \leq N - 1. \end{cases} \tag{2}$$

- Compute the complex-valued discrete-time analytic representation of the IMFs using an N-point inverse DTFT,

$$c_i^A(n) = \frac{1}{N} \sum_{m=0}^{N-1} C_i^A(m)\, e^{\jmath 2\pi mn/N} \tag{3}$$

Then, they can be written in polar form by

$$c_i^A(n) = A_i(n)\, e^{\jmath \theta_i(n)} \tag{4}$$

where $A_i(n)$ and $\theta_i(n)$ are instantaneous amplitude (IA) and phase of the i^{th} IMF, which are computed by

$$A_i(n) = \sqrt{\mathrm{Re}\{c_i^A(n)\}^2 + \mathrm{Im}\{c_i^A(n)\}^2} \tag{5}$$

$$\theta_i(n) = \arctan\left(\frac{\mathrm{Im}\{c_i^A(n)\}}{\mathrm{Re}\{c_i^A(n)\}}\right) \tag{6}$$

After unwrapping the $\theta_i(n)$, IF is estimated by taking the first order difference. Thus, a discrete-time signal is represented in terms of IFs and IAs of its IMFs.

3. HIERARCHICAL CLUSTERING

Hierarchical clustering is one of the methods of cluster analysis that aims to form a hierarchy of clusters. There are two approaches to hierarchical clustering: Agglomerative is a bottom up approach grouping small clusters into larger ones. Divisive is a top down approach splitting big clusters into small ones [15]. In this section, we provide an overview of agglomerative clustering algorithm as follows:

i) Start with each point in a cluster of it.

ii) Until there is only one cluster
(a) Find the closest pair of clusters,
(b) Merge them.

iii) Return the tree of cluster-mergers.

However, this is not the same as how close two data are, or how close two partitions are. Hence, one of the linkage methods, single-linkage [16] suggests that the distance between two clusters, A and B as the minimum distance between their members:

$$d(A, B) = \min_{\vec{x} \in A, \vec{y} \in B} \|\vec{x} - \vec{y}\| \tag{7}$$

The distance metric above may be Euclidean, Mahalanobis, Cosine, Manhattan etc. Here, we use Euclidean distance as a similarity metric among the IF spectra of IMFs:

$$d_{i,j}(IF_i, IF_j) = ||IF_i(n) - IF_j(n)|| \tag{8}$$

where $||.||$ denotes the Euclidean norm, $i = 1, 2, \cdots, M-1$, $j = i+1, i+2, \cdots, M$, and M is the total number of IMFs. Therefore, similarity vector is obtained as follows:

$$D = [d_{12}, d_{13}, \cdots, d_{1M}, d_{23}, d_{24}, \cdots, d_{2M}, \cdots, d_{M-1M}] \quad (9)$$

Finally, after single-linkage, dendrogram is obtained as a result of the clustering that illustrates the arrangement of the clusters.

4. THE PROPOSED METHOD

We assume that several IMFs of a noisy signal may be irrelevant, information free oscillations. The first few IMFs include mostly white noise components compared to others, in case the signal is corrupted by AWGN. However, a reliable metric to determine which IMFs contain mostly noise is a vital problem in a reconstruction algorithm.

In our method, a threshold score is determined as a result of hierarchical clustering. After applying EMD to AWGN corrupted signal, IF spectra of IMFs are obtained by HHT and they are clustered. An adaptive threshold is computed based on the distance metric used in clustering. The steps of the proposed methods are described below:

a) Let $x(n) = s(n) + w(n)$ be observed noisy signal where $x(n)$ and $w(n)$ denote the desired signal and the AWGN respectively.

b) Decompose $x(n)$ into $c_i(n)$, $i = 1, 2, \cdots, M$, where M is the total number of IMFs.

c) Apply HHT to $c_i(n)$ to obtain IF spectra, $IF_i(n)$.

d) Compute Euclidean distance among the $IF_i(n)$, $d_{ij}(IF_i, IF_j) = ||IF_i(n) - IF_j(n)||$, where $i = 1, 2, \cdots, M-1$ and $j = 1, 2, \cdots, M$.

e) Obtain similarity vector, D.

f) Normalize D in the range of [0.1, 1].

g) Determine a threshold as the mean value of D, $\gamma = \frac{1}{L}\sum_{i=1}^{L} D_i$, where $L = M(M-1)/2$ denotes the length of D.

h) Apply single-linkage and plot dendrogram to visualize results.

i) Then reconstruct a denoised signal, $\hat{s}(n)$ as: $\hat{s}(n) = \sum_j c_j(n), \quad j = \{i | D_i < \gamma\}$.

Thus, irrelevant or mostly white noise IMFs can be excluded in the reconstruction of the signal. The proposed method is tested on single channel EEG signals corrupted by different level of AWGN to check its denoising performance.

5. RESULTS

Single channel normal and epileptic EEG recordings [17] with 200 Hz sampling frequency are used to test the proposed method. They are decomposed into IMFs by applying the EMD algorithm introduced by Rato et al. [13]. The recordings and their IMFs for 0.5 sec. epoch time are shown in Fig. 1. Normal and epileptic EEG signals are filtered by

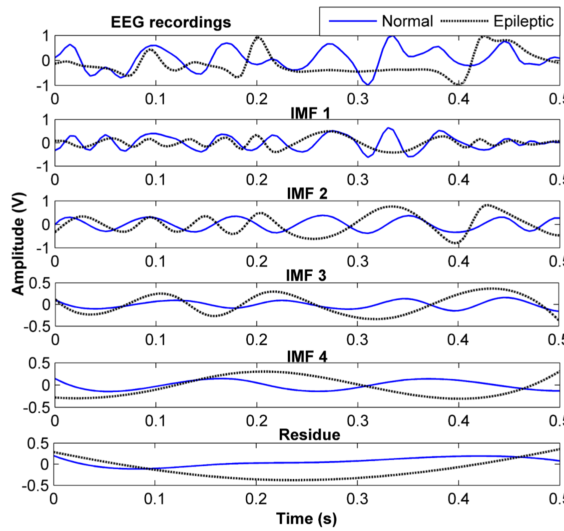

Fig. 1. Original EEG recordings and IMFs.

necessary low pass filter ($f_c = 40Hz$) to eliminate artifacts. Additionally, IF spectra after applying HHT are given in Fig. 2. As shown in Figs. 1 and 2, five IMFs are decomposed for

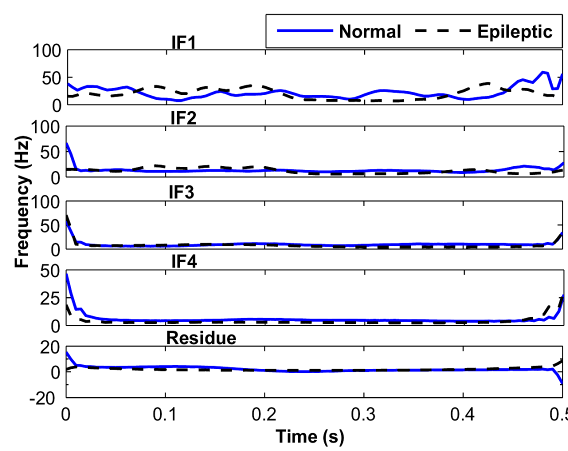

Fig. 2. Hilbert-Huang Transforms of EEG signals.

normal and epileptic EEG signals. EEG signals with 20 dB signal-to-noise ratio (SNR) are analyzed by the above procedure to test the proposed method. The dendrogram based on the normalized similarity vector and the calculated threshold are shown in Fig. 3. An extra IMF is resulted for 20 dB

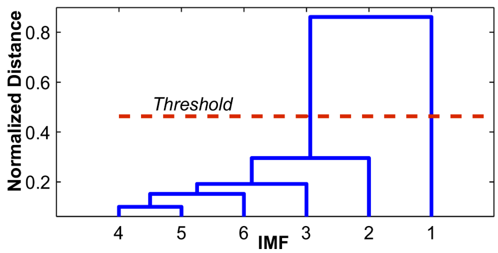

Fig. 3. The dendrogram and threshold for normal EEG recording with 20 dB SNR.

noise added normal EEG signal, and the first IMF is due to the AWGN component shown in Fig. 4 which is detected by the threshold. Therefore, the first IMF which has higher distance score than the computed threshold is eliminated to reconstruct the filtered signal. Then the same procedure is

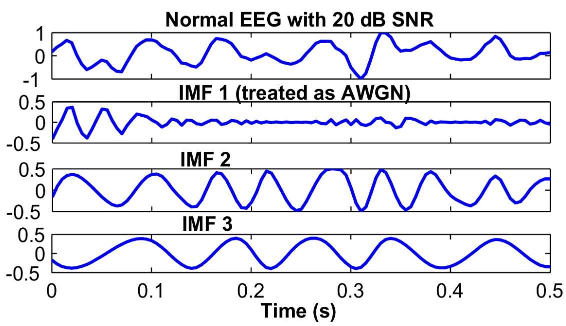

Fig. 4. The three IMFs of the normal EEG with 20 dB SNR.

applied to epileptic EEG, and the result is shown in Figure 5. Reliability of the proposed method is tested for normal

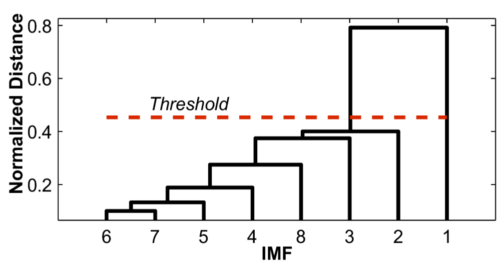

Fig. 5. The dendrogram and threshold for epileptic EEG recording. with 20 dB SNR.

and epileptic EEG signals with 0 dB SNR. The results of the thresholding method for normal and epileptic EEG are shown in Figs. 6 and 7. In case two IMFs are mostly noisy oscillations, they are identified successfully, which have higher distance score than the threshold. First 4 IMFs of the epileptic EEG with 0 dB SNR are given in Fig. 8 to illustrate this point. It is clear that the first two IMFs are noisy components, which should be excluded in reconstruction. Finally, the re-

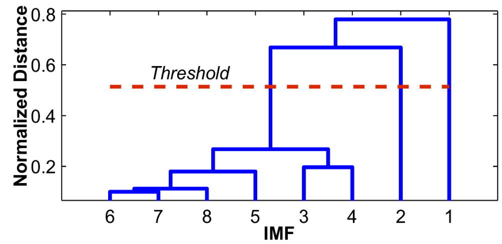

Fig. 6. The dendrogram and threshold for normal EEG recording with 0 dB SNR.

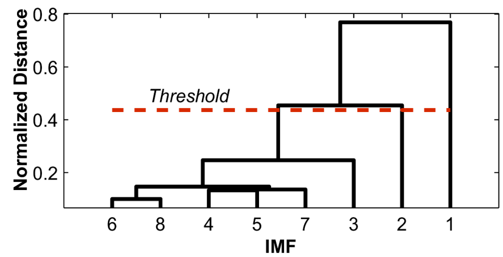

Fig. 7. The dendrogram and threshold for epileptic EEG recording with 0 dB SNR.

construction of EEG signals with 20 and 0 dB SNR using the IMFs with lower distance value than the threshold is given in Figs. 9 and 10. After denoising the signals with 0 dB and 20 dB AWGN, the SNR values are computed as 12.05 dB 27.34 dB for normal, 7.58 dB and 20.08 for epileptic EEG respectively. Hence, the identification of white noise dominant IMFs using the proposed method based on HHT and hierarchical clustering can be used for denoising. However, interference of AWGN into informative IMFs still remains as an artifact due to nature of the EMD algorithm.

6. CONCLUSION

In this study, additive white Gaussian noise (AWGN) removal from normal and epileptic EEG signals is investigated by using the EMD and hierarchical clustering. Instantaneous frequency (IF) spectra of intrinsic mode functions (IMF) which is called Hilbert-Huang spectrum have distinguishing properties between noisy and noise free oscillations. Mean value of the Euclidean distanced similarity vector is used as a threshold to identify the AWGN components. The IMF with higher distance then the threshold on single-linkage based dendrogram is excluded in the reconstruction. Therefore, IMFs including mostly white noise are successfully determined using the proposed method and a filtered version of the given signal is reconstructed.

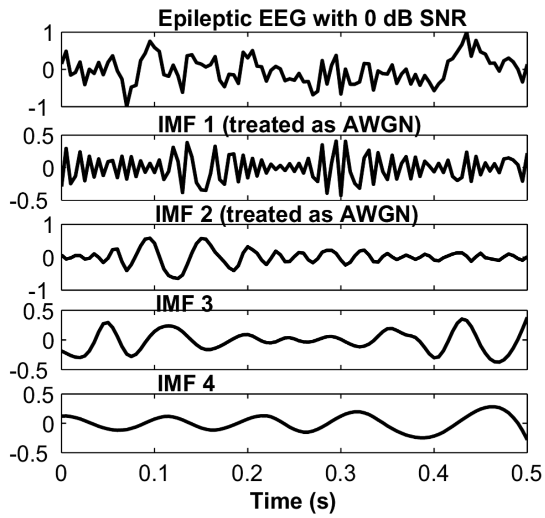

Fig. 8. The four IMFs of the epileptic signal with 0 dB SNR.

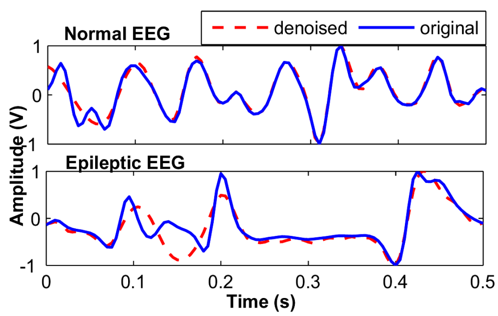

Fig. 9. Reconstruction of EEG signals with 20 dB AWGN.

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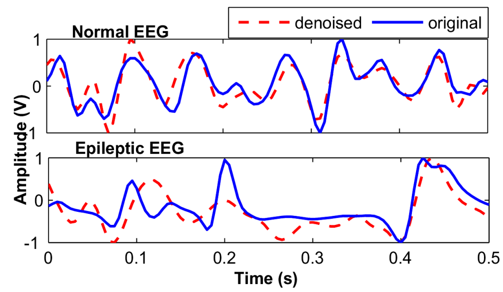

Fig. 10. Reconstruction of EEG signals with 0 dB AWGN.

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