

OPTIMAL QUANTIZATION AND POWER ALLOCATION FOR ENERGY-BASED DISTRIBUTED SENSOR DETECTION

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ABSTRACT

We consider the decentralized detection of an unknown deterministic signal in a spatially uncorrelated distributed wireless sensor network. N samples from the signal of interest are gathered by each of the M spatially distributed sensors, and the energy is estimated by each sensor. The sensors send their quantized information over orthogonal channels to the fusion center (FC) which linearly combines them and makes a final decision. We show how by maximizing the modified deflection coefficient we can calculate the optimal transmit power allocation for each sensor and the optimal number of quantization bits to match the channel capacity.

Index Terms— Distributed detection, soft decision, wireless sensor networks.

1. INTRODUCTION

Wireless sensor networks (WSNs) spatially deployed over a field can monitor many phenomena. Because of their relatively low cost and robustness to node failures they are receiving significant attention. A typical wireless sensor network consists of a fusion center (FC) and a number of geographically distributed sensors. Each individual sensor makes an estimate of a particular quantity (in our case, the energy of the received signal), and then sends a quantized version to the (FC), where all the sensor outputs are optimally combined to arrive at a global detection decision. WSNs have been considered for different applications such as localizing and tracking acoustic targets, voice activity detection, and spectrum sensing for cognitive radios. In such applications, accurate distributed observations are fundamental to reduce detection errors.

The problem of decentralized detection (and estimation) in a WSN assuming error-free communication has been extensively tackled in [2], [3], [4] to name just a few. For a target MSE performance, the authors in [3] proposed the minimization of the summation of sensor transmit powers, while [4] suggested minimization of the Euclidean norm of the transmit powers. In both [3] and [4] the number of bits used for

quantization to transmit data from each sensor to the FC is constrained to be less than channel capacity.

In [10] asymptotic results are provided for distributed detection on joint power constraint in wireless sensor networks while in [11] a finite number of sensors with both individual and joint power constraints is considered for distributed detection over MIMO channels. A decentralized strategy for optimizing the estimation MSE subject to a network rate constraint is presented in [5]. A more recent work in [6] proposed optimum training and data power allocation with inhomogeneous sensors using binary phase shift keying modulated decisions at the FC for distributed detection.

In this work (for a finite number of sensors) we derive analytically the optimal transmit power and number of quantization bits for each sensor, and investigate the detection performance of the sensor network over flat fading transmission links. Our work differs from [3] in that instead of sending the quantized version of the sensor observations, we propose to send the quantized local test statistics (i.e., the sample energy) to the fusion center. In [3] the authors assume full knowledge of the deterministic signal (s) to be detected, while here we derive a scheme to detect an unknown deterministic signal. Moreover, our proposed optimal power and bit allocation scheme is a function of the local SNR at each individual node. We employ a linear fusion rule at the FC and adopt the modified deflection coefficient (MDF) [7] as the detection performance criterion, while [3] uses a matched filter.

2. SYSTEM MODEL

Consider the problem of detecting the presence of an unknown deterministic signal $s(n)$ by a sensor network consisting of M sensors. N samples of the observed signal are gathered and the energy estimation is performed by each sensor. The measurement at each sensor $s_i(n)$ is further corrupted by AWGN $w_i(n) \sim \mathcal{N}(0, \sigma_i^2)$. Each node then sends its information (quantized to L_i bits) to the FC for soft decision combining. The measured signal takes one of the following forms, depending on the underlying hypothesis:

$$\begin{aligned}
H_0 : x_i(n) &= w_i(n) \\
H_1 : x_i(n) &= s_i(n) + w_i(n) \\
&\text{for } i = 1, 2, \dots, M \text{ and } n = 1, 2, \dots, N.
\end{aligned} \quad (1)$$

The i^{th} sensor evaluates

$$T_i = \sum_{n=1}^N (x_i(n))^2, \quad i = 1, 2, \dots, M \quad (2)$$

which for large N can be approximated by a Gaussian distribution [8]. It is not difficult to show that

$$\begin{aligned}
E\{T_i|H_0\} &= N\sigma_i^2, \quad \text{Var}\{T_i|H_0\} = 2N\sigma_i^4 \\
E\{T_i|H_1\} &= N\sigma_i^2(1 + \xi_i), \quad \text{Var}\{T_i|H_1\} = 2N\sigma_i^4(1 + 2\xi_i)
\end{aligned} \quad (3)$$

$$\text{where } \xi_i = \sum_{n=1}^N s_i^2(n) / N\sigma_i^2.$$

3. QUANTIZED SOFT DECISION COMBINING

Here we will investigate linear soft decision combining at the FC. This has superior performance to the hard decision approach, but it entails additional complexity at the FC. Soft decision combining also puts additional demands on both the limited power resources of the sensors and effective utilization of the sensor to FC channel capacity. So here we propose a scheme, where each individual sensor has to quantize its observed test statistic (T_i) to L_i bits. To satisfy the capacity constraint on each sensor to FC channel, we require:

$$L_i \leq \frac{1}{2} \log_2 \left(1 + \frac{p_i h_i^2}{\zeta_0} \right) \text{ bits} \quad (4)$$

where p_i denotes the transmit power of sensor i , h_i is the flat fading gain between sensor node i and the FC, and ζ_0 is the variance of the AWGN at the FC. The quantized test statistic (\hat{T}_i) at the i^{th} sensor can be modeled as

$$\hat{T}_i = T_i + v_i \quad (5)$$

where v_i is quantization noise independent of $w_i(n)$ in (1). Assuming uniform quantization with $T_i \in [0, 2U]$, then

$$\sigma_{v_i}^2 = \frac{U^2}{3 \times 2^{2L_i}}. \quad (6)$$

Linearly combining $\{\hat{T}_i\}_{i=1}^M$ at the FC gives

$$T_f = \sum_{i=1}^M \alpha_i \hat{T}_i \quad (7)$$

where the weights $\{\alpha_i\}_{i=1}^M$ will be optimized in section 4.

Again, for large M , T_f will be approximately Gaussian and we can show that:

$$\begin{aligned}
E\{T_f|H_0\} &= \sum_{i=1}^M \alpha_i (N\sigma_i^2 + U) \\
E\{T_f|H_1\} &= \sum_{i=1}^M \alpha_i (N\sigma_i^2(1 + \xi_i) + U) \\
\text{Var}\{T_f|H_0\} &= \sum_{i=1}^M \alpha_i^2 (2N\sigma_i^4 + \sigma_{v_i}^2) \\
\text{Var}\{T_f|H_1\} &= \sum_{i=1}^M \alpha_i^2 [2N\sigma_i^4(1 + 2\xi_i) + \sigma_{v_i}^2].
\end{aligned} \quad (8)$$

The FC makes the following decisions:

$$\left. \begin{aligned}
&\text{if } T_f < \Lambda_f, \text{ decide } H_0 \\
&\text{if } T_f > \Lambda_f, \text{ decide } H_1
\end{aligned} \right\} \quad (9)$$

where Λ_f is the FC detection threshold. The probabilities of false alarm and detection at the FC are respectively:

$$\begin{aligned}
P_{fa} &= \Pr(T_f > \Lambda_f | H_0) = Q \left(\frac{\Lambda_f - E\{T_f|H_0\}}{\sqrt{\text{Var}\{T_f|H_0\}}} \right) \\
P_d &= \Pr(T_f > \Lambda_f | H_1) = Q \left(\frac{\Lambda_f - E\{T_f|H_1\}}{\sqrt{\text{Var}\{T_f|H_1\}}} \right)
\end{aligned} \quad (10)$$

where $Q(\cdot)$ is the Q -function. And from (10) we can write [9]

$$P_d = Q \left(\frac{Q^{-1}(P_{fa}) \sqrt{\text{Var}\{T_f|H_0\}} - \Psi}{\sqrt{\text{Var}\{T_f|H_1\}}} \right) \quad (11)$$

where

$$\Psi = E\{T_f|H_1\} - E\{T_f|H_0\} = N \sum_{i=1}^M \alpha_i (\sigma_i^2 \xi_i). \quad (12)$$

So using (4), (6) and (8) in (11) we get

$$P_d = Q \left(\frac{Q^{-1}(P_{fa}) \sqrt{\sum_{i=1}^M \alpha_i^2 \left(2N\sigma_i^4 + \frac{U^2}{3 \left(1 + \frac{p_i h_i^2}{\zeta_0} \right)} \right)} - \Psi}{\sqrt{\sum_{i=1}^M \alpha_i^2 \left[2N\sigma_i^4 (1 + 2\xi_i) + \frac{U^2}{3 \left(1 + \frac{p_i h_i^2}{\zeta_0} \right)} \right]}} \right). \quad (13)$$

The formula in (13) imposes a relationship between the probability of detection, the power allocated to each transmission (sensor to the FC) link and the weight (α_i in (7)) for each individual link.

4. OPTIMUM WEIGHT COMBINING AND POWER ALLOCATION

In this section, we would like to find the optimum weighting vector (α_{opt}) and the optimum power allocation vector (\mathbf{p}_{opt}) that maximize (13) (see definition later), under the constraint of a maximum transmit power budget (P_t). However, maximizing (13) w.r.t. α and \mathbf{p} is difficult and no closed form solution can be found. So we will approximate the optimal solution by adopting the modified deflection coefficient [7] as an alternative function to be maximized. This is given as:

$$\tilde{d}^2(\alpha, \mathbf{p}) = \left(\frac{\mathbb{E}\{T_f|H_1\} - \mathbb{E}\{T_f|H_0\}}{\sqrt{\text{Var}\{T_f|H_1\}}} \right)^2 = \frac{(\mathbf{r}^T \alpha)^2}{\alpha^T \mathbf{G} \alpha} \quad (14)$$

where

$$\mathbf{r} = [N\sigma_1^2 \xi_1, N\sigma_2^2 \xi_2, \dots, N\sigma_M^2 \xi_M]^T$$

$$\alpha = [\alpha_1, \alpha_2, \dots, \alpha_M]^T, \mathbf{p} = [p_1, p_2, \dots, p_M]^T$$

$$\mathbf{G} = 2N \text{diag} \left(\sigma_1^4 (1+2\xi_1) + \frac{\sigma_{v_1}^2}{2N}, \dots, \sigma_M^4 (1+2\xi_M) + \frac{\sigma_{v_M}^2}{2N} \right).$$

Note that the dependence of $\tilde{d}^2(\alpha, \mathbf{p})$ on the transmit power vector \mathbf{p} enters (14) through the $\{\sigma_{v_i}^2\}_{i=1}^M$ terms via (4) and (6). Now, our optimization problem is:

$$\begin{aligned} (\alpha_{opt}, \mathbf{p}_{opt}) &= \arg \max_{\alpha, \mathbf{p}} \left(\tilde{d}^2(\alpha, \mathbf{p}) \right) \\ &\text{subject to } \sum_{i=1}^M p_i \leq P_t \text{ for } p_i \geq 0, i = 1, 2, \dots, M. \end{aligned} \quad (P1)$$

We assume that the fusion center (FC) has full knowledge of quantities such as the channel gains (h_i) from sensors to FC, sensing noise variances (σ_i^2) at the different sensors, and prior information about the signal's energy. In the case where the conditions affecting the network do not change fast, the above assumptions are realistic. Furthermore, in the cases where we know the position of the target (i.e., we know where the phenomenon to be detected happens), the assumption for the knowledge of the ξ_i is a valid assumption. Our proposed scheme can be used to detect a spatial resonance in a bridge, to detect a fire event in a factory to name just a few.

4.1. Weight combining optimization

Further, via the transformation $\beta = \mathbf{G}^{1/2} \alpha$, the deflection coefficient (14) becomes:

$$\tilde{d}^2(\beta, \mathbf{0}) = \frac{\beta^T \mathbf{M} \beta}{\|\beta\|^2}, \mathbf{M} = \mathbf{G}^{-T/2} \mathbf{r} \mathbf{r}^T \mathbf{G}^{-1/2}. \quad (15)$$

So $\alpha_{opt} = \mathbf{G}^{-1/2} \beta_{opt} = k \mathbf{G}^{-1} \mathbf{r}$, where $\beta_{opt} = k \mathbf{G}^{-1/2} \mathbf{r}$ is the normalized eigenvector corresponding to the maximum

eigenvalue of \mathbf{M} . Also, we can easily show that:

$$\alpha_{opt} = k \begin{bmatrix} \frac{N\sigma_1^2 \xi_1}{2N\sigma_1^4(1+2\xi_1) + \sigma_{v_1}^2} \\ \frac{N\sigma_2^2 \xi_2}{2N\sigma_2^4(1+2\xi_2) + \sigma_{v_2}^2} \\ \vdots \\ \frac{N\sigma_M^2 \xi_M}{2N\sigma_M^4(1+2\xi_M) + \sigma_{v_M}^2} \end{bmatrix}. \quad (16)$$

So now (16) establishes a relationship between the optimum weighting vector (α_{opt}) and the sensor transmit power (\mathbf{p}) through the $\sigma_{v_i}^2$ quantity (see definition (4) and (6)).

4.2. Optimum power allocation

We now substitute α_{opt} ($k = 1$) back into (14) and we then have the following optimization problem to obtain \mathbf{p}_{opt} :

$$\begin{aligned} \mathbf{p}_{opt} &= \arg \max_{\mathbf{p}} \left(\tilde{d}^2(\alpha_{opt}, \mathbf{p}) \right) \\ &\text{subject to } \sum_{i=1}^M p_i \leq P_t \text{ for } p_i \geq 0, i = 1, 2, \dots, M \end{aligned}$$

which is easily shown to be equivalent to (P3):

$$\begin{aligned} &\text{maximize}_{\mathbf{p}} \left(\sum_{i=1}^M \frac{N^2 \sigma_i^4 \xi_i^2}{2N\sigma_i^4(1+2\xi_i) + \frac{U^2}{3\left(1+\frac{p_i h_i^2}{\zeta_0}\right)}} \right) \\ &\text{subject to } \sum_{i=1}^M p_i \leq P_t, p_i \geq 0, i = 1, 2, \dots, M. \end{aligned} \quad (P3)$$

The aim of solving the above optimization problem is to distribute in an optimum way the total power budget among M distributed sensors such that the probability of detection is maximized. We consider the total power budget constraint in order to investigate the following: given a constant total power budget (fixed cost of our network) how we can maximize the probability of detection at the fusion center by controlling the sensor transmit power (i.e., the number of active sensors)? As it will be shown later in the simulations results by controlling the transmit power in an optimum way we can select a number of active sensors while keeping those that have very poor sensor to FC channels in sleeping mode. In this way, the sensors that require very high power will not transmit and so provide longer battery life. After justifying our constrain choice, (P3) can be solved analytically using the Lagrangian function :

$$\begin{aligned} &\sum_{i=1}^M \frac{N^2 \sigma_i^4 \xi_i^2}{2N\sigma_i^4(1+2\xi_i) + \frac{U^2}{3\left(1+\frac{p_i h_i^2}{\zeta_0}\right)}} \\ &- \lambda_0 \left(\sum_{i=1}^M p_i - P_t \right) + \sum_{i=1}^M \mu_i p_i \end{aligned}$$

and imposing the Karush-Kuhn-Tucker (K.K.T) conditions [13]:

$$\frac{N^2 \sigma_i^4 \xi_i^2}{\left(2N \sigma_i^4 (1+2\xi_i) + \frac{U^2}{3 \left(1 + \frac{p_i h_i^2}{\zeta_0}\right)}\right)^2} \times \frac{U^2 \times \frac{h_i^2}{\zeta_0}}{3 \left(1 + \frac{p_i h_i^2}{\zeta_0}\right)^2} - \lambda_0 + \mu_i = 0 \quad (17)$$

$$\begin{aligned} \lambda_0 \left(\sum_{i=1}^M p_i - P_t \right) &= 0 \\ \sum_{i=1}^M p_i - P_t &\leq 0 \\ \lambda_0 \geq 0, \mu_i p_i &= 0, i = 1, 2, \dots, M \\ \mu_i \geq 0, p_i &\geq 0, i = 1, 2, \dots, M. \end{aligned} \quad (18)$$

Solving the K.K.T conditions in (17) and (18) gives:

$$p_{i,opt} = \left[\frac{1}{\sqrt{\lambda_0}} \left(\frac{\xi_i U \sqrt{3}}{6 \sigma_i^2 (1+2\xi_i) \sqrt{\frac{h_i^2}{\zeta_0}}} \right) - \frac{U^2}{6N \sigma_i^4 (1+2\xi_i) \frac{h_i^2}{\zeta_0}} - \frac{\zeta_0}{h_i^2} \right]^+ \quad (19)$$

where $[x]^+$ equals 0 if $x < 0$, otherwise it equals x , and λ_0 can be evaluated in similar way as in [4] by imposing equality in the constraint $\sum_{i=1}^M p_i = P_t$ in (P3).

5. SIMULATION RESULTS

We will now present some simulations to examine the effectiveness of our approach. For all simulations, we estimate Λ_f in (9) by substituting a fixed value for P_{fa} . We let all the σ_i^2 terms at each sensor be different, such that

$$10 \log_{10} \left(\sum_{i=1}^M \xi_i \right) = -3 \text{ dB}, \text{ where } \xi_i = \frac{\sum_{n=1}^N s_i^2(n)}{N \sigma_i^2}.$$

In addition we let $\zeta_0 = 0.1$ in (4). We will also refer to ‘‘equal weight’’ combining in (7) (i.e., $\alpha_i = \frac{1}{\sqrt{M}}, \forall i$) and use this as a benchmark. Finally, we choose L_i with equality in (4). In Fig. 1, the two lower plots show the sensor transmit power p_i and the number of bits allocated to quantize T_i for the i^{th} sensor to the FC channel respectively. The actual channel coefficients (randomly chosen for $M = 10$) are in the upper plot. Clearly with optimum linear weighting in (7) we allocate more power and bits to the best channels unlike the non-optimum equal weighting. In the case of the optimum combining, nodes that have very bad channels (i.e., nodes that

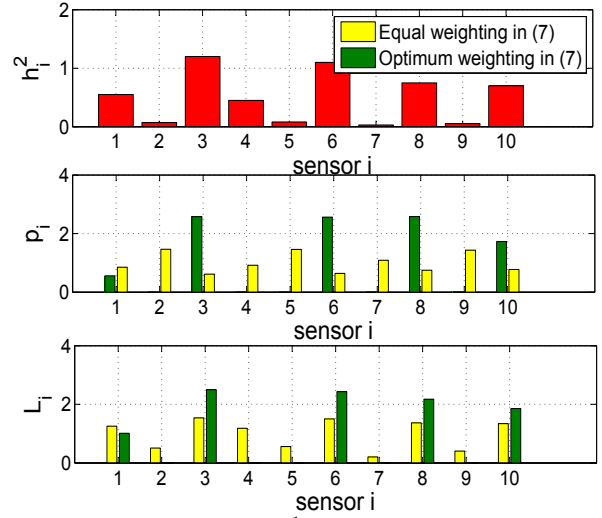


Fig. 1. Equal weight ($\alpha_i = \frac{1}{\sqrt{M}}, \forall i$) and optimal weight combining ($\alpha = \alpha_{opt}$ in (16)) transmit power and channel quantization bits allocation for $P_{fa} = 0.1$, $P_t = 10$, $U = 0.1$ and $M = 10$.

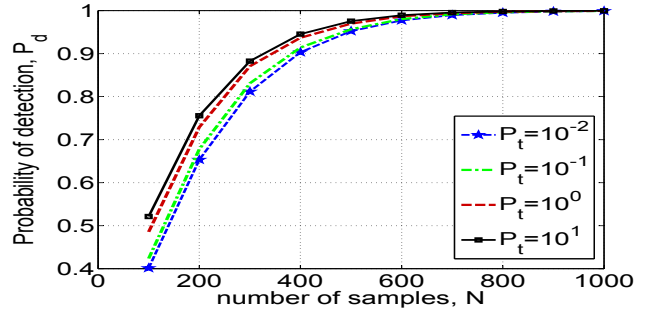


Fig. 2. Probability of detection (P_d) for $P_{fa} = 0.1$, $M = 15$, $U = 0.2$ and optimum weight combining.

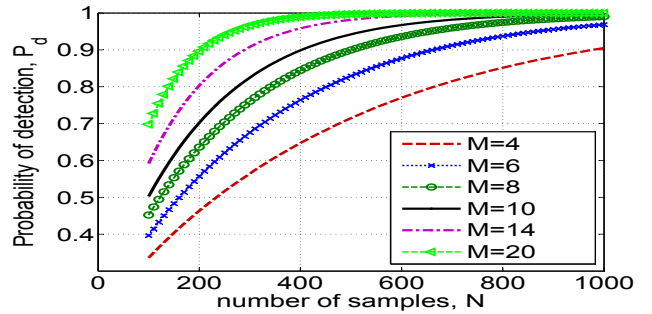


Fig. 3. Probability of detection (P_d) for $P_{fa} = 0.1$, $P_t = 10$, $U = 0.1$ and optimum weight combining.

require very high power to transmit) will be censored (i.e., will not transmit even one bit). In Fig. 2, as expected, increasing either the number of received samples (N) or the maximum power (P_t), improves P_d . In Fig. 3, we illustrate how P_d improves with increasing the number of sensors (M). And in Fig. 4, we re-examine Fig. 3 for $M = 10$

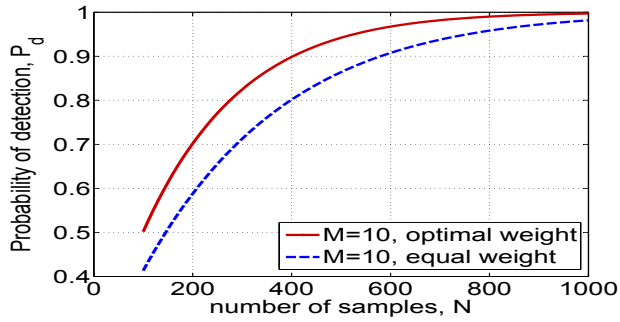


Fig. 4. Probability of detection (P_d) for two different weighting schemes for $P_{fa} = 0.1$, $U = 0.1$ and $P_t = 10$.

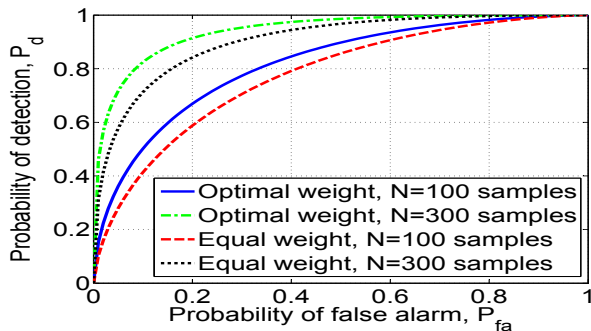


Fig. 5. Receiver operating characteristic with $P_t = 10$, $U = 0.1$ and $M = 10$ for two different weighting schemes.

and compare optimal and non-optimal weighting, showing the advantage of optimal weighting over equal weighting in (7). Finally, Fig. 5 shows the receiver operating characteristic (ROC) parametrized against the number of samples (N) for both optimal and equal weighting.

6. CONCLUSION

We have shown how to perform distributed detection, via sensors transmitting a quantized version of the received energy test statistic to the FC. In addition we have calculated the optimal linear combining coefficients at the FC and the optimal transmit power for each sensor in order to maximize P_d . Although we maximized the modified deflection coefficient (as an approximation to maximizing P_d), the simulations have shown that this approach still allocates sensor transmit powers and quantization bits in a intuitively optimal way. Future work will investigate a general (non-linear) optimal combining strategy at the FC.

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