EXPLOITING TIME AND FREQUENCY INFORMATION FOR DELAY/DOPPLER ALTIMETRY

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ABSTRACT

Delay/Doppler radar altimetry is a new technology that has been receiving an increasing interest, especially since the launch of Cryosat-2 in 2010, the first altimeter using this technique. The Delay/Doppler technique aims at reducing the measurement noise and increasing the along-track resolution in comparison with conventional pulse limited altimetry. A new semi-analytical model with five parameters has been recently introduced for this new technology. However, two of these parameters are highly correlated resulting in bad estimation performance when estimating all parameters. This paper proposes a new strategy improving estimation performance for delay/Doppler altimetry. The proposed strategy exploits all the information contained in the delay/Doppler domain. A comparison with other classical algorithms (using the temporal samples only) allows to appreciate the gain in estimation performance obtained when using both temporal and Doppler data.

***Index Terms*—** SAR altimetry, delay/Doppler altimetry, least squares estimation, antenna mispointing.

1. INTRODUCTION

Delay/Doppler altimetry (DDA) is a new technology that aims at reducing the measurement noise and increasing the along-track resolution in comparison with conventional pulse limited altimetry [1]. The noise reduction is achieved by increasing the number of observations while the resolution improvement results from using the Doppler information contained in the observed data. An interesting semi-analytical model was recently proposed in [2] for DDA. This model generalizes the model developed in [3] by considering the effect of antenna mispointing. More precisely, the analytical expression for the flat surface impulse response (FSIR) considers antenna mispointing angles, a circular antenna pattern, no vertical speed effect and a uniform scattering. A two dimensional delay/Doppler map (DDM) is then obtained by a numerical convolution between this analytical FSIR expression, the probability density function (PDF) of the heights of the specular scatterers and the time/frequency point target response (PTR) of the radar. Appropriate processing, including range migration and temporal multi-looking, is generally applied to the resulting DDM yielding the time delay multi-look echo [4, 5]. The resulting echo depends on five parameters that are the epoch τ, the significant wave height SWH, the amplitude P_u, the along-track mispointing angle ξ_{al} and the across-track mispointing angle ξ_{ac}. The analysis of this temporal echo shows high correlation between two of its parameters, P_u and ξ_{al}, which reduces the quality of the estimated parameters [2, 6].

This paper proposes a new estimation strategy, based on a least squares approach as in [2,3], mitigating the bad effects of estimating highly correlated parameters. The proposed strategy exploits not only the time delay data but also the Doppler frequency information in order to separate the impact of the five parameters on the DDA model. More precisely, it will be shown that the effect of the along-track mispointing is more important on the Doppler multi-look echo. This property allows a better estimation of this parameter and of the amplitude P_u by using both temporal and Doppler signals. The proposed estimation strategy is also compared with the classical temporal based estimation algorithms for DDA. The obtained results are very promising and show an improved estimation performance.

The paper is organized as follows. Section 2 summarized the semi-analytical model introduced in [2] for DDA. The effects of the mispointing angles on this model are analyzed in Section 3. The proposed least squares estimation algorithm is then introduced in Section 4. Section 5 validates and compares the proposed estimation algorithm with state-of-the-art methods. Conclusions and future work are finally reported in Section 6.

2. A GENERALIZED DELAY/DOPPLER MODEL

The delay/Doppler altimeter was proposed in order to increase the along-track resolution by considering the Doppler effect resulting from the satellite velocity. As in conventional altimetry, the mean power of a delay/Doppler echo can be expressed as the convolution of three terms: the flat surface impulse response (FSIR), the probability density function (PDF) of the heights of the specular scatterers and

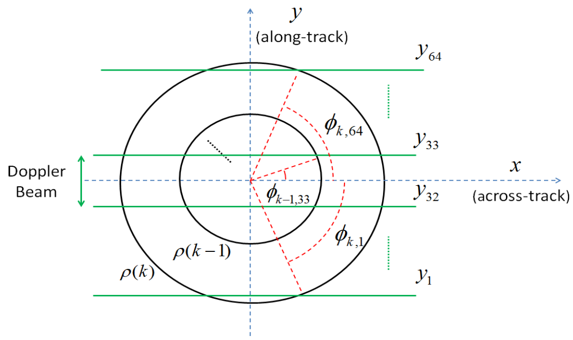

Fig. 1: Propagation circles (in black) and Doppler beams (in green) for conventional altimetry and DDA.

the time/frequency point target response (PTR) of the radar as follows [3, 7, 8]

$$P(t, f) = \text{FSIR}(t, f) * \text{PDF}(t) * \text{PTR}(t, f) \tag{1}$$

with

$$\begin{cases} \text{PDF}(t) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{t^2}{2\sigma_s^2}\right) \\ \text{PTR}(t, f) = \text{PTR}_T(t)\text{PTR}_F(f) \\ \text{PTR}_T(t) = \left|\frac{\sin\left(\pi\frac{t}{T}\right)}{\pi\frac{t}{T}}\right|^2 \\ \text{PTR}_F(f) = \left|\frac{\sin\left(\pi\frac{f}{F}\right)}{\pi\frac{f}{F}}\right|^2 \end{cases} \tag{2}$$

where t is the two-way incremental ranging times, i.e., $t = t' - \frac{2h}{c}$, with t' the travel time of the echo from the instant of transmission, h is the altitude of the satellite, c is the speed of light, T is the sampling period, F is the frequency resolution and σ_s is linked to the significant wave height SWH by the equation $\sigma_s = \frac{\text{SWH}}{2c}$. An analytical model for FSIR that considers antenna mispointing has been recently proposed in [2]. This FSIR expression is obtained by integrating the reflected energy from the observed delay/Doppler footprint (integrating from each Doppler beam as shown in Fig. 1). The resulting expression is given by [2]

$$\begin{aligned} \text{FSIR}(t, n) &= \frac{P_u}{\pi}\left(1 + \frac{ct}{2h}\right)^{-3} U(t)\, I_0\left(\frac{b}{2}\right) \\ &\times \exp\left[-\frac{4}{\gamma}\left(1 - \frac{\cos^2(\xi)}{1+\epsilon^2(t)}\right) + \frac{b}{2}\right] \\ \times &\left[I_0(a)\left(\phi_{t,n+1} - \phi_{t,n}\right) + \sum_{k=1}^{m} \frac{1}{k} I_k(a)\, h_{k,n}(\tilde{\phi})\right] \end{aligned} \tag{3}$$

where

$$h_{k,n}(\tilde{\phi}) = \begin{cases} 2\cos\left(k\tilde{\phi}\right)\left[\sin\left(k\phi_{t,n+1}\right) - \sin\left(k\phi_{t,n}\right)\right], \\ \text{for even } k \\ \\ -2\sin\left(k\tilde{\phi}\right)\left[\cos\left(k\phi_{t,n+1}\right) - \cos\left(k\phi_{t,n}\right)\right], \\ \text{for odd } k \end{cases} \tag{4}$$

and where γ is an antenna beam width parameter, P_u is the waveform amplitude, $\phi_{t,n} = \text{Re}\left[\arctan\left(\frac{y_n}{\sqrt{\rho^2(t) - y_n^2}}\right)\right]$, $y_n = \frac{h\lambda}{2v_s} f_n$ is the coordinate of the nth along-track beam, λ is the wavelength, v_s is the satellite velocity, $f_n = (n - 32N_f - 0.5)\frac{F}{N_f}$ is the nth Doppler frequency (with $n \in \{1, \cdots, 64N_f\}$), N_f is the frequency oversampling factor, $\epsilon(t) = \frac{\rho(t)}{h} = \sqrt{\frac{ct}{h}}$, $\rho(t)$ is the radius of propagation circles, ξ (resp. $\tilde{\phi}$) is the mispointing angle with respect to the z axis (resp. the x axis [3]), $U(.)$ is the Heaviside function, I_k is the kth order modified Bessel function of the first kind, m is a fixed number that controls the desired FSIR precision (see [2]), $a = a(t, \xi)$ and $b = b(t, \xi)$ are given by

$$a(t, \xi) = \frac{4\epsilon(t)}{\gamma}\frac{\sin(2\xi)}{1+\epsilon^2(t)}, \; b(t, \xi) = \frac{4\epsilon^2(t)}{\gamma}\frac{\sin^2(\xi)}{1+\epsilon^2(t)}. \tag{5}$$

The resulting signal is shifted by the time instant τ which is known as the epoch parameter (related to the range between the satellite and the observed surface).

The reflected power $P(t, f)$ (known as a delay/Doppler map) is finally obtained by a numerical computation of the double convolution in (1) using the FSIR formula given in (3). The classical way to exploit the matrix $P(t, f)$ is to compute a time delay "multi-look" altimetric waveform [4]. This waveform is obtained after applying a delay compensation operation to each Doppler beam followed by the sum of these beams [4, 5]. The resulting temporal multi-look signal can be written

$$s(t) = \sum_{n=1}^{N} P\left(t - \delta t_n - \tau, f_n\right) = \sum_{n=1}^{N} m\left(t, f_n\right) \tag{6}$$

where δt_n is the delay compensation expressed in seconds (this operation is summarized in Fig. 2). Note that the final altimetric signal depends on the following parameter vector $\boldsymbol{\theta} = (\text{SWH}, P_u, \tau, \xi_{\text{ac}}, \xi_{\text{al}})^T$ where $\xi_{\text{al}} = \xi\sin(\tilde{\phi})$ and $\xi_{\text{ac}} = \xi\cos(\tilde{\phi})$ are the along-track and across-track mispointing angles. Note finally that the discrete multi-look echo is gathered in the vector $\boldsymbol{s} = (s_1, \cdots, s_K)^T$, where $K = 128$ samples (or gates) and $s_k = s\left(kT\right)$.

3. ANALYSIS VERSUS MISPOINTING ANGLES

The antenna mispointing is introduced by means of two variables ξ and $\tilde{\phi}$ which are directly related to the along-track and across-track mispointing angles. This antenna mispointing has a different effect depending on its direction. Consider first the across-track mispointing. Fig. 3 shows an energy migration from the low time gates to the high time gates (because of the move of the antenna gain in the x axis when ξ_{ac} increases). This across-track mispointing reduces the amplitude of the temporal multi-look echo as shown in Fig. 3 (right-top) but it also affects the shape of the waveform as

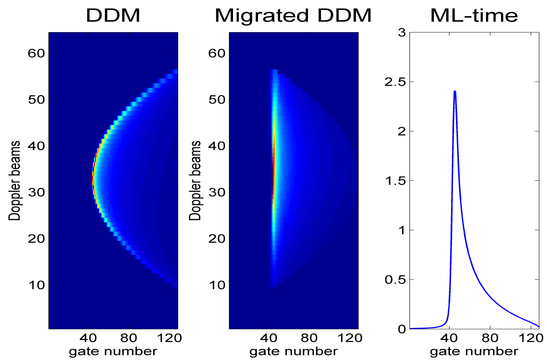

Fig. 2: Construction of a multi-look waveform. (left) a delay/Doppler map (DDM), (middle) DDM after range migration, and (right) temporal multi-look (M-L) echo.

shown in Fig. 3 (right-bottom) representing the same normalized waveforms. Consider now the along-track mispointing. When increasing this parameter, the energy moves from the lower Doppler beams to the higher ones (because each along-track ground position is related to a Doppler frequency [1] and due to the move of the antenna gain along the y axis). This behavior reduces the amplitude of the temporal multi-look echo as shown in Fig. 4 (right-top) while it does not change the shape of the waveform as shown in Fig. 4 (right-bottom) representing the same normalized waveforms. Therefore, the along-track mispointing angle ξ_{al} has a similar effect as the amplitude P_u when only considering the temporal multi-look echo. This property results in bad estimation of the five parameters when considering this echo.

In order to deal with this issue, one has to introduce additional information that allows the distinction between ξ_{al} and P_u. This can be done by considering the whole DDM matrix or more conveniently the Doppler or frequential multi-look echo[1] which is obtained by summing the energy of temporal gates for each Doppler beam (i.e., summing the columns of the DDM after range migration) as follows

$$d(f) = \sum_{k=1}^{K} m(k, f). \tag{7}$$

Indeed, Fig. 5 shows the effect of antenna mispointing on the Doppler multi-look echo. While ξ_{ac} only reduces the amplitude of this echo (see Fig. 5 (right)), ξ_{al} introduces an asymmetric behavior on it (see Fig. 5 (left)) because of the move of the energy according to Doppler beams. Note that positive or negative signs of the across-track angle have the same consequence on the echo (no difference on the Doppler multi-look echoes) while the sign of the along-track mispointing angle can be determined by the shift of the Doppler multi-look echoes (to the right for $\xi_{\text{al}} > 0$ and to the left for $\xi_{\text{al}} < 0$). This behavior provides a solution to isolate the effect of ξ_{al} and by the way to better estimate it, as we will see in the next sections.

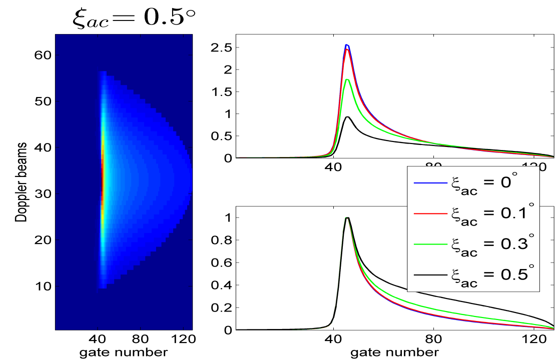

Fig. 3: DDM (left), temporal multi-look echoes (Right-Top) and the same normalized temporal multi-look echoes (Right-Bottom) for different across-track mispointing angles.

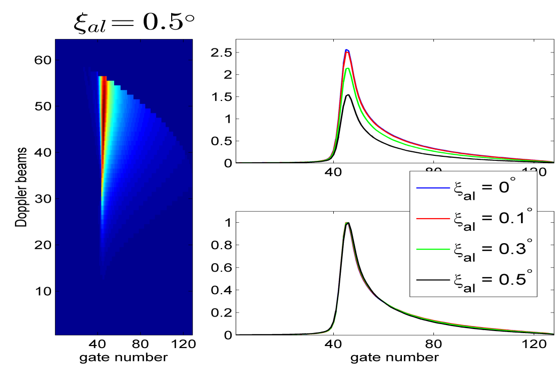

Fig. 4: DDM (left), temporal multi-look echoes (Right-Top) and the same normalized temporal multi-look echoes (Right-Bottom) for different along-track mispointing angles.

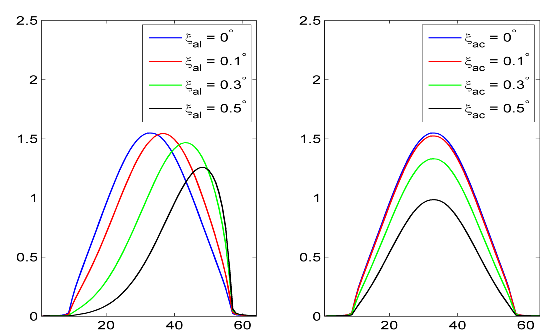

Fig. 5: Doppler multi-look echoes for different along-track mispointing angles (left) and across-track mispointing angles (right).

[1]Processing the whole delay/Doppler map would require to handle matrices of size 64×128 requiring a high computational complexity

4. PROPOSED ESTIMATION STRATEGY

As in [3,5], we propose to estimate the parameters of the generalized DDA model (temporal or both temporal and Doppler echo) by using the following least-squares strategy

$$\hat{\boldsymbol{\theta}}_{\mathrm{LS}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{2} \sum_{h=1}^{H} g_h^2(\boldsymbol{\theta}) \tag{8}$$

where $g_h(\boldsymbol{\theta}) = y_h - x_h(\boldsymbol{\theta})$ is the vector of residues, $\boldsymbol{y} = (y_1, \ldots, y_H)^T$ is the observed altimetric waveform, $x_h(\boldsymbol{\theta}) = [x_1(\boldsymbol{\theta}), \ldots, x_H(\boldsymbol{\theta})]^T$ is the theoretical model which depends on the parameter vector of interest $\boldsymbol{\theta}$ and H is the length of the considered echo ($H = K$ when considering the classical temporal echo $\boldsymbol{x} = \boldsymbol{s}$). In this paper, we propose to solve (8) using the Levenberg-Marquardt algorithm [9]. This algorithm uses a gradient descent approach to update the vector of parameters $\boldsymbol{\theta}$ as follows

$$\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)} - \left[\boldsymbol{J}^T\boldsymbol{J} + \mu \boldsymbol{I}_J\right]^{-1} \boldsymbol{J}^T \boldsymbol{g}\left(\boldsymbol{\theta}^{(i)}\right) \tag{9}$$

where $\boldsymbol{\theta}^{(i)}$ is the estimate of $\boldsymbol{\theta}$ at the ith iteration, $\boldsymbol{J} = \boldsymbol{J}\left(\boldsymbol{\theta}^{(i)}\right) = \left[\frac{\partial \boldsymbol{g}(\boldsymbol{\theta}^{(i)})}{\partial \theta_1}, \cdots, \frac{\partial \boldsymbol{g}(\boldsymbol{\theta}^{(i)})}{\partial \theta_J}\right]$ is an $H \times J$ matrix such that $\frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \theta_j} = \left[\frac{\partial g_1(\boldsymbol{\theta})}{\partial \theta_j}, \cdots, \frac{\partial g_H(\boldsymbol{\theta})}{\partial \theta_j}\right]^T$, J is the number of parameters to estimate, $\boldsymbol{I}_J$ is the $J \times J$ identity matrix and μ is a regularization parameter.

The new estimation strategy considered in this paper is based on the temporal/Doppler signal $\boldsymbol{x} = \left(\boldsymbol{s}^T, \boldsymbol{d}^T\right)^T$ where $\boldsymbol{d} = (d_1, \cdots, d_N)^T$, with $d_n = d(nF)$. In this case, the length of the vector $\boldsymbol{x}$ is $H = K + N$. The signal $\boldsymbol{x}$ is used to estimate the five altimetric parameters $\boldsymbol{\theta} = (\mathrm{SWH}, P_u, \tau, \xi_{\mathrm{ac}}, \xi_{\mathrm{al}})^T$ resulting in a so-called generalized DDA5 (denoted as G-DDA5). This estimation strategy is compared to the following classical temporal based estimation algorithms

- DDA3 proposed in [3]. This approach does not consider mispointing angles ($\xi_{\mathrm{al}} = \xi_{\mathrm{ac}} = 0°$),
- DDA4 proposed in [2] where the along-track mispointing angle has been fixed to an a priori estimated value (for real echoes, this can be done thanks to the star-trackers on-board the satellite) to avoid the correlation problems between ξ_{al} and P_u,
- DDA5 proposed in [6]. This approach estimates 5 parameters from the temporal multi-look echo and suffers from the correlation between ξ_{al} and P_u.

5. SIMULATION RESULTS

The different estimation strategies are evaluated with noisy synthetic echoes with known parameters (in our simulations, we have chosen $P_u = 1$, $\tau = 31$ gates, $\xi_{\mathrm{al}} = 0°$ when varying SWH $\in [1, 8]$ m and $\xi_{\mathrm{ac}} \in [0, 0.7]$ degrees). The noise has been generated as described in [5]. All results presented in this section have been averaged using $N_{\mathrm{MC}} = 500$ Monte Carlo runs (with the same parameters but with different noise realizations). Figs. 6 and 7 show examples of estimated waveforms in the presence of across-track and along-track mispointing angles obtained with the G-DDA5 algorithm.

In a second step, we evaluate the performance of the proposed estimation method by analyzing the root mean square errors (RMSEs) of the estimated model parameters when varying SWH and ξ_{ac}. Indeed, while the obtained RMSEs depend on the whole set of parameters (P_u, τ, SWH, ξ_{al}, ξ_{ac}), a focus is done on their behavior as a function of SWH (high sensitivity) and as a function of ξ_{ac} (the new estimated parameter). The results are displayed in Figs. 8 and 9, respectively. In absence of mispointing, Fig. 8 shows that all the methods perform similarly except DDA5 which is very sensitive to the correlation between the parameters ξ_{al} and P_u. Fig. 9 shows that the DDA3 algorithm is very sensitive to the presence of mispointing and thus that its performance decreases significantly when ξ_{ac} increases (i.e., the parameter RMSEs increase when ξ_{ac} increases). Moreover, the proposed algorithm G-DDA5 allows the performance of DDA4 to be reached while being able to estimate the along-track mispointing angle (which is supposed to be equal to its true value for DDA4).

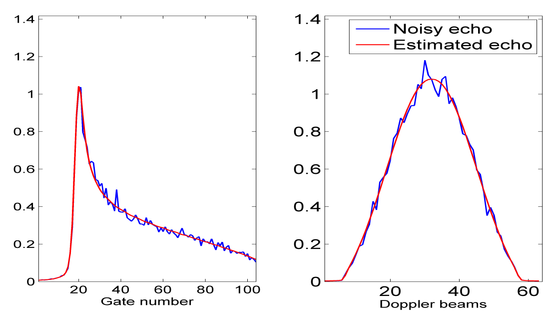

Fig. 6: Example of estimated waveforms using the G-DDA5 model (a) temporal signal (b) Doppler signal (The parameters are $P_u = 1$, $\tau = 31$ gates, SWH $= 2$ m, $\xi_{\mathrm{al}} = 0°$ and $\xi_{\mathrm{ac}} = 0.5°$). The parameter estimates are $P_u = 1.02$, $\tau = 31.1$ gates, SWH $= 2.23$ m, $\xi_{\mathrm{al}} = -2 \times 10^{-4}$ and $\xi_{\mathrm{ac}} = 0.50°$.

6. CONCLUSIONS

The first contribution of this paper was to analyze the effects of antenna mispointing angles on the temporal and Doppler signals resulting from a semi-analytical model recently introduced for delay/Doppler altimetry. We can conclude that the along-track mispointing angle mainly affects the amplitude of the temporal waveform whereas it changes the global shape of the Doppler waveform. Conversely, the across-track mispointing angle mainly affects the amplitude of the Doppler

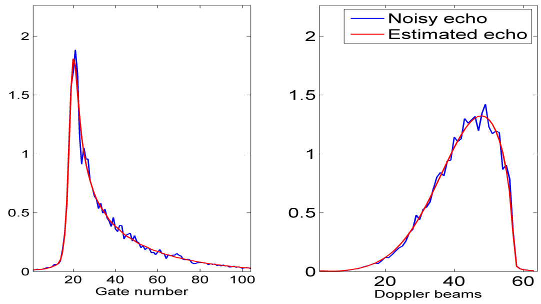

Fig. 7: Example of estimated waveforms using the G-DDA5 model (a) temporal signal (b) Doppler signal (The parameters are $P_u = 1$, $\tau = 31$ gates, SWH $= 2$ m, $\xi_{\text{al}} = 0.5°$ and $\xi_{\text{ac}} = 0°$). The parameter estimates are $P_u = 0.97$, $\tau = 31.15$ gates, SWH $= 2.05$ m, $\xi_{\text{al}} = 0.48°$ and $\xi_{\text{ac}} = 0.05°$.

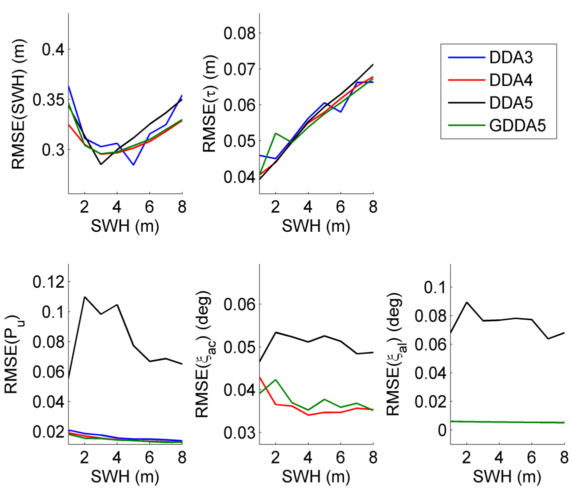

Fig. 8: Parameter RMSEs versus SWH when considering echoes without mispointing estimated with DDA3, DDA4, DDA5 and G-DDA5. The simulation has been obtained using 500 Monte-Carlo realizations with the parameters $P_u = 1$, $\tau = 31$ gates and $\xi_{\text{al}} = \xi_{\text{ac}} = 0°$.

signal whereas its changes the shape of the temporal waveform. Based on these considerations, we introduced a new estimation algorithm for delay/Doppler altimetry which is the second contribution of this work. This algorithm is based on a least squares approach applied on both temporal and Doppler signals. The performance resulting from this new estimation strategy is very satisfactory and showed that the mispointing angles can be estimated jointly with the other altimetric parameters without performance degradation. Prospects include the derivation of Cramér-Rao lower bounds associated with the proposed delay/Doppler model. Generalizing the proposed strategy to a weighted least squares approach (as in [5]) is also an interesting issue that is currently under

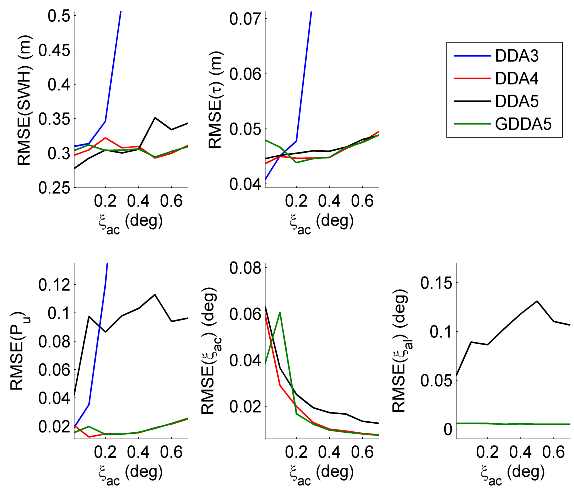

Fig. 9: Parameter RMSEs versus ξ_{ac} when considering echoes estimated with DDA3, DDA4, DDA5 and G-DDA5. The simulation has been obtained using 500 Monte-Carlo realizations with the parameters $P_u = 1$, SWH $= 2$ m, $\tau = 31$ gates and $\xi_{\text{al}} = 0°$.

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