ENERGY EFFICIENCY IN MIMO LARGE SCALE TWO-TIER NETWORKS WITH BEAMFORMING AND ADAPTIVE MODULATION

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ABSTRACT

In this paper, the energy efficiency in the downlink of a two-tier network consisting of macro- and femtocells using beamforming is analyzed. Orthogonal subchannel allocation is used in order to eliminate inter-tier interference. The random locations of the interferers in both tiers are modeled via a Poisson Point Process. Improvements in the energy efficiency of the system (in b/J/Hz), when several femtocells are deployed in a network, are observed under different scenarios. Finally, using realistic implementation parameters, we examine how the energy efficiency is affected by different antenna configurations, and we also obtain the optimal configurations.

Index Terms— Femtocell, heterogeneous networks, energy efficiency, maximum ratio transmission (MRT), Poisson point processes, Rayleigh fading.

1. INTRODUCTION

The ever increasing demands for better coverage and higher data rates from cellular system users have created a need for efficient and cost effective solutions. Heterogeneous networks, consisting of smaller base stations overlaid with the traditional macro base station deployment, is a promising solution to provide the users with the required quality of service. However, a large scale and unplanned deployment of small base stations can render the network inefficient in terms of energy, as the power consumed can be increased significantly. According to recent studies, around 2% of the total CO_2 emissions into the atmosphere comes from the telecommunications industry, and this is related to the network energy consumption [1]. So making the system more energy efficient is a very important issue for next generation networks.

In the case of femtocells deployed over an existing macro station network some algorithms and models for the energy consumption have been proposed. However most of them:

- Assume there is a single representative macrocell and/or femtocell;
- Assume the locations of the femtocells are deterministic and known;
- Are based only on simulations without any underpinning analytical work.

These assumptions reduce considerably the scope of previous analyses. The use of Poisson Point Processes (PPPs) to model an infrastructureless wireless network such as ad-hoc networks or femtocells, has proven to be an effective tool, as it can provide an insight into the behavior of a random placement of nodes over a given area. Moreover, in recent works the use of point processes to model a macrocell tier has also been reported as the typical deterministic grid model is a very unrealistic assumption to find in practice [2], [3]. On the other hand, the use of MIMO techniques to improve the network performance is a requirement for future cellular systems. But the energy aspects of deploying a larger number of antennas has not been very seriously analyzed, in particular, beyond the case of a point-to-point link.

Another important aspect of this paper is that we are motivated to use Maximum Ratio Transmission (MRT) given the fact that no cooperation between cells is assumed. This is because femtocells are power limited devices, and therefore, non complex and energy efficient algorithms are expected to be deployed. MRT consisting of beamforming at the transmitter and Maximum Ratio Combining (MRC) at the receiver [4], is a simple but efficient technique, to achieve high diversity gains. So in this work, the energy efficiency is analyzed in the downlink of a two-tier network consisting of macro- and femtocells using MRT, where both tiers are assumed to follow a PPP under a Rayleigh fading environment.

The rest of the paper is organized as follows. Section 2 introduces the system model. Section 3 describes the analysis of the coverage for both tiers. The expected throughput in each tier is derived in Section 4. The energy efficiency metric and its optimization are described in Section 5. The simulation results are presented in Section 6. Finally, the conclusions are given in Section 7.

Throughout the paper the following notation is used. Boldface capital and lower case letters represent matrices

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and vectors respectively. $E\left(x\right)$ stands for the expected value of the random variable x. $\mathbf{A}^{H}$ represents the conjugate transpose of the matrix $\mathbf{A}$. A matrix following a circularly symmetric complex Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ is expressed as $\mathbf{A} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Finally, $|\mathbf{A}|$ denotes the determinant of matrix $\mathbf{A}$.

2. SYSTEM MODEL

Consider the downlink of an interference limited OFDMA (such as LTE) two-tier network consisting of femtocell access points (FAPs) and macrocell base sations (MBSs). The effect of noise will be neglected as interference dominates the overall performance of the network. Assume also that the total number of available subchannels (S) is divided between tiers, assigning orthogonal subchannels to each one in a given time slot. So we will have $S_m < S$ subchannels assigned to the macrocell tier and $S_f = S - S_m$ subchannels assigned to the femtocell tier, such that the inter-tier interference is completely avoided. So the only sources of interference are base stations belonging to the same tier.

Femtocell users are assumed to be located indoors and so a wall partition loss (L_w) must be considered. The tiers are modeled by two independent homogeneous PPPs ($\Phi_i,\ i \in \{f, m\}$), where f and m stand for femtocell and macrocell tier respectively. The intensity characterizing the number of base stations per unit area is λ_i. The number of base stations in a given tier (n_i) is a Poisson distributed random variable with $n_i \sim \text{Pois}(N_i = \lambda_i A)$, where A is the area of the network and N_i is the mean number of base stations. In the case of femtocells, slotted ALOHA is considered as the Medium Access Control (MAC) scheme with MAP ρ_f and so the effective intensity of the transmitting femtocells is given by $\rho_f \lambda_f$. Macrocells are assumed to be sectorized with N_S sectors and so the effective intensity of the interferers in this tier is considered as $\frac{\lambda_m}{N_S}$.

The propagation model is assumed to be a composite of Rayleigh flat-fading and path loss $l(R_i^{j,k}) = \left(R_i^{j,k}\right)^{-\alpha_i}$, where $R_i^{j,k}$ is the distance from the j-th transmitter to the k-th user in tier i and α_i is the path loss exponent. In the femtocell tier we use different values for the path loss exponent of the desired link (α_0) and the path loss exponent of an interferer link (α_f) as the later can experience different propagation scenarios. Finally, the mean total transmitted power of a base station in tier $i \in \{f, m\}$ is denoted as P_i^{tx}.

A MIMO system is assumed where the base stations in tier i use M_i^t antennas for transmission and M_i^r antennas for reception. Let $\mathbf{H}_i^{j,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ denote the $M_i^r \times M_i^t$ channel matrix between the j-th base station and k-th user in tier i. Moreover, assume that the BSs in both tiers use MRT, and that the channel state information (CSI) is known at both the transmitter and the receiver. Therefore the complex symbol to be sent, $s_i^{j,k}$, with $E\left(\left|s_i^{j,k}\right|^2\right) = 1$, is first precoded at the transmitter with an $M_i^t \times 1$ beamforming vector $\mathbf{v}_i^{j,k}$ which is the eigenvector corresponding to the maximum eigenvalue (Λ_{max}) of the Wishart matrix $\left(\mathbf{H}_i^{j,k}\right)^H \mathbf{H}_i^{j,k}$ [4]. Using Slivnyak's theorem [2], then without loss of generality we place a typical user at the origin and obtain its statistics. The received signal vector is then given by

$$\begin{aligned} \mathbf{y}_f = &\sqrt{P_f^{tx}\, l\left(R_f^{0,0}\right)}\ \mathbf{H}_f^{0,0}\mathbf{v}_f^{0,0}s_f^{0,0} \\ &+ \sum_{j\in\Phi_f} \sqrt{P_f^{tx}\ l\left(R_f^{j,0}\right) L_w}\ \mathbf{H}_f^{j,0}\mathbf{v}_f^{j,j}s_f^{j,j} \end{aligned} \tag{1}$$

$$\begin{aligned} \mathbf{y}_m = &\sqrt{P_m^{tx}\, l\left(R_m^{0,0}\right)}\ \mathbf{H}_m^{0,0}\mathbf{v}_m^{0,0}s_m^{0,0} \\ &+ \sum_{j\in\Phi_f} \sqrt{P_m^{tx}\, l\left(R_m^{j,0}\right)}\ \mathbf{H}_m^{j,0}\mathbf{v}_m^{j,j}s_m^{j,j}. \end{aligned} \tag{2}$$

At the receiver MRC is used and a $1 \times M_i^r$ weight vector $\left(\mathbf{w}_i^{j,k}\right)^H$ is applied to the received signal before decoding the symbols, i.e., the signal to be decoded is given by $\overset{\wedge}{y_i} = \left(\mathbf{w}_i^{0,0}\right)^H \mathbf{y}_i$, where $\mathbf{w}_i^{j,k} = \mathbf{H}_i^{j,k}\mathbf{v}_i^{j,k}$. Now, the figure of merit from which other calculations can be derived is the Signal to Interference Ratio (SIR) at the typical user, which is given by

$$SIR_f = \frac{\Lambda_{max}\, l\left(R_f^{0,0}\right)}{\sum\limits_{j\in\Phi_f} g^{j,0} L_w\, l\left(R_f^{j,0}\right)} = \frac{\Lambda_{max}\, l\left(R_f^{0,0}\right)}{I_{\Phi_f}} \tag{3}$$

$$SIR_m = \frac{\Lambda_{max}\, l\left(R_m^{0,0}\right)}{\sum\limits_{j\in\Phi_m} g^{j,0}\, l\left(R_m^{j,0}\right)} = \frac{\Lambda_{max}\, l\left(R_m^{0,0}\right)}{I_{\Phi_m}} \tag{4}$$

where $g^{j,0}$ represents the fading power coefficient for the link between the desired user and the $j\text{-}th$ source of interference with $g^{j,0} \sim \text{Exp}(1)$. The SIRs in (3) and (4) follow from the fact that with MRC, the resulting interference is a weighted combination of complex Gaussian random variables which is again Gaussian. This makes the power of the interference a sum of exponential random variables, just as in the case of a SISO system [5].

In the femtocell tier each femtocell user is assumed to be associated with a femtocell at a fixed distance $R_f^{j,j}$ at the edge of the femtocell coverage area, while in the macrocell tier a user is associated with the closest BS. This means that $R_m^{j,j}$ is a random variable following the distribution of the distance (D) to the closest base station, which for a homogeneous PPP was proven in [2] to be $f_D(r) = e^{-2\lambda\pi r^2}2\lambda\pi r$. The scenario previously described is depicted in Fig. 1 where the blue dots correspond to MBSs and the red crosses are the FAPs located across the area. Note that under the closest association scheme used for the macrocell tier, the cells form a Voronoi tesselation.

$$P_f^c(\beta) = \sum_{p=1}^{t}\sum_{m=u-t}^{(u+t)p-2p^2}\sum_{k=0}^{m} d_{p,m}\frac{(-p)^k}{k!}\frac{d^k}{dp^k} e^{\left(-\rho_f\lambda_f\left(p\left(R_f^{0,0}\right)^{\alpha_0}\beta L_w\right)^{\delta_f}\frac{\pi^2\delta_f}{\sin\left(\pi\delta_f\right)}\right)} \tag{5}$$

$$P_m^c(\beta) = \sum_{p=1}^{t}\sum_{m=u-t}^{(u+t)p-2p^2}\sum_{k=0}^{m} d_{p,m}\frac{(-p)^k}{k!}\frac{d^k}{dp^k}\left(\frac{1}{1+\left(\frac{2\beta p}{N_S(\alpha_m-2)}\; {}_2F_1\left(1,1-\delta_m;2-\delta_m;-\beta p\right)\right)}\right). \tag{6}$$

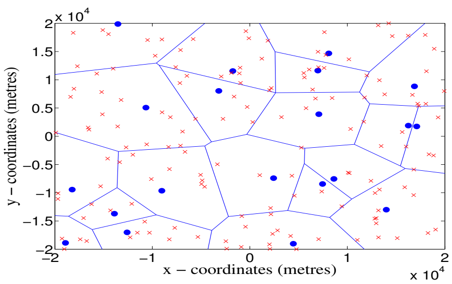

Fig. 1: Two tier network consisting of femto- and macrocells.

3. COVERAGE

The coverage of each tier $P_i^c(\beta_i), i \in \{f, m\}$ (superscript "c" for "coverage") is defined as the probability that the received SIR is above a certain threshold (β_i), which depends on the required QoS, i.e., $P_i^c(\beta) = P(SIR_i > \beta_i)$. Without loss of generality, we assume that the thresholds for each tier are the same ($\beta_f = \beta_m = \beta$). Now from (3) and (4), this coverage is related to the CDF of the maximum eigenvalue (Λ_{max}) of a Wishart matrix, which was originally obtained in [6] as $F_{\Lambda_{max}}(x) = \frac{|\boldsymbol{\Psi}(x)|}{\left[\prod_{i=1}^{t}(t-i)!\prod_{i=1}^{u}(u-i)!\right]}$, where $t = \min\left(M_i^t, M_i^r\right)$, $u = \max\left(M_i^t, M_i^r\right)$ and $\boldsymbol{\Psi}(x)$ is a Hankel matrix whose elements are given by $\{\boldsymbol{\Psi}(x)\}_{i,j} = \gamma(i+j-1, x)$ with $\gamma(a, b)$ being the lower incomplete Gamma function. In [5] an alternative expression was found as a sum of exponential functions. Using this alternative expression and applying the definition in (3), the coverage probability in the femtocell tier is given as in (5), where $d_{p,m}$ is a coefficient which can be obtained from $|\boldsymbol{\Psi}(x)|$, α_0 is the path loss exponent of the desired link and $\delta_f = \frac{2}{\alpha_f}$. In a similar way for the macrocell tier, using the alternative definition and integrating (4) with respect to the nearest neighbour distribution as in [2], the coverage probability can then be expressed as (6), where ${}_2F_1(a, b; c; d)$ stands for the Gauss hypergeometric function and $\delta_m = \frac{2}{\alpha_m}$.

4. THROUGHPUT

The use of adaptive modulation is assumed in which, depending on the channel conditions, the symbols to be transmitted are chosen from a finite set of available modulations. Assuming L modulation schemes, in a given transmission the normalized data rate that this system handles is given by $r_i^o = \log\left(1 + \frac{\beta_i^o}{G}\right)$ bps/Hz if $SIR_i \in \left[\beta_i^o, \beta_i^{o+1}\right)$, with $o = 1, 2, ..., L$, $i \in \{f, m\}$ and G is the Shannon gap of adaptive modulation. Considering integer data rates, the average throughput per base station in each tier can be expressed as

$$T_f(\rho_f) = \rho_f \sum_{o=1}^{L} P_f^c(\beta_f^o) \quad \text{bps/Hz} \tag{7}$$

$$T_m = \sum_{o=1}^{L} P_m^c(\beta_m^o) \quad \text{bps/Hz.} \tag{8}$$

By substituting (5) and (6) into (7) and (8) respectively, the throughput for each tier can be obtained. Depending on the particular scenario there is an optimal MAP value $\left(\rho_f^\star\right)$ which maximizes the throughput of femtocells, i.e., $\rho_f^\star = \arg\max_{\rho_f} T_f(\rho_f)$. Now the selection of slotted ALOHA as the MAC strategy is justified given the fact that the femtocells are limited in power, and so simpler algorithms are expected. Given the complexity of the resulting expression for the throughput, we find $\rho_f^\star$ numerically. On the other hand, for the macrocell tier, note that the increase in the number of BSs does not affect the throughput, i.e., (8) is independent of λ_m. This comes from the fact that in this model the typical macro-user connects to the closest macro BS and so increasing the number of macro stations will increase the probability of this user being served by a BS. But it so happens that the increase in interference exactly offsets this gain (when noise is neglected) [2].

5. ENERGY EFFICIENCY

We used the power consumption model presented in [7] for both macro and femtocells: $P_i = a_i P_i^{tx} + b_i, i \in \{f, m\}$. Here a_i is a parameter dependent on the transmitted power of the base station (P_i^{tx}), which is related to the efficiency of the power amplifier, and b_i is a parameter independent of the transmission power which deals with the power spent in signal processing, cooling effects of the site and battery backup. A power penalty for the CSI acquisition is not considered, as the scope of this work is to quantify the energy gains of MIMO technology and its inclusion is left for future work. The macro and femto tier total power consumption models are then given by

$$P_m^T = N_m N_S\left(a_m P_m^{tx} + M_m^t b_m\right) \quad \text{Watts} \tag{9}$$

$$P_f^T = N_f\left(\rho_f^\star a_f P_f^{tx} + M_f^t b_f\right) \quad \text{Watts.} \tag{10}$$

Note that $a_i, i \in \{f, m\}$ in (9) and (10) are not scaled by the number of antennas, given that the total power radiated from all the antennas is equal to P_i^{tx}. We assume typical values for the components of the power consumption model ([7], [8]), presented in Table 1.

Table 1: SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
$R_f^{j,j}$	30 m	u_m	30
α_m	4	u_f	2
α_{f_0}	3	P_f^{tx}	100 mW
α_f	3.5	P_m^{tx}	10 W
L_w	4 dB	a_f	4
G	3 dB	a_m	3.77
L	8	b_f	9.6
N_S	3	b_m	68.73
q	0.5	S	1

Now, a metric commonly used to characterize the energy efficiency of the system is given in [1] as

$$EE = \frac{T}{P}\ b/J \tag{11}$$

where T is the effective throughput of the network in bps and P is the total power consumption in Watts. The problem to be addressed in this work is an optimization problem such as the one presented in [9] but with the emphasis on the energy efficiency. That is, the optimization problem is finding the amount of spectrum allocated to each tier that maximizes the energy efficiency of the network. In addition we use a constraint on the relationship between the throughput per user ($T_{m,u}$) in the macro tier and the throughput per user $\left(T_{f,u}\left(\rho_f^\star\right)\right)$ in the femto tier. (Note that $T_{f,u}\left(\rho_f^\star\right) > T_{m,u}$ and the subscript "u" refers to "user"). So formally, the problem is defined as

$$\begin{aligned} S_m^\star &= \arg\max_{S_m} \underbrace{\frac{S_m N_m T_m + (S - S_m) N_f T_f\left(\rho_f^\star\right)}{P_m^T + P_f^T}}_{EE\ \text{as in (11)}} \\ &\text{s.t. } T_{m,u} \geq q\, T_{f,u}\left(\rho_f^\star\right), \\ &\text{where } T_{m,u} = \frac{S_m T_m}{u_m} \text{ and } T_{f,u} = \frac{(S - S_m)\, T_f\left(\rho_f^\star\right)}{u_f}. \end{aligned} \tag{12}$$

Note that $q \in [0, 1]$ is a quality of service requirement ensuring that a user in the macro tier experiences at least a fraction (q) of the throughput of a user in the femto tier and u_i ($i \in \{f, m\}$) is the number of users served by each base station. Without loss of generality we assume $S = 1$, and so the assignment is a percentage of the available spectrum. Given the fact that (12) is a linear combination with constraints on the minimum throughput requirement, the optimum S_m value ($S_m^\star$) is found when the constraints are satisfied with equality.

6. NUMERICAL RESULTS

Simulation results are now presented in Figs. 2-4 for both Monte-Carlo simulations (i.e., circles, with 5 x10^4 runs for each point) and the analysis (i.e., lines). Note that the simulations lie exaclty on the analytical plots. The parameters used for the simulations are given in Table 1 and we chose them similar to other publications [7], [9]. Given the fact that the user equipment is comprised of battery limited devices, the scenarios simulated in this work consider a maximum of $M_i^r = 2$ antennas per user, whereas the number of antennas in the BSs can be up to $M_i^t = 3$.

In Fig. 2, the effect of the MAP (ρ_f) on the femtocell throughput $(T_f(\rho_f))$ is investigated. The results deal with the achievable throughput when the transmitted power (P_i^{tx}) is the same for all antenna configurations and they do not take into account the effect of the power consumed related to signal processing, cooling effects, etc. It can be seen that there is an optimum value that maximizes the throughput and that it is different depending upon which antenna configuration is used. As increasing the number of antennas makes the system more robust against interference, the MAP (ρ_f) that maximizes the throughput increases with the number of antennas.

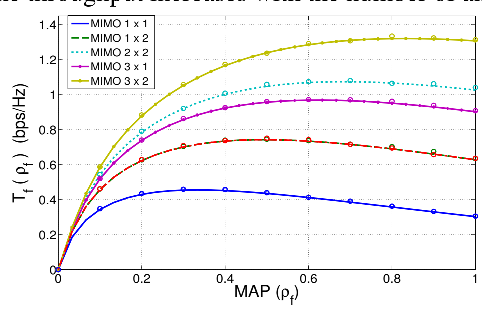

Fig. 2: The effect of MAP (ρ_f) on the femtocell throughput $(T_f(\rho_f)$ in (7)) for different MIMO configurations.

In Figs. 3 and 4, the energy efficiency of the system is shown with a different number of femtocells deployed in the mean area of a macrocell when λ_m is kept constant. In Fig. 3 the energy efficiency of the system is presented when only the power related to transmission is considered (i.e., $b_f = b_m = 0$ in (9) and (10)). This scenario is important when the main concern in the system is the amount of transmit power. It can be seen that the use of multiple antennas has the direct effect of increasing the energy efficiency. So regardless of the number of femtocells deployed, the use of more antennas is always desirable.

In Fig. 4 the energy efficiency of the system is obtained when we consider the total power (i.e., transmit power plus all other power components). The configurations with the highest achieved energy efficiency, along with results for a SISO system, are presented. It can be seen that the increase in the number of femtocells increases the energy efficiency of the system up to a certain threshold after which the energy consumed by the femtocells and its effect on the interference outweigh the gain in throughput, thus reducing the energy ef-

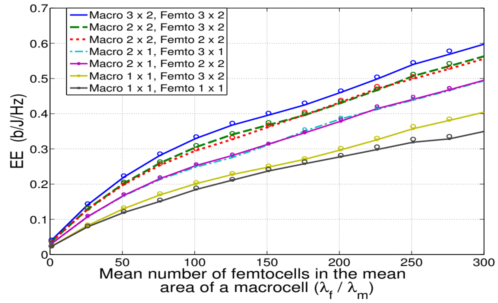

Fig. 3: Energy efficiency (see EE in (12)) versus average number of femtocells in the mean area of a macrocell $\left(\frac{\lambda_f}{\lambda_m}\right)$ with $b_f = b_m = 0$ in (9) and (10) (i.e., only transmit power considered).

ficiency. It can also be seen that the maximum energy efficiency is provided by a system with antenna configurations $M_m^t = 1$, $M_m^r = 2$ and $M_f^t = 1$, $M_f^r = 2$. However, as the number of femtocells increases beyond 100, an $M_m^t = 2$, $M_m^r = 2$ and $M_f^t = 1$, $M_f^r = 2$ system becomes more energy efficient. For a large number of femtocells the energy efficiency of an $M_m^t = 3$, $M_m^r = 2$ and $M_f^t = 1$, $M_f^r = 2$ system can be compared with the previous ones. Note that only for a small number of femtocells, can a SISO system be more energy efficient than other configurations. However, as the number of femtocells increases beyond 50, it rapidly becomes less efficient than other combinations. Furthermore, for each antenna configuration there is a mean number of femtocells which maximizes the energy efficiency of that configuration.

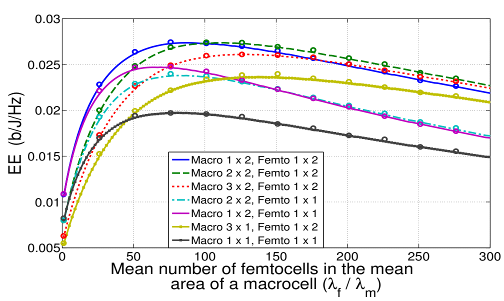

Fig. 4: Energy efficiency (see EE in (12)) versus average number of femtocells in the mean area of a macrocell $\left(\frac{\lambda_f}{\lambda_m}\right)$ with $b_f \neq b_m \neq 0$ in (9) and (10) (i.e., both transmit power and other sources included).

7. CONCLUSIONS

In two-tier systems (macro and femtocells), these results show that using MRT and multiple antennas increases both the overall throughput and the energy efficiency of the network if only the power of data transmission is considered (assuming same transmitted power in all antenna configurations). However when other contributions to the overall network power consumption are also considered, not all the configurations show advantages from an energy efficiency point of view. So a direct increase in the number of antennas does not necessarily result in increased energy efficiency. The optimal antenna configurations were obtained for realistic parameters found in practice, and as we vary these parameters, (e.g., propagation exponent, wall partition loss and MBS density) the optimal configurations vary. It is worthwhile mentioning that SISO systems can be more energy efficient under certain specific conditions. So depending upon the scenario single antenna systems could be preferred from an energy efficiency point of view. Finally, the results illustrate the tradeoff between the energy consumption and the performance expected in terms of overall throughput.

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