

## DISTRIBUTED DIFFERENTIAL SPACE-TIME CODING FOR TWO-WAY RELAY NETWORKS USING ANALOG NETWORK CODING

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### ABSTRACT

Differential distributed space-time coding (DSTC) techniques using several transmission protocols have recently been proposed to overcome the overhead involved with the acquisition of channel state information (CSI) required for coherent processing. In this paper, a novel amplify-and-forward differential DSTC technique for two-way wireless relay networks (TWRNs) is proposed, that uses the concept of analog network coding and which does not require CSI at any node to decode the information symbols. In our transmission scheme, the relays do not waste power to transmit known information. This is achieved by combining the symbol vectors of both terminals at the relays into a single symbol vector without decoding the information symbols. Furthermore, in our scheme, the direct link between the communicating terminals can be easily incorporated to further improve the diversity gain. Simulations show a substantially improved performance in terms of bit error rate (BER) of the proposed technique as compared to the known techniques.

*Index Terms*— Two-way wireless relay networks, distributed space-time coding, differential space-time coding, cooperative diversity, analog network coding.

### I. INTRODUCTION

The overall system performance in terms of bit error rate (BER) and throughput in fading environments can be improved by exploiting cooperative multiple-antenna techniques which can efficiently be applied to combat the effect of multi-path fading and overcome various channel impairments [1]–[3]. In one-way relay networks, spatial diversity gain over conventional single-antenna systems is provided by multiple relays that cooperate to forward their received signals from the source terminal to the destination terminal.

Recently, various powerful cooperative diversity techniques have been proposed that exploit the spatial diversity offered by the relays, exhibit reasonable encoding and decoding complexity, and improve the performance gains in terms of BER and data rate [2]–[8]. Among them, distributed space-time coding (DSTC) techniques have recently been considered [4]–[10] in which the relays encode their received signals in the spatial domain, over multiple antennas, and in time domain, over multiple time slots in order to improve the overall system performance in terms of BER and achievable data rate at no additional cost of bandwidth or transmitted power and without requiring CSI at the transmitting nodes [2], [3].

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Based on the role of the relays, several transmission protocols have been developed that specify how the received signals are processed at the relays and how the transmissions are carried out [2], [3]. The simplest and the most common protocol is the amplify and forward (AF) protocol, where each relay receives a noisy version of the information signal which it simply amplifies and retransmits to the destination.

A common, yet often impractical assumption in distributed relay networks, is that perfect CSI is available at a central processing node and that the signaling overhead associated with the distribution of, e.g., channel- or beamformer coefficients is negligible [11], [12]. DSTC techniques require perfect CSI only at the receiving nodes [5], [13]–[15]. The recently proposed differential techniques have been designed based on more realistic assumption of CSI available neither at the terminals nor at the relays. These techniques which do not require CSI at any node can significantly reduce the system complexity and increase the spectral efficiency of network by reducing the overhead involved with the acquisition of CSI [8]–[10], [15]–[18].

TWRNs have been considered where two communicating terminals mutually exchange their information via distributed relay networks [15]–[22]. Based on the number of phases required for the information exchange, TWRN protocols can be classified into three popular classes: the four-phase protocols, the three-phase protocols, and the two-phase protocols. Due to the increase in symbol rate associated with the two- and three-phase protocols, it has been shown that the techniques using the two- and three-phase protocols outperform the conventional techniques using the four-phase protocols [15], [17]–[19].

In this paper, we propose a novel differential amplify-and-forward DSTC technique using the three-phase protocol for TWRNs where the direct link between the communicating terminals can be used to further improve the diversity and hence reliability of the communication. The proposed technique combines the transmitted symbol vectors of both terminals at the relays into a single symbol vector using a simple differential encoding scheme, such that each terminal can decode the transmitted symbol vector of the other terminal using the information of its own transmitted symbol vector. Interestingly, in contrast to symbol combination schemes that rely on extended modulation [18], the proposed distributed differential encoding scheme performed at the relays is not associated with any wasted in power for transmitting information signals known at any receiver resulting in improved BER performance at both terminals.

### II. WIRELESS RELAY NETWORK MODEL

We consider a half-duplex TWRN with  $R + 2$  single-antenna nodes as shown in Fig. 1 of two terminals  $\mathcal{T}_1$  and  $\mathcal{T}_2$  that exchange their information symbols via  $R$  nodes ( $\mathcal{R}_1, \dots, \mathcal{R}_R$ ) acting as distributed relays for the signals transmitted from both terminals. We denote the channels from  $\mathcal{T}_1$  to  $\mathcal{T}_2$ , from  $\mathcal{T}_1$  to the  $r$ th relay, and from  $\mathcal{T}_2$  to the  $r$ th relay as  $f_0$ ,  $f_r$ , and  $g_r$ , respectively. We assume channel reciprocity for the transmission from  $\mathcal{T}_1$  to  $\mathcal{T}_2$  and vice versa. Further, we consider the extended block fading channel model, for which it is assumed that the channels remain approximately constant over 3 consecutive symbol

blocks consisting of  $3T$  symbols and to slowly evolve outside this time interval. We further consider that the relays are perfectly synchronized in terms of carrier frequency and symbol timing and the CSI is not available at any node. The nodes  $\mathcal{T}_1, \mathcal{T}_2, \mathcal{R}_1, \dots, \mathcal{R}_R$  have limited average transmit powers  $P_{\mathcal{T}_1}, P_{\mathcal{T}_2}, P_{\mathcal{R}_1}, \dots, P_{\mathcal{R}_R}$ , respectively.

Throughout this paper,  $(\cdot)^H, |\cdot|, (\cdot)^*, \|\cdot\|, (\cdot)^T, \mathbf{I}_T, \mathbf{e}_r, \sigma^2, [\mathbf{a}]_i$ , and  $\mathbb{E}\{\cdot\}$  denote the Hermitian transpose, the absolute value, the complex conjugate, the Frobenius norm, the matrix transpose, the  $T \times T$  identity matrix, the  $r$ th column of  $\mathbf{I}_T$ , the noise variance, the  $i$ th entry of a vector  $\mathbf{a}$ , and the statistical expectation, respectively.

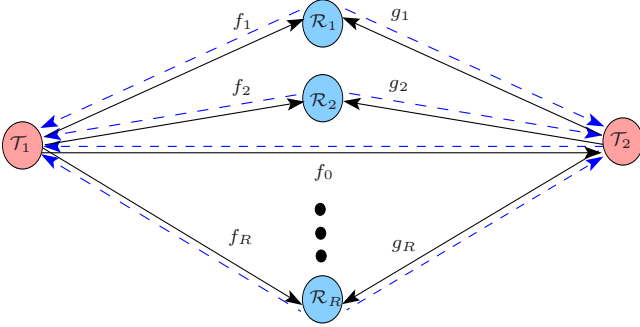


Fig. 1. TWRN with  $R + 2$  nodes.

### III. THREE-PHASE TWO-WAY DIFFERENTIAL DSTC TECHNIQUE

Let us assume that  $\mathbf{s}_{\mathcal{T}_1}^{(k)}$  and  $\mathbf{s}_{\mathcal{T}_2}^{(k)}$  denote the  $T \times 1$  vectors containing the  $k$ th block of information symbols of terminal  $\mathcal{T}_1$  and  $\mathcal{T}_2$ , respectively, where the symbols are taken from a  $M$ -PSK constellation denoted by set  $\mathcal{S}_{\mathcal{T}_t}$ , and  $T$  is the number of time slots in each phase. In the  $t$ th phase of the  $k$ th block, terminal  $\mathcal{T}_t$  computes the differentially encoded space-time block coding (STBC) matrix  $\mathbf{X}_{\mathcal{T}_t}^{(k)}$ , given by

$$\mathbf{X}_{\mathcal{T}_t}^{(k)} = \mathbf{S}_{\mathcal{T}_t}^{(k)} \mathbf{X}_{\mathcal{T}_t}^{(k-1)} \quad (1)$$

where  $\mathbf{X}_{\mathcal{T}_t}^{(0)} = \mathbf{I}_T$  and  $\mathbf{S}_{\mathcal{T}_t}^{(k)}$  is an orthogonal STBC (OSTBC) matrix containing the  $k$ th information symbol vector of terminal  $\mathcal{T}_t$  defined as

$$\begin{aligned} \mathbf{S}_{\mathcal{T}_t}^{(k)} &= \mathcal{X}(\mathbf{s}_{\mathcal{T}_t}^{(k)}) \\ &= [\mathbf{A}_1 \mathbf{s}_{\mathcal{T}_t}^{(k)} + \mathbf{B}_1 (\mathbf{s}_{\mathcal{T}_t}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{s}_{\mathcal{T}_t}^{(k)} + \mathbf{B}_T (\mathbf{s}_{\mathcal{T}_t}^{(k)})^*] \end{aligned} \quad (2)$$

with STBC precoding matrices  $\mathbf{A}_1, \dots, \mathbf{A}_T$  and  $\mathbf{B}_1, \dots, \mathbf{B}_T$  for  $\mathbb{E}\{|\mathbf{s}_{\mathcal{T}_t}^{(k)}|_i^2\} = 1$  and  $t = 1, 2$ . We remark that (1) describes the OSTBC matrix as it is used in a conventional multiple-antenna system with  $R$  antennas at the transmitter  $\mathcal{T}_t$ . In the context of this paper, we however assume that each transmitter consists of a single antenna and the OSTBC matrix in (1) is transmitted with the help of  $R$  relays [8], [27]. Throughout this paper we assume that the OSTBC precoding matrices exhibit the mutual exclusivity property that either  $\mathbf{A}_r = \mathbf{0}_T$  or  $\mathbf{B}_r = \mathbf{0}_T$  which is valid for a large number of commonly used OSTBCs. For instance let us consider a wireless relay network with two relay nodes  $R = 2$  that has also been addressed in the simulation section IV, for which the Alamouti scheme can be applied [8], [15], [17], [25] where  $\mathbf{A}_r$  and  $\mathbf{B}_r$  are chosen as

$$\begin{aligned} \mathbf{A}_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{B}_1 = \mathbf{0}_T, \\ \mathbf{A}_2 &= \mathbf{0}_T, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \end{aligned} \quad (3)$$

During the first phase from time slot 1 to  $T$  of the  $k$ th block, terminal  $\mathcal{T}_1$  transmits the first column of the differentially encoded OSTBC matrix  $\mathbf{X}_{\mathcal{T}_1}^{(k)}$  defined in (1) to the relays, hence

$$\begin{aligned} \mathbf{x}_{\mathcal{T}_1}^{(k)} &= \mathbf{X}_{\mathcal{T}_1}^{(k)} \mathbf{e}_1 \\ &= \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{X}_{\mathcal{T}_1}^{(k-1)} \mathbf{e}_1 \\ &= \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k-1)} \end{aligned} \quad (4)$$

where  $\mathbf{x}_{\mathcal{T}_1}^{(0)} = \mathbf{e}_1$  defines the initial symbol vector in the first transmission that can be used as a reference at the receiver to start the differential decoding procedure without requiring CSI. Note that the initial symbol vector  $\mathbf{x}_{\mathcal{T}_1}^{(0)}$  is only sent at the beginning of the transmission. In the next block of symbols, the transmitted vector of the previous block can be used as a reference in the decoding procedure. During the second phase from time slot  $T + 1$  to  $2T$ , terminal  $\mathcal{T}_2$  transmits in the  $k$ th block the differentially encoded  $T \times 1$  symbol vector  $\mathbf{x}_{\mathcal{T}_2}^{(k)}$  to the relays, given by

$$\begin{aligned} \mathbf{x}_{\mathcal{T}_2}^{(k)} &= \mathbf{X}_{\mathcal{T}_2}^{(k)} \mathbf{e}_1 \\ &= \mathbf{S}_{\mathcal{T}_2}^{(k)} \mathbf{X}_{\mathcal{T}_2}^{(k-1)} \mathbf{e}_1 \\ &= \mathbf{S}_{\mathcal{T}_2}^{(k)} \mathbf{x}_{\mathcal{T}_2}^{(k-1)} \end{aligned} \quad (5)$$

where  $\mathbf{x}_{\mathcal{T}_2}^{(0)} = \mathbf{e}_1$ . Since  $[\mathbf{s}_{\mathcal{T}_t}^{(k)}]_i$  is taken from a  $M$ -PSK constellation,  $\mathbf{S}_{\mathcal{T}_t}^{(k)}$  and  $\mathbf{X}_{\mathcal{T}_t}^{(k)}$  are unitary matrices. In the first phase of the  $k$ th block, from time slot 1 to  $T$ , the  $T \times 1$  vector received at the  $r$ th relay is given by

$$\mathbf{y}_{\mathcal{R}_1, r}^{(k)} = \sqrt{3P_{\mathcal{T}_1}} f_r^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k)} + \mathbf{n}_{\mathcal{R}_1, r}^{(k)} \quad (6)$$

where  $f_r^{(k)}$  denotes the flat fading channel from terminal  $\mathcal{T}_1$  to the  $r$ th relay in the  $k$ th block, and  $\mathbf{n}_{\mathcal{R}_1, r}^{(k)}$  denotes the  $T \times 1$  noise vector of the  $k$ th block at the  $r$ th relay in the first phase. We assume that the noise vector can be modeled as a spatially white independently and identically distributed complex circular Gaussian random variable with zero mean and covariance  $\sigma^2 \mathbf{I}_T$ . Similarly, in the second phase of the  $k$ th block, from time slot  $T + 1$  to  $2T$ , the  $T \times 1$  vector received at the  $r$ th relay is given by

$$\mathbf{y}_{\mathcal{R}_2, r}^{(k)} = \sqrt{3P_{\mathcal{T}_2}} g_r^{(k)} \mathbf{x}_{\mathcal{T}_2}^{(k)} + \mathbf{n}_{\mathcal{R}_2, r}^{(k)} \quad (7)$$

where  $g_r^{(k)}$  denotes the channel from terminal  $\mathcal{T}_2$  to the  $r$ th relay in the  $k$ th block and  $\mathbf{n}_{\mathcal{R}_2, r}^{(k)}$  denotes the  $T \times 1$  noise vector at the  $r$ th relay in the second phase of the  $k$ th block.

The  $r$ th relay recovers the OSTBC matrix of the first terminal up to channel and noise effects, such that

$$\begin{aligned} \mathbf{X}_{\mathcal{R}_1, r}^{(k)} &= [\mathbf{A}_1 \mathbf{y}_{\mathcal{R}_1, r}^{(k)} + \mathbf{B}_1 (\mathbf{y}_{\mathcal{R}_1, r}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{y}_{\mathcal{R}_1, r}^{(k)} + \mathbf{B}_T (\mathbf{y}_{\mathcal{R}_1, r}^{(k)})^*] \\ &= \sqrt{3P_{\mathcal{T}_1}} [\mathbf{A}_1 \mathbf{x}_{\mathcal{T}_1}^{(k)} f_r + \mathbf{B}_1 (\mathbf{x}_{\mathcal{T}_1}^{(k)})^* f_r^*, \dots, \mathbf{A}_T \mathbf{x}_{\mathcal{T}_1}^{(k)} f_r + \mathbf{B}_T (\mathbf{x}_{\mathcal{T}_1}^{(k)})^* f_r^*] \\ &\quad + [\mathbf{A}_1 \mathbf{n}_{\mathcal{R}_1}^{(k)} + \mathbf{B}_1 (\mathbf{n}_{\mathcal{R}_1}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{n}_{\mathcal{R}_1}^{(k)} + \mathbf{B}_T (\mathbf{n}_{\mathcal{R}_1}^{(k)})^*] \\ &= \sqrt{3P_{\mathcal{T}_1}} \mathbf{X}_{\mathcal{T}_1}^{(k)} \Delta_{f, r} + \mathbf{N}_{\mathcal{R}_1, r}^{(k)} \\ &= \sqrt{3P_{\mathcal{T}_1}} \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{X}_{\mathcal{T}_1}^{(k-1)} \Delta_{f, r} + \mathbf{N}_{\mathcal{R}_1, r}^{(k)}, \end{aligned} \quad (8)$$

$$\mathbf{N}_{\mathcal{R}_1, r}^{(k)} = [\mathbf{A}_1 \mathbf{n}_{\mathcal{R}_1}^{(k)} + \mathbf{B}_1 (\mathbf{n}_{\mathcal{R}_1}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{n}_{\mathcal{R}_1}^{(k)} + \mathbf{B}_T (\mathbf{n}_{\mathcal{R}_1}^{(k)})^*] \quad (9)$$

where  $\mathbf{S}_{T_1}^{(k)}$  is defined in (2) and  $\Delta_{f,r} = \text{diag}\{\mathbf{f}_r\}$  for  $\mathbf{f}_r$  denoting a  $R \times 1$  vector with the  $i$ th element defined as

$$[\mathbf{f}_r]_i = \begin{cases} f_r, & \text{if } \mathbf{B}_i = \mathbf{0}_T; \\ f_r^*, & \text{if } \mathbf{A}_i = \mathbf{0}_T. \end{cases} \quad (10)$$

Similar to (8), the  $r$ th relay can also essentially recover the OSTBC matrix of the second terminal, such that

$$\begin{aligned} \mathbf{X}_{\mathcal{R}_{2,r}}^{(k)} &= [\mathbf{A}_1 \mathbf{y}_{\mathcal{R}_{2,r}}^{(k)} + \mathbf{B}_1 (\mathbf{y}_{\mathcal{R}_{2,r}}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{y}_{\mathcal{R}_{2,r}}^{(k)} + \mathbf{B}_T (\mathbf{y}_{\mathcal{R}_{2,r}}^{(k)})^*] \\ &= \sqrt{3P_{T_2}} [\mathbf{A}_1 \mathbf{x}_{T_2}^{(k)} g_r + \mathbf{B}_1 (\mathbf{x}_{T_2}^{(k)})^* g_r^*, \dots, \mathbf{A}_T \mathbf{x}_{T_2}^{(k)} g_r + \mathbf{B}_T (\mathbf{x}_{T_2}^{(k)})^* g_r^*] \\ &\quad + [\mathbf{A}_1 \mathbf{n}_{\mathcal{R}_2}^{(k)} + \mathbf{B}_1 (\mathbf{n}_{\mathcal{R}_2}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{n}_{\mathcal{R}_2}^{(k)} + \mathbf{B}_T (\mathbf{n}_{\mathcal{R}_2}^{(k)})^*] \\ &= \sqrt{3P_{T_2}} \mathbf{X}_{T_2}^{(k)} \Delta_{g,r} + \mathbf{N}_{\mathcal{R}_{2,r}}^{(k)}, \\ &= \sqrt{3P_{T_2}} \mathbf{S}_{T_2}^{(k)} \mathbf{X}_{T_2}^{(k-1)} \Delta_{g,r} + \mathbf{N}_{\mathcal{R}_{2,r}}^{(k)}, \end{aligned} \quad (11)$$

$$\mathbf{N}_{\mathcal{R}_{2,r}}^{(k)} = [\mathbf{A}_1 \mathbf{n}_{\mathcal{R}_2}^{(k)} + \mathbf{B}_1 (\mathbf{n}_{\mathcal{R}_2}^{(k)})^*, \dots, \mathbf{A}_T \mathbf{n}_{\mathcal{R}_2}^{(k)} + \mathbf{B}_T (\mathbf{n}_{\mathcal{R}_2}^{(k)})^*] \quad (12)$$

where  $\mathbf{S}_{T_2}^{(k)}$  is defined in (2) and  $\Delta_{g,r} = \text{diag}\{\mathbf{g}_r\}$  for  $\mathbf{g}_r$  denoting the  $R \times 1$  vector with the  $i$ th element defined as

$$[\mathbf{g}_r]_i = \begin{cases} g_r, & \text{if } \mathbf{B}_i = \mathbf{0}_T; \\ g_r^*, & \text{if } \mathbf{A}_i = \mathbf{0}_T. \end{cases} \quad (13)$$

To provide coherent superposition of the signals transmitted from the relays at the respective destinations, the  $r$ th relay applies soft decoding of the signals transmitted in each time slot. Hence, the  $r$ th relay computes the  $r$ th column of the respective distributed STBC matrices as

$$\tilde{\mathbf{y}}_{\mathcal{R}_{1,r}}^{(k)} = \beta_{\mathcal{R}_{1,r}}^{(k)} \left( \mathbf{X}_{\mathcal{R}_{1,r}}^{(k)} (\mathbf{X}_{\mathcal{R}_{1,r}}^{(k-1)})^H \right) \mathbf{e}_r \quad (14)$$

$$\tilde{\mathbf{y}}_{\mathcal{R}_{2,r}}^{(k)} = \beta_{\mathcal{R}_{2,r}}^{(k)} \left( \mathbf{X}_{\mathcal{R}_{2,r}}^{(k)} (\mathbf{X}_{\mathcal{R}_{2,r}}^{(k-1)})^H \right) \mathbf{e}_r \quad (15)$$

where  $\beta_{\mathcal{R}_{1,r}}^{(k)}$  and  $\beta_{\mathcal{R}_{2,r}}^{(k)}$  are general scaling factors that adjust the transmitted power. Let us assume, without loss of generality, that

$$\beta_{\mathcal{R}_{1,r}}^{(k)} = \frac{1}{\|(\mathbf{X}_{\mathcal{R}_{1,r}}^{(k)} (\mathbf{X}_{\mathcal{R}_{1,r}}^{(k-1)})^H) \mathbf{e}_r\|^2}, \quad (16)$$

$$\beta_{\mathcal{R}_{2,r}}^{(k)} = \frac{1}{\|(\mathbf{X}_{\mathcal{R}_{2,r}}^{(k)} (\mathbf{X}_{\mathcal{R}_{2,r}}^{(k-1)})^H) \mathbf{e}_r\|^2}. \quad (17)$$

However, another possible choice would be to select scaling factors based on averaging all signal vectors instead of averaging only the current signal vectors. For sufficiently large SNR, we observe from (16) and (17) that

$$\beta_{\mathcal{R}_{1,r}}^{(k)} \approx \frac{1}{3P_{T_1} |f_r^{(k)}|^2}, \quad (18)$$

$$\beta_{\mathcal{R}_{2,r}}^{(k)} \approx \frac{1}{3P_{T_2} |g_r^{(k)}|^2}. \quad (19)$$

From (18) and (19), equations (14) and (15) can be approximated as

$$\tilde{\mathbf{y}}_{\mathcal{R}_{1,r}}^{(k)} \approx \mathbf{S}_{T_1}^{(k)} \mathbf{e}_r + \mathbf{v}_{\mathcal{R}_{1,r}}^{(k)}, \quad (20)$$

$$\tilde{\mathbf{y}}_{\mathcal{R}_{2,r}}^{(k)} \approx \mathbf{S}_{T_2}^{(k)} \mathbf{e}_r + \mathbf{v}_{\mathcal{R}_{2,r}}^{(k)} \quad (21)$$

where

$$\mathbf{v}_{\mathcal{R}_{1,r}}^{(k)} = \mathbf{V}_{\mathcal{R}_{1,r}}^{(k)} \mathbf{e}_r, \quad (22)$$

$$\begin{aligned} \mathbf{V}_{\mathcal{R}_{1,r}}^{(k)} &= \beta_{\mathcal{R}_{1,r}}^{(k)} \left( \mathbf{X}_{T_1}^{(k)} \Delta_{f,r} (\mathbf{N}_{\mathcal{R}_{1,r}}^{(k-1)})^H \right. \\ &\quad \left. + \mathbf{N}_{\mathcal{R}_{1,r}}^{(k)} (\mathbf{X}_{T_1}^{(k-1)} \Delta_{f,r})^H + \mathbf{N}_{\mathcal{R}_{1,r}}^{(k)} (\mathbf{N}_{\mathcal{R}_{1,r}}^{(k-1)})^H \right), \end{aligned} \quad (23)$$

$$\mathbf{v}_{\mathcal{R}_{2,r}}^{(k)} = \mathbf{V}_{\mathcal{R}_{2,r}}^{(k)} \mathbf{e}_r, \quad (24)$$

$$\begin{aligned} \mathbf{V}_{\mathcal{R}_{2,r}}^{(k)} &= \beta_{\mathcal{R}_{2,r}}^{(k)} \left( \mathbf{X}_{T_2}^{(k)} \Delta_{g,r} (\mathbf{N}_{\mathcal{R}_{2,r}}^{(k-1)})^H \right. \\ &\quad \left. + \mathbf{N}_{\mathcal{R}_{2,r}}^{(k)} (\mathbf{X}_{T_2}^{(k-1)} \Delta_{g,r})^H + \mathbf{N}_{\mathcal{R}_{2,r}}^{(k)} (\mathbf{N}_{\mathcal{R}_{2,r}}^{(k-1)})^H \right). \end{aligned} \quad (25)$$

Hence, the  $r$ th relay is assigned to particular STBC precoding matrices  $\mathbf{A}_1, \dots, \mathbf{A}_R, \mathbf{B}_1, \dots, \mathbf{B}_R$  that are used to linearly encode the received signal vectors  $\mathbf{y}_{\mathcal{R}_{1,r}}^{(k)}$  and  $\mathbf{y}_{\mathcal{R}_{2,r}}^{(k)}$  or their conjugate before broadcasting the resulting vector to both terminals. In the last reformulations of (8) and (11), we have made use of the mutual exclusivity property that either  $\mathbf{A}_r = \mathbf{0}_T$  or  $\mathbf{B}_r = \mathbf{0}_T$ . We observe from (20) and (21) that the  $r$ th relay recovers the OSTBC matrix of both terminals with a noise component.

Next, we propose an encoding strategy at the relays that facilitates simple signal separation at the destination, without however involving decoding of the signals received at the relays. The  $r$ th relay encodes the signal vector of the first terminal in (20) using the OSTBC precoding scheme introduced in (2), hence it computes the OSTBC matrix

$$\mathbf{S}_{\mathcal{R}_{1,r}}^{(k)} = \mathcal{X}(\tilde{\mathbf{y}}_{\mathcal{R}_{1,r}}^{(k)}) = \mathbf{S}_{T_1}^{(k)} + \mathbf{V}_{\mathcal{R}_{1,r}}^{(k)} \quad (26)$$

where  $\mathbf{S}_{T_1}^{(k)}$  is defined in (2) and  $\mathbf{V}_{\mathcal{R}_{1,r}}^{(k)}$  combines the noise terms defined in (23). Similar to (26), the  $r$ th relay can also encode the signal vector of the second terminal in (21) using the OSTBC precoding scheme defined in (2), such that

$$\mathbf{S}_{\mathcal{R}_{2,r}}^{(k)} = \mathcal{X}(\tilde{\mathbf{y}}_{\mathcal{R}_{2,r}}^{(k)}) = \mathbf{S}_{T_2}^{(k)} + \mathbf{V}_{\mathcal{R}_{2,r}}^{(k)} \quad (27)$$

where  $\mathbf{S}_{T_2}^{(k)}$  is defined in (2) and  $\mathbf{V}_{\mathcal{R}_{2,r}}^{(k)}$  denotes the noise terms defined in (25). During the third phase of the  $k$ th block, the  $r$ th relay combines the received signal vectors into a single  $T \times 1$  signal vector, such that

$$\begin{aligned} \mathbf{s}_{\mathcal{R}_{3,r}}^{(k)} &= \left( \mathbf{S}_{\mathcal{R}_{2,r}}^{(k)} \mathbf{S}_{\mathcal{R}_{1,r}}^{(k)} \mathbf{e}_r \right) \\ &= \mathbf{S}_{\mathcal{R}}^{(k)} \mathbf{e}_r + \mathbf{v}_{\mathcal{R}_{3,r}}^{(k)} \end{aligned} \quad (28)$$

where

$$\mathbf{S}_{\mathcal{R}}^{(k)} = \mathbf{S}_{T_2}^{(k)} \mathbf{S}_{T_1}^{(k)}, \quad (29)$$

$$\mathbf{v}_{\mathcal{R}_{3,r}}^{(k)} = \left( \mathbf{S}_{T_1}^{(k)} \mathbf{V}_{\mathcal{R}_{2,r}}^{(k)} + \mathbf{V}_{\mathcal{R}_{1,r}}^{(k)} \mathbf{S}_{T_2}^{(k)} + \mathbf{V}_{\mathcal{R}_{1,r}}^{(k)} \mathbf{V}_{\mathcal{R}_{2,r}}^{(k)} \right) \mathbf{e}_r. \quad (30)$$

From (28) and (29), we observe that the  $r$ th relay combines the symbol vectors of both terminals  $\mathbf{s}_{T_1}^{(k)}$  and  $\mathbf{s}_{T_2}^{(k)}$  into a single symbol vector  $\mathbf{s}_{\mathcal{R}_{3,r}}^{(k)}$  using a specific type of differential encoding scheme that does not require decoding of the symbols at the relays. This combining scheme enables each terminal to decode the transmitted symbols of the opposite terminal using the information of its own transmitted symbols. As a consequence, using the proposed differential encoding strategy the relays do not waste power to transmit known information to either side resulting in improved overall system performance in terms of BER as compared to straightforward combination schemes in [18]. From equation (28), we observe that the combined symbol vectors at the relays  $\mathbf{s}_{\mathcal{R}_{3,r}}^{(k)}$  are approximately identical. This means that the relay network can be considered as a "virtual" (centralized) multiple-input multiple-output (MIMO) network in which CSI is not available at the transmitter. In this case, STBC techniques designed for MIMO systems can straightforwardly be applied. The  $R$  relays then jointly transmit a distributed STBC matrix defined by  $R$  respective symbol vectors  $\mathbf{s}_{\mathcal{R}_{3,1}}^{(k)}, \mathbf{s}_{\mathcal{R}_{3,2}}^{(k)}, \dots, \mathbf{s}_{\mathcal{R}_{3,R}}^{(k)}$  where the  $r$ th relay transmits the symbol vector  $\mathbf{s}_{\mathcal{R}_{3,r}}^{(k)}$  defined in (28).

In the following, we only consider the received signals at terminal  $\mathcal{T}_2$ . The signal received at terminal  $\mathcal{T}_1$  can be computed correspondingly. The received signal vector at terminal  $\mathcal{T}_2$  in the  $k$ th block is given by

$$\begin{aligned} \mathbf{y}_{\mathcal{T}_2}^{(k)} &= \sum_{r=1}^R \gamma_r g_r^{(k)} \mathbf{s}_{\mathcal{R}_{3,r}}^{(k)} + \mathbf{n}_{\mathcal{T}_2}^{(k)} \\ &= \left( \sum_{r=1}^R \gamma_r g_r^{(k)} \mathbf{S}_{\mathcal{R}}^{(k)} \mathbf{e}_r \right) + \mathbf{v}_{\mathcal{T}_2}^{(k)} \end{aligned} \quad (31)$$

where

$$\mathbf{v}_{\mathcal{T}_2}^{(k)} = \sum_{r=1}^R \gamma_r g_r^{(k)} \mathbf{v}_{\mathcal{R}_{3,r}}^{(k)} + \mathbf{n}_{\mathcal{T}_2}^{(k)}, \quad (32)$$

$\mathbf{s}_{\mathcal{R}_{3,r}}^{(k)}$  is defined in (28),  $\mathbf{n}_{\mathcal{T}_2}^{(k)}$  denotes the  $T \times 1$  vector containing the receiver noise of the  $k$ th block at terminal  $\mathcal{T}_2$ , and  $\gamma_r$  is a scaling factor to satisfy the power constraint at the  $r$ th relay, i.e.,  $\gamma_r = \sqrt{P_{\mathcal{R}_r}}$  when  $\beta_{\mathcal{R}_{1,r}}^{(k)}$  and  $\beta_{\mathcal{R}_{2,r}}^{(k)}$  are defined in (16) and (17). For simplicity and without loss of generality, it is assumed that  $\gamma_1 = \gamma_2 = \dots = \gamma_R = \gamma$ . From (31), the received signal vector at terminal  $\mathcal{T}_2$  during the third phase of the  $k$ th block can be expressed as

$$\begin{aligned} \mathbf{y}_{\mathcal{T}_2}^{(k)} &= \gamma \mathbf{S}_{\mathcal{R}}^{(k)} \mathbf{g}^{(k)} + \mathbf{v}_{\mathcal{T}_2}^{(k)} \\ &= \gamma \mathbf{S}_{\mathcal{T}_2}^{(k)} \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{g}^{(k)} + \mathbf{v}_{\mathcal{T}_2}^{(k)} \end{aligned} \quad (33)$$

where

$$\mathbf{g}^{(k)} = [g_1^{(k)}, \dots, g_R^{(k)}]^T, \quad (34)$$

and  $\mathbf{v}_{\mathcal{T}_2}^{(k)}$  is defined in (32). After multiplying (33) at the  $(k-1)$ th block by  $(\mathbf{S}_{\mathcal{T}_2}^{(k-1)})^H \hat{\mathbf{S}}_{\mathcal{T}_1}^{(k-1)}$ , we have

$$\begin{aligned} \tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)} &= (\mathbf{S}_{\mathcal{T}_2}^{(k-1)})^H \hat{\mathbf{S}}_{\mathcal{T}_1}^{(k-1)} \mathbf{y}_{\mathcal{T}_2}^{(k-1)} \\ &= \gamma \mathbf{g}^{(k)} + (\mathbf{S}_{\mathcal{T}_2}^{(k-1)})^H \hat{\mathbf{S}}_{\mathcal{T}_1}^{(k-1)} \mathbf{v}_{\mathcal{T}_2}^{(k)}. \end{aligned} \quad (35)$$

Making use of the extended block fading assumption where  $\mathbf{g}^{(k-1)} = \mathbf{g}^{(k)} = \mathbf{g}$ , the decoder at terminal  $\mathcal{T}_2$  can be expressed as

$$\hat{\mathbf{S}}_{\mathcal{T}_1}^{(k)} = \arg \min_{\mathbf{S}_{\mathcal{T}_1}^{(k)}} \left\| \mathbf{y}_{\mathcal{T}_2}^{(k)} - \mathbf{S}_{\mathcal{T}_2}^{(k)} \mathbf{S}_{\mathcal{T}_1}^{(k)} \tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)} \right\|^2. \quad (36)$$

In the case that the direct link between the two terminals  $\mathcal{T}_1$  and  $\mathcal{T}_2$  is available, the received signal vector at terminal  $\mathcal{T}_2$  during the first transmission phase is given by

$$\begin{aligned} \mathbf{y}_{\mathcal{T}_2,\text{dl}}^{(k)} &= \sqrt{3P_{\mathcal{T}_1}} f_0^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k)} + \mathbf{n}_{\mathcal{T}_2}^{(k)} \\ &= \sqrt{3P_{\mathcal{T}_1}} f_0^{(k)} \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k-1)} + \mathbf{n}_{\mathcal{T}_2}^{(k)} \end{aligned} \quad (37)$$

where  $\mathbf{n}_{\mathcal{T}_2}^{(k)}$  denotes the  $T \times 1$  vector containing the receiver noise of the  $k$ th block at terminal  $\mathcal{T}_2$  and  $\mathbf{x}_{\mathcal{T}_1}^{(k)}$  is defined in (4). Making use of the symbol vector  $\mathbf{y}_{\mathcal{T}_2,\text{dl}}^{(k)}$  defined in (37), the decoder at terminal  $\mathcal{T}_2$  can be expressed as:

$$\mathbf{S}_{\mathcal{T}_1}^{(k)} = \arg \min_{\mathbf{S}_{\mathcal{T}_1}^{(k)}} \left\| \mathbf{y}_{\mathcal{T}_2}^{(k)} - \mathbf{S}_{\mathcal{T}_2}^{(k)} \mathbf{S}_{\mathcal{T}_1}^{(k)} \tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)} \right\|^2 + \left\| \mathbf{y}_{\mathcal{T}_2,\text{dl}}^{(k)} - \mathbf{S}_{\mathcal{T}_1}^{(k)} \mathbf{y}_{\mathcal{T}_2,\text{dl}}^{(k-1)} \right\|^2. \quad (38)$$

A similar decoding procedure can be applied at  $\mathcal{T}_1$ . Note that to not waste power in transmitting known information at either side, in the proposed technique the  $r$ th relay combines the unitary STBC matrices

of both terminals into a single unitary STBC matrix without hard-decoding them before broadcasting the  $r$ th column of the combined unitary STBC matrix to the destination. At the destinations, each terminal can decode the transmitted STBC matrix of the other terminal from its received STBC matrix of the relays using the information of its own transmitted STBC matrix.

In the case that general STBCs are used, sphere decoding [26] can be applied or in the case of orthogonal DSTCs, a symbol-wise decoder can be used to decode the received symbols at terminal  $\mathcal{T}_2$  [5], [8].

We remark that the use of the direct link between the communicating terminals increases the decoding complexity at the destination terminal and generally a full search over all possible STBC matrices  $\mathbf{S}_{\mathcal{T}_1}^{(k)}$  is required. This increase is accompanied by the increase in the diversity order provided by the direct link.

Other transmission protocols like DF protocol can also be applied to the proposed technique where each relay perform hard decision decoding of the information symbol vectors  $\mathbf{s}_{\mathcal{T}_1}^{(k)}$  and  $\mathbf{s}_{\mathcal{T}_2}^{(k)}$  of the first and second terminal, respectively, defined in (4) and (5) without requiring CSI. In this case, similarly as in (29), the decoded and re-encoded unitary STBC matrices are combined at the  $r$ th relay into a single unitary STBC matrix. In the next phase, the  $r$ th relay broadcasts the  $r$ th column of the encoded unitary matrix to both terminals. The proposed technique using the DF protocol enjoys a substantially lower relay decoding complexity as compared to the previously proposed two-phase DSTC techniques which have the order of  $|\mathcal{S}_{\mathcal{T}_1}| |\mathcal{S}_{\mathcal{T}_2}|$  [19]. To apply DSTC on the relay nodes using the DF protocol, we consider error free decoding at the relays, i.e.,  $\mathbf{s}_{\mathcal{R}_{3,1}}^{(k)} = \mathbf{s}_{\mathcal{R}_{3,2}}^{(k)} = \dots = \mathbf{s}_{\mathcal{R}_{3,R}}^{(k)} = \mathbf{s}_{\mathcal{R}_3}^{(k)}$ . Thus, the relays should decode the information symbols correctly in order to achieve full diversity. Otherwise the decoder at the destination terminal suffers from a poor error performance. To reduce these effects, the use of cyclic redundancy check (CRC) at the relay nodes is proposed in [19] to identify decoding errors at the relays and prevent error propagation.

## IV. SIMULATION RESULTS

In our simulations, we have assumed a TWRN with four single-antenna relay nodes consisting of two terminals and two relays, independent flat Rayleigh fading channels and a power distribution equal to  $P_{\mathcal{T}_1} = P_{\mathcal{T}_2} = \sum_{r=1}^R P_{\mathcal{R}_r}$ . For fair comparison of the BER performance of all techniques, the same total transmitted power ( $P_T = P_{\mathcal{T}_1} + P_{\mathcal{T}_2} + \sum_{r=1}^R P_{\mathcal{R}_r}$ , where  $P_{\mathcal{R}_1} = P_{\mathcal{R}_2} = \dots = P_{\mathcal{R}_R}$ ) and transmission rate is used.

In Fig. 2, the BER at terminal  $\mathcal{T}_2$  is displayed versus the SNR and the proposed differential DSTC technique using 8-PSK modulation is compared with the two-phase differential DSTC technique proposed in [18] using 4-PSK modulation, the three-phase differential DSTC technique using 8-PSK modulation proposed in [27], and the four-phase differential distributed Alamouti space-time coding technique using 16-PSK modulation [8] for a total rate of 1 bpcu. The acronym ‘‘DL’’ stands for the use of the direct link in the technique and ‘‘The proposed scheme’’ stands for the proposed technique using  $\gamma_r = \sqrt{P_{\mathcal{R}_r}}$ ,  $\beta_{\mathcal{R}_{1,r}}^{(k)} = \frac{1}{\|(\mathbf{x}_{\mathcal{R}_{1,r}}^{(k)} \mathbf{x}_{\mathcal{R}_{1,r}}^{(k-1)})^T \mathbf{e}_r\|^2}$  and  $\beta_{\mathcal{R}_{2,r}}^{(k)} = \frac{1}{\|(\mathbf{x}_{\mathcal{R}_{2,r}}^{(k)} \mathbf{x}_{\mathcal{R}_{2,r}}^{(k-1)})^T \mathbf{e}_r\|^2}$  defined in (16) and (17).

From Fig. 2, it can be observed that the proposed technique outperforms the best known two-, three-, and four-phase techniques and also allows the communicating terminals to use the direct link between them to increase the diversity order. From Fig. 2, the techniques which use the direct link between the communicating terminals outperform those without direct link since the direct link provides additional diversity gain.

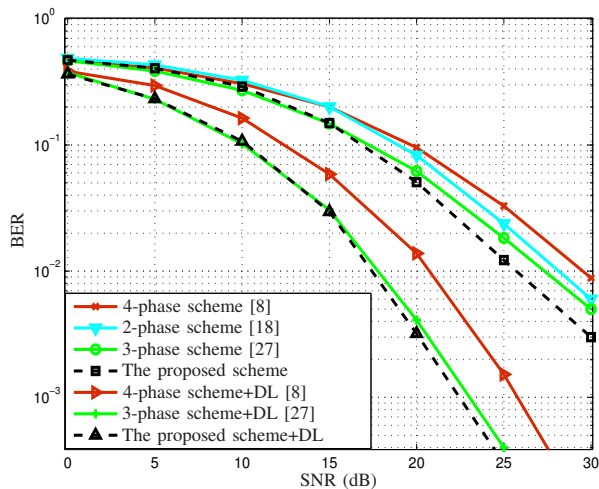


Fig. 2. BER versus SNR for several differential schemes with  $R = 2$  and a rate of 1 bpcu.

## V. CONCLUSION

In this paper, we propose a novel non-coherent distributed space-time coding technique for two-way wireless relay networks using the three-phase protocol. The proposed technique does not require CSI at the terminals or relays and uses the concept of analog network coding to offer higher coding gain than the state-of-the-art techniques by combines the transmitted symbol vectors of both terminals into a single symbol vector without hard decision decoding the information symbols. Furthermore, our proposed technique allows the communicating terminals to use the direct link between them to increase the diversity order.

## VI. REFERENCES

- [1] A. Amah and A. Klein, "Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying," *Eurasip Journal on Wireless Communications and Networking*, vol. 2011, Jul 2011.
- [2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity Part-I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.
- [3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity Part-II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939-1948, Nov. 2003.
- [4] T. Unger and A. Klein, "On the performance of distributed space-time block codes," *IEEE Communications Letters*, vol. 11, No. 5, pp. 411-413, May 2007.
- [5] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.
- [6] S. Yiu, R. Schober, and L. Lampe, "Distributed space-time block coding," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1195-1206, Jul. 2006.
- [7] B. Maham and A. Hjørungnes, "Power Allocation Strategies for Distributed Space-Time Codes in Amplify-and-Forward Mode," *Eurasip Journal in Signal Processing*, vol. 2009, July 2009.
- [8] Y. Jing and H. Jafarkhani, "Distributed differential space-time coding in wireless relay networks," *IEEE Trans. Commun.*, vol. 56, no. 7, pp. 1092-1100, Jul. 2008.
- [9] T. Wang, Y. Yao, and G. B. Giannakis, "Non-coherent distributed space time processing for multiuser cooperative transmissions," *IEEE Trans. On Wireless Comm.*, vol. 5, no. 12, pp. 3339-3343, Dec. 2006.
- [10] F. Oggier and B. Hassibi, "A coding strategy for wireless networks with no channel information," in *Proc. Allerton Conf. Communications, Control, and Computing*, Monticello, IL, pp. 113-117, Sep. 2006.
- [11] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. on Information Theory*, vol. 55, pp. 2499-2517, June, 2009.
- [12] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. on Signal Proc.*, vol. 58, no. 3, pp. 1238-1250, Mar. 2010.
- [13] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: Space-time transmission and iterative decoding," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 362-371, Feb. 2004.
- [14] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524-3536, Dec. 2006.
- [15] S. J. Alabed, J. M. Paredes, and A. B. Gershman, "A simple distributed space-time coded strategy for two-way relay channels," *IEEE Transactions on Wireless Communications*, pp. 1260-1265, vol. 11, no. 4, April, 2012.
- [16] S. J. Alabed and M. Pesavento, "A simple distributed differential transmit beamforming technique for two-way wireless relay networks," in *the 16th International IEEE/ITG Workshop on Smart Antennas (WSA 2012)*, pp. 243-247, Dresden, Germany, Mar. 2012.
- [17] S. J. Alabed, M. Pesavento, and A. B. Gershman, "Distributed differential space-time coding techniques for two-way wireless relay networks," in *Proceedings of the Fourth IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP 11)*, pp. 221-224, San Juan, Puerto Rico, December 2011.
- [18] Z. Utkovski, G. Yammine, and J. Lindner, "A distributed differential space-time coding scheme for two-way wireless relay networks," *ISIT 2009*, Seoul, Korea, pp. 779-783, Jun. 2009.
- [19] T. Cui, F. Gao, T. Ho, and A. Nallanathan, "Distributed space-time coding for two-way wireless relay networks," *IEEE Trans. Signal Processing*, vol. 57, pp. 658-671, May 2009.
- [20] T. Unger and A. Klein, "Applying Relay Stations with Multiple Antennas in the One- and Two-Way Relay Channel," in *Proc. International Symposium on Personal, Indoor and Mobile Radio Communications*, Athens, Greece, Sep 2007.
- [21] T. Unger and A. Klein, "On the performance of two-way relaying with multiple-antenna relay stations," in *Proc. IST Mobile and Wireless Communications Summit*, Budapest, Hungary, Jul 2007.
- [22] S. Berger, T. Unger, M. Kuhn, A. Klein, and A. Wittneben, "Recent advances in amplify-and-forward two-hop relaying," *IEEE Communications Magazines*, vol. 47, pp. 50-56, Jul 2009.
- [23] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, pp. 1-4, Jan. 2001.
- [24] S. J. Alabed, J. M. Paredes, and A. B. Gershman, "A low complexity decoder for quasi-orthogonal space-time block codes," *IEEE Transactions on Wireless Communications*, vol. 10, no. 3, March 2011.
- [25] Y. Jing and H. Jafarkhani, "Using orthogonal and quasi-orthogonal designs in wireless relay networks," *IEEE Trans. Infom. Theory*, vol. 53, no. 11, pp. 4106-4118, Nov. 2007.
- [26] E. Agrell, T. Eriksson, A. Vardy and K. Zeger, "Closest point search in lattices," *IEEE Trans. Inform. Theory*, vol. 48, pp. 2201-2214, Aug. 2002.
- [27] S. Alabed, M. Pesavento, and A. Klein "Non-coherent distributed space-time coding techniques for two-way wireless relay networks," *EURASIP special issue on Sensor Array Processing*, Feb. 2013, DOI: 10.1016/j.sigpro.2012.12.001.