IMPROVING SCALAR QUANTIZATION FOR CORRELATED PROCESSES 
USING ADAPTIVE CODEBOOKS ONLY AT THE RECEIVER

Sai Han and Tim Fingscheidt

Institute for Communications Technology, Technische Universität Braunschweig
Schleinitzstr. 22, 38106 Braunschweig, Germany
{ s.han, t.fingscheidt } @ tu-bs.de

ABSTRACT

Lloyd-Max quantization (LMQ) is a widely used scalar non-uniform quantization approach targeting for the minimum mean squared error (MMSE). Once designed, the quantizer codebook is fixed over time and does not take advantage of possible correlations in the input signals. Exploiting correlation in scalar quantization could be achieved by predictive quantization, however, for the price of a higher bit error sensitivity. In order to improve the Lloyd-Max quantizer performance for correlated processes without encoder-sided prediction, a novel scalar decoding approach utilizing the correlation of input signals is proposed in this paper. Based on previously received samples, the current sample can be predicted a priori. Thereafter, a quantization codebook adapted over time will be generated according to the prediction error probability density function. Compared to the standard LMQ, distinct improvement is achieved with our receiver in error-free and error-prone transmission conditions, both with hard-decision and soft-decision decoding.

Index Terms— Lloyd-Max quantization, correlated process, predictive quantization, probability density function, soft-decision decoding

1. INTRODUCTION

Quantization plays a key role in digital communications [1]. In [2] three categories of quantizers are identified: (1) Scalar quantization [3, 4], or vector quantization [5]; (2) quantizers with fixed-rate coding having the same codeword length for each quantization index [6, 7], or quantizers with variable-rate coding [8, 9]; (3) memoryless quantizers with the same quantization codebook over time [3, 4, 10], or quantizers with memory which means that the current quantization reconstruction levels are dependent on past samples [1, 11], with the memory referring to statistical properties of the source process.

When the input signal is correlated, memoryless scalar quantization is robust but inefficient, since it is designed in the same way for correlated as for uncorrelated processes. Therefore, the redundancy in terms of the correlation of source signals cannot be exploited. In contrast, utilizing the correlation in signals, predictive quantization [12, 13] and transform coding using an orthogonal transform matrix to decorrelate the source signals [14] are two major approaches of scalar quantization with memory.

Predictive quantization requires a quantizer inside a prediction loop at the transmitter and the same predictor is used to decode the signal at the receiver. The difference between the actual signal and its predictive estimate — the latter being based on previously reconstructed samples — is actually quantized. This is known from differential pulse code modulation (DPCM) [11], adaptive differential pulse code modulation (ADPCM) [15], and ADPCM-coded speech and audio [16]. However, due to error propagation, predictive coding is known to be sensitive to bit errors.

In order to achieve better performance in adverse transmission conditions, an additional error concealment is often desirable [17–19]. Hard-decision decoding and soft-decision decoding are two further alternative approaches in the receiver applicable to any of the above mentioned three major categories of quantizers. If soft information in the form of log-likelihood ratios (LLRs) containing channel reliability information is available, a soft-decision decoder could be used in the receiver [20–23], instead of error concealment which is typically based only on hard-decided bits.

In this paper, stemming from the idea of scalar quantization with memory, we present how to take advantage of the correlation in the source process, explicitly excluding vector quantization due to delay constraints. Different to predictive quantization which requires predictors both at the transmitter and the receiver side, we assume a non-predictive encoder and employ a predictor only at the receiver side. We utilize an instantaneous prediction error probability density function to compute a quantizer codebook with modified reconstruction levels in the receiver, adapted over time. Moreover, we compare the performance between standard LMQ and our new approach both with hard-decision and soft-decision decoding at the receiver. A particular advantage of our approach is its system-compatible use in decoders for robust non-predictive scalar quantizers in the encoder.

The paper is structured as follows: In Section 2, we de-
The reconstruction level sample probability density function (PDF) is depicted in Fig. 1. A first order autoregressive process (AR(1)) with the i.i.d. zero mean Gaussian innovation \( \varepsilon = (\varepsilon_n, \varepsilon_1, \ldots, \varepsilon_n, \ldots) \) is used as input into the model for the (unquantized) correlated samples \( \tilde{\mathbf{u}} = (\tilde{u}_0, \tilde{u}_1, \ldots, \tilde{u}_n, \ldots) \) satisfying \( \tilde{u}_n = \tilde{e}_n + \rho \cdot \tilde{u}_{n-1} \), with \( n \in \mathbb{N} \) being the time index and \( \rho \) being the correlation coefficient of the AR(1) process. After Lloyd-Max quantization (LMQ), each quantized sample \( u_n \) is represented by a quantizer bit combination in bipolar notation \( x_n = \{-1, +1\}^M \) corresponding to an \( M \) bit quantization index \( i \in \mathcal{I} = \{0, 1, \ldots, 2^M - 1\} \). Without loss of generality, the channel in this paper is described as an equivalent channel model comprising binary phase-shift keying (BPSK) modulation without any channel coding. For hard-decision decoding (HD), the received hard-decided bipolar bit combination \( \tilde{x}_n = \{-1, 1\}^M \) is transformed to a received quantization index \( \tilde{i}_n \in \mathcal{I} \), which is further utilized at the receiver. In contrast, the log-likelihood ratios (LLRs) \( L(x_n) = (L(\tilde{x}_n(0)), L(\tilde{x}_n(1)), \ldots, L(\tilde{x}_n(M - 1))) \in \mathbb{R}^M \) of each received bit \( \tilde{x}_n(m) \in \{-1, 1\} \) are expected by a receiver employing soft-decision decoding (SD) [21], with bit index \( m \in \{0, 1, \ldots, M - 1\} \).

2. IMPROVED LLOYD-MAX DECODING

2.1. Overview

The block diagram of the transmission system in our paper is depicted in Fig. 1. A first order autoregressive process (AR(1)) with the i.i.d. zero mean Gaussian innovation \( \varepsilon = (\varepsilon_n, \varepsilon_1, \ldots, \varepsilon_n, \ldots) \) is used as input into the model for the (unquantized) correlated samples \( \tilde{\mathbf{u}} = (\tilde{u}_0, \tilde{u}_1, \ldots, \tilde{u}_n, \ldots) \) satisfying \( \tilde{u}_n = \tilde{e}_n + \rho \cdot \tilde{u}_{n-1} \), with \( n \in \mathbb{N}^0 \) being the time index and \( \rho \) being the correlation coefficient of the AR(1) process. After Lloyd-Max quantization (LMQ), each quantized sample \( u_n \) is represented by a quantizer bit combination in bipolar notation \( x_n \) corresponding to an \( M \) bit quantization index \( i \in \mathcal{I} \). Without loss of generality, the channel in this paper is described as an equivalent channel model comprising binary phase-shift keying (BPSK) modulation without any channel coding. For hard-decision decoding (HD), the received hard-decided bipolar bit combination \( \tilde{x}_n \) is transformed to a received quantization index \( \tilde{i}_n \), which is further utilized at the receiver. In contrast, the log-likelihood ratios (LLRs) \( L(x_n) = (L(\tilde{x}_n(0)), L(\tilde{x}_n(1)), \ldots, L(\tilde{x}_n(M - 1))) \in \mathbb{R}^M \) of each received bit \( \tilde{x}_n(m) \) are expected by a receiver employing soft-decision decoding (SD) [21], with bit index \( m \in \{0, 1, \ldots, M - 1\} \).

2.2. New Adaptive LMQ Decoder Codebook

The core of our approach is to generate a new decoder quantization codebook adapted over time and depending on the past received samples. While decision levels are standard LMQ ones (as in the encoder), reconstruction levels are adaptively modified.

The well-known LMQ reconstruction levels for quantization index \( i \) read [1]

\[
u^{(i)} = \frac{\int_{d_i}^{d_{i+1}} \tilde{u} \cdot p_G(\tilde{u}) \, d\tilde{u}}{\int_{d_i}^{d_{i+1}} p_G(\tilde{u}) \, d\tilde{u}}.
\]

The reconstruction level \( u^{(i)} \) is the centroid of the area of the sample probability density function (PDF) \( p_G(\tilde{u}) \) in the \( i \)-th quantization interval (i.e., the area between decision levels \( d_i \) and \( d_{i+1} \)), which is shown as black dot in the upper plot of Fig. 2.

However, for correlated samples, after estimation of the previous sample \( \tilde{u}_{n-1} \) in the receiver, a respective predicted sample \( \hat{u}_n \) at current time index \( n \) can be obtained according to (in the AR(1) case)

\[
\hat{u}_n = \rho \cdot \tilde{u}_{n-1},
\]

adopting the correlation coefficient \( \rho \) from the unquantized signal \( \tilde{u} \). Note that the predictor memory is initialized with zeros, meaning \( \tilde{u}_{n-1} = 0 \) for \( n \leq 0 \).

If we are interested in \( u^{(i)} \) for a fixed received \( i = \tilde{i}_n \) at time \( n \), given the predicted sample \( \hat{u}_n \) (representing the past received samples), (1) becomes

\[
u^{(i)} = \frac{\int_{d_i}^{d_{i+1}} \hat{u} \cdot p_G(\hat{u}) \, d\hat{u}}{\int_{d_i}^{d_{i+1}} p_G(\hat{u}) \, d\hat{u}}.
\]

For the Gaussian AR(1) process from Fig. 1 we easily see that for a given \( \hat{u}_n \), the PDF of \( \tilde{u}_n \) results in \( p_G(\tilde{u}_n | \hat{u}_n) = p_G(\tilde{e}_n = \tilde{u}_n - \hat{u}_n) = f(\tilde{u}_n) \), which is the transmitter-sided prediction error PDF \( p_G(\hat{u}) \) shifted by \( \hat{u}_n \).

Applying this to the receiver and replacing \( \hat{u}_n \) by \( \hat{u} \) as it is given by (2), we can write

\[
p_G(\tilde{u}_n | \hat{u}_n) = p_G(\tilde{e}_n = \tilde{u}_n - \hat{u}_n) = f(\tilde{u}_n),
\]

with the shifted receiver-sided prediction error PDF \( p_G(\tilde{e}_n = \tilde{u}_n - \hat{u}_n) \) being sketched in the lower graph of Fig. 2. Note that \( p_G(\hat{u}) \) in general has a lower variance than \( p_G(\tilde{u}) \). Also note that the shape of \( p_G(\hat{u}) \) in quantization interval \([d_i, d_{i+1}]\) is quite different from \( p_G(\tilde{u}) \). The optimal reconstruction values are then

\[
u^{(i)} = \frac{\int_{d_i}^{d_{i+1}} \hat{u} \cdot p_G(\hat{e}_n = \hat{u}_n - \hat{u}_n) \, d\hat{u}_n}{\int_{d_i}^{d_{i+1}} p_G(\hat{e}_n = \hat{u}_n - \hat{u}_n) \, d\hat{u}_n},
\]

while the decision levels \( d_i \) and \( d_{i+1} \) from the standard LMQ are kept according to the given received index \( i = \tilde{i}_n \).
Fig. 3: Block diagram of the newly proposed receiver with hard-decision decoding (HD).

The shifted prediction error PDF as used in (4) is given by:

$$p_E(e_n = \hat{u}_n - \hat{u}_n^+) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\hat{u}_n - \mu)^2}{2\sigma^2}\right),$$

with the mean $\mu = \hat{u}_n^+$, and variance $\sigma^2$. Note that equation (4) can be easily solved numerically with the help of the error function.

Finally, the estimated sample $\hat{u}_n$ is given by computing (4) for each time $n$ using the received index $i_n$ (hard-decision decoding). For soft-decision decoding it must be done for all $i \in I$, followed by further computations.

3. FURTHER DECODER OPTIONS

The receiver can either employ conventional hard-decision decoding (HD) or soft-decision decoding (SD). While SD is mostly applicable to mobile transmission, HD can always be employed, particularly in voice over IP (VoIP).

3.1. Using Hard Decisions

Fig. 3 shows the diagram of the new decoder employing conventional hard decisions. Note that the quantization index $i$ in (4) is replaced by the received quantization index $i_n$.

3.2. Using Soft Decisions

For the new decoder employing soft decisions as depicted in Fig. 4, a new LMQ codebook and a posteriori probabilities $P(x_n^{(i)} | x_n^{(j)})$ (APPS) of a possibly transmitted bit combination $x_n^{(i)}$ with respect to all the received bit combinations $x_n = (x_0, x_1, \ldots, x_n)$ are needed for the sample estimation [21].

Assuming an additive white Gaussian noise (AWGN) channel, the log-likelihood ratios (LLRs) being input into the receiver with soft-decision decoding in Fig. 4, can be obtained by $L(\tilde{x}_n(m)) = \frac{E_b}{N_0} \cdot \tilde{x}_n(m)$, with $\tilde{x}_n(m)$ being the real-valued signal observed at the (noisy) transmission channel output satisfying $\tilde{x}_n(m) = \text{sign}(L(\tilde{x}_n(m))) = \text{sign}(\tilde{x}_n(m))$.

Fig. 4: Block diagram of the newly proposed receiver with soft-decision decoding (SD).

the signal energy per bit $E_b$, and $N_0$ being the noise power spectral density. Accordingly, a bit error probability is given by

$$\text{BER}_n(m) = \frac{1}{1 + \exp(\frac{L(\tilde{x}_n(m))}{\rho})}.$$ 

Thereafter, assuming a memoryless channel, the probabilities of the transition from a possibly transmitted bit combination $x_n^{(i)}$ to a received bit combination $\hat{x}_n$ (i.e., the transition probabilities) is formulated by

$$P(\hat{x}_n(m) | x_n^{(i)}) = \prod_{m=0}^{M-1} P(\hat{x}_n(m) | x_n^{(i)}(m)),$$

with $\hat{x}_n(m)$ being a possibly transmitted bit and the conditional bit probability

$$P(\hat{x}_n(m) | x_n^{(i)}(m)) = \begin{cases} 1 - \text{BER}_n(m), & \text{if } \hat{x}_n(m) = x_n^{(i)}(m), \\ \text{BER}_n(m), & \text{else.} \end{cases}$$

3.2.1. A Posteriori Probabilities (APPS)

Not only the transition probabilities, but also some a priori knowledge is employed to compute the a posteriori probabilities $P(x_n^{(i)} | x_n^{(j)})$. A large training database is required to be processed beforehand to derive the a priori knowledge. The quantized parameters could be regarded as the output of a zeroth-order Markov process leading to zeroth-order a priori knowledge $P(x_n^{(i)} | \hat{x}_n)$ (AK0), or as a first-order Markov process resulting in first-order a priori knowledge $P(x_n^{(i)} | x_{n-1}^{(j)})$ (AK1). The occurrence frequency for different pairs of quantizer output symbols is counted and normalized to obtain the joint probability $P(x_n^{(i)} | x_{n-1}^{(j)})$, thereafter, the AK0 term is obtained from

$$P(x_n^{(i)}) = \sum_{j \in I} P(x_n^{(i)} | x_{n-1}^{(j)}).$$

According to the chain rule, the AK1 term is computed by

$$P(x_n^{(i)} | x_{n-1}^{(j)}) = \frac{P(x_n^{(i)} | x_{n-1}^{(j)})}{\sum_{k \in I} P(x_n^{(i)} | x_{n-1}^{(k)})}.$$ 

Applying the AK0 or AK1 term, the a posteriori probabilities can be obtained either by [21]

$$P(x_n^{(i)} | \hat{x}_n) = \frac{1}{C_n} \cdot P(\hat{x}_n | x_n^{(i)}) \cdot P(x_n^{(i)}),$$
4.2. Discussion

The simulation results of using the standard LMQ and our new approach are compared in Figs. 5 and 6, with HD denot-
In this paper we present a new approach to improve the decoding performance of scalar Lloyd-Max quantizers (LMQs) for correlated processes, while being compatible with standard LMQ encoders.

5. CONCLUSIONS

In consequence, the SNR of using the standard LMQ and the new LMQ decoder in this case converge to be the same.

REFERENCES


