PERFORMANCE ANALYSIS OF COOPERATIVE SPECTRUM SENSING FOR COGNITIVE RADIO USING STOCHASTIC GEOMETRY

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ABSTRACT

In this paper, we analyse (censored) cooperative spectrum sensing for cognitive radios (CRs) where, in order to save power, each CR only transmits its test statistic to a fusion center (FC) if the test statistic is greater than some threshold (ξ). One problem with this approach is that, it is possible that no test statistics may be transmitted. The solution that we propose here is to choose an optimum test statistic threshold ($\xi = \xi_{opt}$) to avoid (probabilistically) no test statistic being transmitted to the FC. We also propose the additional power saving approach where only if the required CR transmit power (P_i, to achieve a required SNR at the FC) is less than some optimum transmit threshold ($P_t = P_{topt}$) do we transmit the test statistic. We then show the effect of (ξ_{opt}, P_{topt}) on both the detection performance at the FC (using selection combining) and the total CRs transmit power. Finally, simulation results show this approach mitigates the original problem (in an energy efficient way) of no CR test statistic being transmitted to the FC.

I. INTRODUCTION

Cognitive radio (CR) is a very effective technology for dealing with a spectrum scarcity that has resulted from the rapid development of wireless communication technology [1] and [2]. The unlicensed device/secondary user attempts to harness the licensed band/primary band opportunistically in a manner that the primary receiver is protected from any harmful interference. But local spectrum sensing is not enough when there is a fading channel. So in the literature cooperative spectrum sensing (CSS) has been proposed [3], [4] and [5]. In CSS, each cognitive radio reports its measurement/test statistic to the FC. The reported test statistics consume power and this power consumption might be significant if the number of cognitive radios is large. Thus power consumption needs to be considered in CSS design.

Many papers currently deal with the issue of power consumption. For example, in [6] and [7], the concept of a censoring (sending only test statistics that are larger than a local threshold (ξ)) was introduced to reduce the number of transmitted test statistics and thus save energy. This approach showed a slight performance degradation compared with uncensored cooperative detection. However in [6] and [7], the geometry of the CRs (i.e., the spatial distribution of CRs with respect to the primary user or the FC) was not considered. Also, the number of CRs was assumed known. All those assumptions are not realistic.

In censored cooperative sensing, problems can arise if the FC does not receive any test statistic from the CRs because the threshold ξ is set too high. As a result, the detection performance at the FC might degrade. Also this this issue has not been taken into account in [6] and [7].

To further reduce energy consumption we will also introduce an additional parameter, P_t - the transmit power threshold. We will only transmit a test statistic from the CR if $P_i \leq P_t$, where P_i is the CR transmit power necessary to acheive a required SNR at the FC. But with a poor channel, P_i will be large and it may not satisfy $P_i \leq P_t$. So once again, no test staistics may be received at the FC.

To address this issue, we propose to examine the activity probability (p_a) which is the probability that at least one test statistic is received by the FC. In this paper, we aim to find the optimum test statistic threshold ($\xi = \xi_{opt}$) and the optimum transmit power threshold ($P_t = P_{topt}$) so $p_a \to 1$. Simulation results will show that this approach mitigates the original problem of no test statistic being transmitted (in an energy efficient way).

So the contributions of this paper can be summarised as follows. First, we find by simulation both the ξ_{opt} and the P_{topt} such that $p_a \to 1$ (with a considerable saving power). Second, the detection performance of a selection combining based FC for an unknown number of CRs (with small-scale fading and pathloss) is theoretically derived. Finally, the average power that is needed to transmit the test statistics to the FC is obtained analytically.

The rest of this paper is organized as follows. The system model is introduced in Section II. Cooperative spectrum sensing (CSS) is presented in Section III. In Section IV, we derive both probability of false alarm and detection for selection combining based data fusion is described. Power consumption is analyzed in Section V. Results and discussion are given in Section VI. Finally, in Section VII we present our conclusions.

II. SYSTEM MODEL

II-A. Primary and secondary network models

We consider that the secondary users/cognitive radios are distributed in a plane according to a homogeneous Poisson point process (PPP), i.e., Φ with intensity λ. The probability of n secondary users being inside an area $\mathcal{A} \in R^2$ is characterized by

$$\text{Prob}\{n \text{ users in } \mathcal{A}\} = \frac{(\lambda\mathcal{A})^n}{n!}\, e^{-\lambda\mathcal{A}}, \quad n \geq 0. \quad (1)$$

The location of the ith cognitive radio is denoted by (x_i, y_i) and it is uniformly distributed inside the area $\mathcal{A}$. The cognitive radios are supervised by the FC located in the center of the area $\mathcal{A}$. For the primary network, we consider a single primary user located at the origin. We also assume that the FC is located at a distance R_x from the primary user.

II-B. Channel model

The channel between the ith cognitive radio and the primary user is modeled by $H_i l(x_i, y_i)$, where H_i is an exponential random variable with a unit mean (modeling flat fading) and $l(x_i, y_i)$ is the path loss between the location (x_i, y_i) and the primary user. This can also be written in terms of the path loss exponent α, and the frequency dependent constant κ, i.e., $l(x_i, y_i) = \frac{\kappa}{(x_i^2+y_i^2)^{0.5\alpha}}$. For simplicity, we assume $\kappa = 1$.

II-C. Received signal model

The ith cognitive radio inside the area $\mathcal{A}$ receives either noise $(\mathcal{H}_0)$ or a primary signal plus noise $(\mathcal{H}_1)$ dependent on the activity of the primary network. So mathematically the signal received by the ith cognitive radio is given by the following two hypotheses:

$$\begin{aligned} \mathcal{H}_0: \quad y_i(m) &= v_i(m) \\ \mathcal{H}_1: \quad y_i(m) &= \sqrt{H_i l(x_i, y_i)}\, s(m) + v_i(m) \end{aligned} \quad (2)$$

where $m = 0, 1, 2, ..., M-1$; M is the number of samples collected by the ith cognitive radio; $y_i(m)$ is the signal received by ith cognitive radio; $v_i(m)$ is i.i.d. circularly symmetric complex Gaussian noise, $(CN(0, \sigma_v^2))$; $s(m)$ is the (unit power) primary signal which is randomly and independently drawn from a complex constellation with unit power.

III. COOPERATIVE SPECTRUM SENSING

We consider that the ith cognitive radio employs an energy detector. It compares the test statistic (T_{EDi}) with the local threshold ξ, where $T_{EDi} = \sum_{m=0}^{M-1} |y_i(m)|^2/M$. Only if $(T_{EDi} > \xi)$ is satisfied will the test statistic to be sent to the FC. In this paper we will evaluate the performance of a selection combining data fusion center. Thus a global test statistic (T_{max}) will be chosen as follows:

$$T_{max} = \max_{\substack{(x_i,y_i)\in\Phi \\ T_{EDi}>\xi}} \left(T_{EDi}\right) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \tau \quad (3)$$

where τ is the global threshold. The idea behind the local threshold at each CR is to save power by transmitting only the most ‘robust’ test statistics to the FC.

To save additional power we introduce another parameter which is a transmit power threshold (P_t). To send the test statistic T_{EDi} to the FC, the required transmit power P_i for the ith cognitive radio should satisfy $P_i \leq P_t$ where

$$P_i = \frac{P_{ref}}{G_i j(x_i, y_i)}. \quad (4)$$

Note that (4) is to guarantee that the received power at the FC is P_{ref}. Here $j(x_i, y_i)$ is the path loss between the ith cognitive radio and the FC and G_i is a unit power exponential random variable representing the small-scale fading between the ith cognitive radio and the FC. So the test statistic in (3) with the condition in (4) now becomes

$$T_{max} = \max_{\substack{(x_i,y_i)\in\Phi \\ T_{EDi}>\xi \\ P_i \leq P_t}} \left(T_{EDi}\right) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \tau. \quad (5)$$

As we mentioned earlier, there exists a probability (because of the choice of ξ and P_t) that no test statistic might be sent to the FC and so the detection performance could be degraded. So we define the activity probability (p_a) as the probability that at least one test statistic is received by the FC under $\mathcal{H}_1$. Here we need $p_a \to 1$ to avoid any degradation in the detection performance, and so we will examine how the choice of ξ and P_t affects p_a.

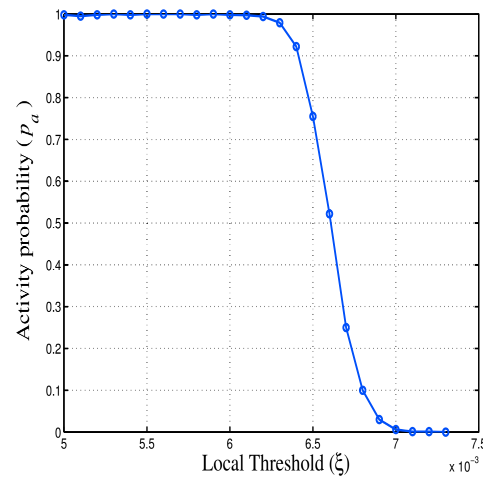

Fig. 1. The activity probability (p_a) versus the local threshold (ξ) for $P_t = 300$.

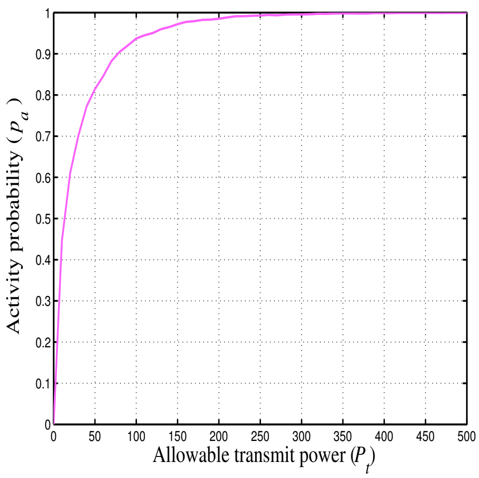

Fig. 2. The activity probability (p_a) versus the transmit power threshold (P_t) for $\xi = 6.1 \times 10^{-3}$.

Fig. (1)[1] shows a plot of p_a versus the ξ. If ξ is very small then $p_a \rightarrow 1$ and so a significant number of test statistics will be sent to the FC. Accordingly, a lot of power would be consumed. Also from Fig. 1, if ξ is very large, it means $p_a \rightarrow 0$ and this is desirable from a power saving point of view but it is undesirable from a detection point of view. Thus ξ should be chosen as large as possible such that $p_a \approx 1$. We call this maximum threshold an optimum local threshold $(\xi = \xi_{opt})$.

The choice of parameter P_t also effects p_a. This can also be seen in Fig. 2[1] which shows a plot of p_a versus the P_t. We can see that when P_t is small p_a is also small. In terms of saving power this is desirable but in terms of detection performance it is not. However, if P_t is large, then $p_a \rightarrow 1$ and this will increase the power consumption. But for the detection performance it is desirable. Motivated by the above explanation, then P_t should be chosen as small as possible such that $p_a \approx 1$. We call this minimum transmit power threshold the optimum transmit power threshold $(P_t = P_{topt})$. Therefore, ξ_{opt} and P_{topt} should satisfy the following condition:

$$(\xi_{opt}, P_{topt}) = \big\{\max \xi \text{ and } \min P_t \text{ such that } p_a(\xi_{opt}, P_{topt}) \simeq 1\big\}. \quad (6)$$

IV. DETECTION PERFORMANCE ANALYSIS (NO POWER CONSTRAINT)

In this section we derive the false alarm probability (p_{FA}) and the detection probability (p_D) performance for (3) (for (5) the analysis is not tractable) in the case when (6) is satisfied. When the CRs send their test statistics $(T_{EDi} > \xi_{opt})$ to the FC it applies selection combining so that the p_{FA} can be written as

$$p_{FA} = \text{Prob}(T_{max} > \tau \big| \mathcal{H}_0) \quad (7)$$

because all the secondary users are independent. Now (7) can be written as

$$p_{FA} = 1 - \mathbb{E}_{H_i,\Phi}\Big[\prod_{\substack{(x_i,y_i)\in\Phi \\ T_{EDi}>\xi_{opt}}} \text{Prob}(T_{EDi} < \tau \big| \mathcal{H}_0)\Big]. \quad (8)$$

where T_{EDi} under $\mathcal{H}_0$ is a sum of the squares of $2M$ Gaussian random variables with zero mean, . Therefore, T_{EDi} follows a central chi-square distribution with $2M$ degrees of freedom. So $\text{Prob}(T_{EDi} < \tau \big| \mathcal{H}_0) = 1 - \frac{\Gamma(M, \frac{2\tau}{\sigma_v^2})}{\Gamma(M)}$, where $\Gamma(.)$ and $\Gamma(.,.)$ are gamma function and incomplete gamma function respectively. Thus (8) can be written as

$$p_{FA} = 1 - \mathbb{E}_{H_i,\Phi}\Big[\prod_{\substack{(x_i,y_i)\in\Phi \\ T_{EDi}>\xi_{opt}}} \Big(1 - \frac{\Gamma(M, \frac{2\tau}{\sigma_v^2})}{\Gamma(M)}\Big)\Big] \quad (9)$$

and by applying the generating functional of the Poisson process in (9) [see [8], eq. (4.3.8)] so we arrive at

$$p_{FA} = 1 - \exp\big(-\lambda\mathcal{A}\frac{\Gamma(M, \frac{2\tau}{\sigma_v^2})}{\Gamma(M)}\big). \quad (10)$$

Note that (10) is independent from ξ_{opt} and so it will not affect the detection performace as we will see in the simulation results.

For the probability of detection (p_D), it can be obtained in a similar manner as p_{FA}. Thus the test statistic T_{EDi} under $\mathcal{H}_1$ will have a noncentral chi square distribution with $2M$ degrees of freedom and a non centrality parameter $\nu = \frac{2|H_i|^2 l(x_i,y_i)}{\sigma_v^2}$. So $\text{Prob}(T_{EDi} < \tau \big| \mathcal{H}_1) = 1 - Q_M(\sqrt{v}, \sqrt{\frac{2\tau}{\sigma_v^2}})$, where $Q_M(.,.)$ is the generalized Marcum Q-function (conditioned on the channels and the pathloss) defined as follows,

$$Q_M(a,b) = \int_b^\infty \frac{x^M}{a^{M-1}} \exp\left(\frac{x^2+a^2}{2}\right) dx.$$

So the probability of detection is

$$p_D = 1 - \mathbb{E}_{H_i,\Phi}\Big[\prod_{\substack{(x_i,y_i)\in\Phi \\ T_{EDi}>\xi_{opt}}} \Big(1 - Q_M(\sqrt{v}, \sqrt{\frac{2\tau}{\sigma_v^2}})\Big)\Big]. \quad (11)$$

Then by applying the generating functional of the Poisson process in (11) [see [8], eq. (4.3.8)], we arrive at

$$p_D = 1 - \exp\Big(-\lambda \int_{\mathcal{A}} \int_0^\infty Q_M(\sqrt{v}, \sqrt{\frac{2\tau}{\sigma_v^2}}) dH\, dxdy\Big). \quad (12)$$

The inner integral (the average p_D over flat fading) is derived in [[9], equation (20)]. So after substituting [[9], equation (20)] into (12) (conditioned on distance) it can be evaluated in one integral instead of two.

[1]Figs. 1 and 2 are ploted for the system parameters defined in Section VI.

V. AVERAGE TOTAL POWER CONSUMPTION

In this section, we derive the average total power consumption $\mathbb{E}[\triangle(\xi, P_t)]$, where $\triangle(\xi, P_t)$ is the secondary network's total power that is needed to transmit the test statistics to the FC. We derive the average total power for two scenarios.

- Scenario I: The first scenario is when the primary user is absent. In this scenario, the total power is given by

$$\Delta_0(\xi, P_t) = \sum_{\substack{(x_i,y_i)\in\Phi_p \\ T_{EDi}\big|\mathcal{H}_0>\xi}} P_i. \tag{13}$$

  Here (Φ_p) is the set of transmitting secondary users that satisfy $P_i \leq P_t$. In this scenario, $\text{Prob}(T_{EDi} > \xi\big|\mathcal{H}_0) = \frac{\Gamma(M,\frac{2\xi}{\sigma_v^2})}{\Gamma(M)}$. Also, the transmitting secondary users Φ_p constitutes a non-homogeneous PPP with an intensity

$$\lambda_0(x,y) = \frac{\lambda\Gamma(M,\frac{2\xi}{\sigma_v^2})}{\Gamma(M)}\text{Prob}(P < P_t)f(G)$$

  where the subscript 'i' is dropped from P_i; and $f(G)$ is the probability density function for the small-scale (power) fading of the channel gain, G. Thus the average total power when the primary user is absent is given by

$$\mathbb{E}\big[\triangle_0(\xi, P_t)\big] = \lambda P_{ref}\frac{\Gamma(M,\frac{2\xi}{\sigma_v^2})}{\Gamma(M)}\int_{\mathcal{A}}\frac{\mathcal{G}}{j(x,y)}d\mathcal{A} \tag{14}$$

  where $\mathcal{G} = \int_{\frac{P_{ref}}{j(x,y)P_t}}^{\infty} G^{-1}\text{exp}(-G)dG$ and we employed the following result [see [8], eq. (4.2.4)].
- Scenario II: The second scenario is when the primary user is present. In this scenario, the total power is given by

$$\Delta_1(\xi, P_t) = \sum_{\substack{(x_i,y_i)\in\Phi_p \\ T_{EDi}\big|\mathcal{H}_1>\xi}} P_i \tag{15}$$

where the transmitting secondary users Φ_p constitute a non-homogeneous PPP with an intensity

$$\lambda_1(x,y) = \lambda Q_M(\sqrt{v}, \sqrt{\frac{2\xi}{\sigma_v^2}})\text{Prob}(P < P_t)f(G).$$

Thus the average total power when the primary user is present is given by

$$\mathbb{E}\big[\triangle_1(\xi, P_t)\big] = \lambda P_{ref}\int_{\mathcal{A}}\int_0^{\infty}\frac{\mathcal{G}Q_M(\sqrt{v}, \sqrt{\frac{2\xi}{\sigma_v^2}})}{j(x,y)}dH\,d\mathcal{A}. \tag{16}$$

If we denote the activity of the secondary network by $P(\mathcal{H}_0)$, then the average total power of the secondary network for sending the test statistics to the FC is given by

$$\mathbb{E}[\triangle(\xi, P_t)] = P(\mathcal{H}_0)\mathbb{E}\big[\triangle_0(\xi, P_t)\big] + P(\mathcal{H}_1)\mathbb{E}\big[\triangle_1(\xi, P_t)\big] \tag{17}$$

where $P(\mathcal{H}_0) = 1 - P(\mathcal{H}_1)$.

VI. RESULTS AND DISCUSSION

In this paper, due to space limitations, we cannot show the effect of all the parameters on the detection performance or the power consumption. So we present some selected results to highlight the effect of choosing ξ and P_t on both the detection performance and the average total power consumption. We define the average signal to noise ratio at the FC by SNR $= 10\text{log}_{10}(\frac{1}{\sigma_v^2 R_x^\alpha})$ dB. $(\xi_{opt}, P_{topt}) = (6.1\times10^{-3}, 300)$ (from Fig. 1 and Fig. 2). Also, we consider the following system parameters $R_x = 30$, $\alpha = 3$, $P_{ref} = 1$, $\lambda = 0.01$, 10^5 Monte Carlo runs, $P(\mathcal{H}_1) = 0.2$ and $\mathcal{A} = 30\times30$.

First, Fig. 3 presents the detection performance of CSS for the case of no power constraint see ((10) and (12)). For $\xi \leq \xi_{opt}$ means that (6) is satisfied (and probabilistically we avoid the case of no test statistic being transmitted). For $\xi > \xi_{opt}$, (6) is not satisfied and the curve is ploted by simulation. Now it can be observed that the detection performance does not change for $\xi \leq \xi_{opt}$ and detection degradation appears for $\xi = 0.007$ because p_a is low (as can be seen from Fig. 1). Note that for $\xi = 0.007$ the curve stops at $p_{FA} = 0.05$, because no test statistics (as we previously discussed) are sent to the FC (which then decides 'no primary use'). So the curve stops at $p_{FA} = 0.05$.

Second, Fig. 4 shows the detection performance of CSS for no power constraint see ((10) and (12)) and for power constraint (simulation). It can be observed that the detection performance is best when there is no constraint on the CR transmit power, but in practice this cannot occur. Also, we can see the degradation of detection performance for $P_t = 50$ and this is because p_a is low (as can be seen from Fig. 2). Note that for the plot for $P_t = 50$ in Fig. (3) the curve stops at around $p_{FA} = 0.2$. This is again because of the no test statistic transmitted problem. Also, we can see that the detection performance improves with P_t.

Finally, we plot $\mathbb{E}[\triangle(\xi, P_t)]$ versus (ξ) (see - (17)) for different values of P_t. Now $P_t = P_{topt}$ is the most energy efficient parameter choice compared with other P_t values. Moreover, it is easily seen that as the ξ increases as $\mathbb{E}[\triangle(\xi, P_t)]$ decreases. In practice, the designer could choose $\xi = \xi_{opt}$ and even $P_t > P_{topt}$ to have a reasonable detection performance and the problem of no test statistic being transmitted to the FC is overcome with a considerable saving of transmit power.

VII. CONCLUSION

We have investigated, both analytically and via simulation, the performance of censored cooperative spectrum sensing (CSS) using stochastic geometry to model the secondary users. In particular, we examined how those of the CR test statistic threshold (ξ) and the transmit power threshold (P_t)

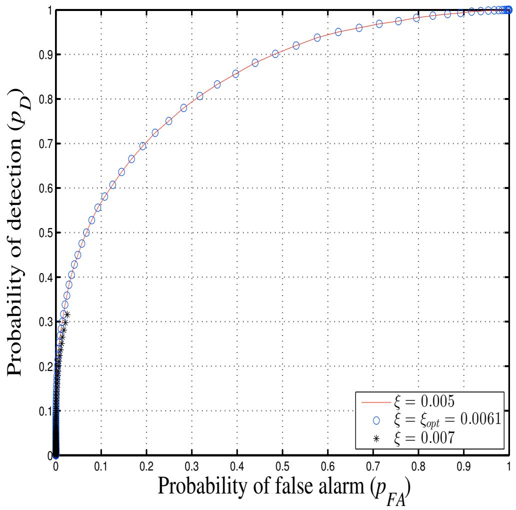

Fig. 3. The probability of detection (p_D) versus the probability of false alarm (p_{FA}) for different values of ξ. In all cases, $\mathrm{SNR} = -10\mathrm{dB}$.

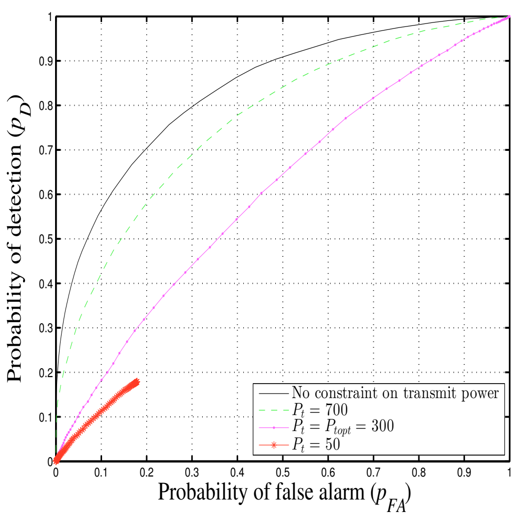

Fig. 4. The probability of detection (p_D) versus the probability of false alarm (p_{FA}) for different values of P_t. In all cases, $\mathrm{SNR} = -10\mathrm{dB}$.

effect overall performance in terms of energy consumption, p_{FA} and p_D. By optimizing (ξ, P_t) we have also shown (for the first time) how to avoid (in an energy effecient way) the limitation of censored CSS where by no test statistic might be transmitted to the FC.

VIII. REFERENCES

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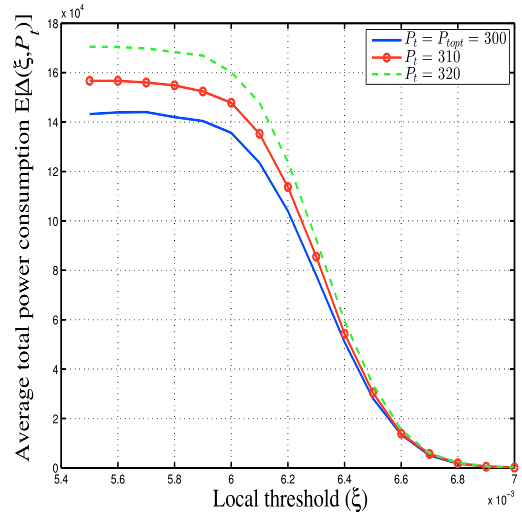

Fig. 5. The average total power consumption $\mathbb{E}[\triangle(\xi, P_t)]$ versus the local threshold ξ for different values of P_t. In all cases, $\mathrm{SNR} = -10\mathrm{dB}$.

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