

# A MULTI-THRESHOLD FEEDBACK SCHEME FOR COGNITIVE RADIO NETWORKS BASED ON OPPORTUNISTIC BEAMFORMING

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## ABSTRACT

Cognitive radio is a promising technique for efficient spectrum utilization in wireless systems. In multi-user Multiple-Input Multiple-Output (MIMO) system, a large amount of feedback information has to be used to achieve multi-user diversity. In this paper, in order to reduce the feedback amount and hence the wasted energy, we propose a novel scheduling scheme of secondary users (SUs) for an underlay cognitive radio network. Our scheme is based on opportunistic beamforming and employs multiple feedback thresholds. The lowest threshold is chosen to insure a predefined allowed scheduling outage probability. A scheduling outage event occurs if at least one transmit beam has no feedback information. The other thresholds are chosen in order to reduce the number of SUs feeding back their maximum signal to interference plus noise ratio (SINR) (and hence the wasted energy) and the delay due to the number of attempts. We show via simulations that a significant gain in terms of energy is obtained at the price of a reasonable delay.

**Index Terms**—Cognitive radio, opportunistic beamforming, secondary users scheduling, multi-threshold feedback scheme

## 1. INTRODUCTION

Cognitive radio (CR) was recently proposed as a promising method to make more efficient use of the limited radio spectrum. In this paper, we consider an underlay cognitive radio network (CRN) [1]. The cognitive base station (CBS), equipped with  $M$  transmitting antennas, schedules a large number  $K$  of secondary users (SUs). Each SU is equipped with a single receiving antenna. The CRN coexists with a primary network consisting of a primary base station (PBS), with a single transmitting antenna and one primary user (PU) with a single receiving antenna. The SUs are permitted to transmit regardless of the on/off status of the PU transmissions provided that their resulted signal power levels at the primary receiver are kept below some predefined interference threshold  $I_{th}$ .

In multiple-antenna multiple-user systems beamforming

has been utilized as a well-known technique for providing high signal to interference plus noise ratio (SINR) to an intended user while minimizing the interference at non-intended users. In cognitive radio networks, assuming different level of channel state information (CSI) available at the secondary transmitter, various beamforming techniques have been developed that find optimal beamformers while maintaining the interference to the primary networks within an acceptable level (see for example [2]- [4]). In practice, obtaining full CSI is difficult (specially when the number of SUs is high and when the frequency division duplex (FDD) mode is used), and often, this may result in high feedback load. To reduce the amount of feedback, opportunistic beamforming [5] can be used. The transmitter generates random beams. Then, each user calculates the SINRs and feeds back its maximum SINR and the index of the corresponding beam. The transmitter schedules transmissions to the users with the highest SINRs. In [6], we proposed a two-steps scheduling scheme based on opportunistic beamforming [5]. We assumed that the CBS has an imperfect estimate of the interference channel (between the CBS and the primary receiver). In order to reduce the interference to the PU, we proposed to generate, in the first step, orthogonal beams to the interference channel estimate. In the second step, the CBS selects for transmission the users with the highest SINRs and assigns to each of these users the beam corresponding to the highest SINR. In order to further reduce the feedback from SUs, we proposed to apply a threshold to the SINR feedback. Notice that other works [7, 8] dealing with multi-user diversity also proposed to apply a threshold on the SINR to reduce the feedback. Indeed, a user does not need to feedback when its maximum SINR value is below the threshold. In [6], we derived an analytical expression of the SINR distribution and gave a close bound of the loss in terms of secondary system throughput due to the thresholding. Notice that with only one threshold, the number of SUs feeding back their maximum SINR can be much greater than the number of beams which leads to a wasting of the energy. To reduce the wasted energy, we propose in this paper a novel scheduling scheme which employs multiple feedback thresholds. The thresholds are denoted as  $S_1 < S_2 < \dots < S_n$ . The lowest threshold  $S_1$ , is chosen

to insure a fixed scheduling outage probability. An outage occurs if at least one transmit beam has no feedback information. The outage probability is calculated using the analytical expression of the SINR distribution given in [6]. To initiate the feedback process, the CBS requests feedback from SUs whose SINR is above  $S_n$ . If the system is in scheduling outage, the threshold is successively lowered to  $S_{n-1}$ ,  $S_{n-2}$  down to  $S_1$ . Consequently, the thresholds are sequentially lowered and the CBS requests again feedback until a successful scheduling (the system is not in outage) is obtained. The thresholds  $S_t$  for  $2 \leq t \leq n$  are chosen according to two criteria:

- reduce the number of secondary users feeding back their maximum SINR, to reduce the wasted energy,
- reduce the average number of attempts that the CBS has to perform. The number of attempts is the number of thresholds considered in the feedback procedure until successful scheduling occurs.

The rest of the paper is organized as follows. In Section 2, the system model and the problem formulation are introduced. In Section 3, we recall the two-steps scheduling method of [6] and the analytical expression of the SINR distribution. In Section 4, we describe the proposed multi-threshold feedback scheme. In Section 5, simulation results are given. Finally, conclusions are drawn in Section 6.

Throughout the paper, the norms of vectors are denoted by  $\|\cdot\|$ . The operators  $(\cdot)^T$ ,  $(\cdot)^H$  and  $E(\cdot)$  stand for the transposition, the transconjugation and the expectation, respectively.

## 2. SYSTEM MODEL

The system model of the cognitive radio system considered in this paper is illustrated in Figure 1. We adopt the system model described in [6], where a cognitive radio network coexists with a primary network. The primary network consists of a PBS with a single transmitting antenna and one PU with a single receiving antenna. The cognitive network comprises  $K$  SUs, with a single receiving antenna each, and a CBS with  $M$  transmitting antennas. Throughout this paper, we assume that  $M \ll K$  and FDD mode for both primary and secondary links. We consider the downlink of the cognitive radio network in which the CBS transmits independent signals to  $N_s$  scheduled secondary users,  $1 \leq N_s \leq M - 1$  (the scheduling will be explained in section 3). We denote by  $\mathbb{S}$  the set of the  $N_s$  selected cognitive users. Since the same carrier frequency is used within the primary and the secondary networks, the received signal at the PU is corrupted by the signals transmitted by the CBS. Similarly, the received signals at the SUs are corrupted by the signal transmitted by the PBS.

During the data transmission period, the received signal at the PU is given by:

$$y_{pu} = \sqrt{P_{pu}}g_{pu}x_{pu} + \sqrt{P_s}\mathbf{h}_{pu} \sum_{i \in \mathbb{S}} \mathbf{w}_i x_i + n_{pu}, \quad (1)$$

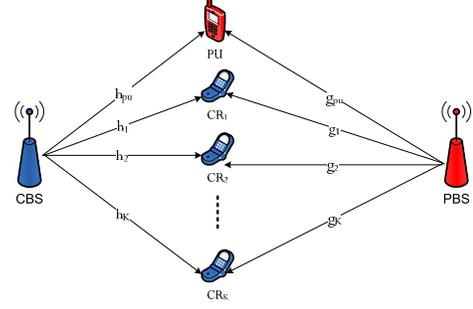


Fig. 1. System model

where  $g_{pu}$  denotes the channel tap gain between the PU and the PBS,  $P_s$  and  $P_{pu}$  denote the transmitted power for each selected cognitive user and for the primary user, respectively. In this work, fixed power allocation for all selected users is adopted. The quantities  $x_{pu}$  and  $x_i$  denote the transmitted data from the PBS to the PU and from the CBS to the  $i$ -th SU, respectively,  $n_{pu}$  denotes the complex Gaussian noise at the PU receiver with zero mean and variance  $\sigma_{pu}^2$ , and  $\mathbf{h}_{pu} = [h_{pu,1}, h_{pu,2}, \dots, h_{pu,M}]$  denotes the channel vector between the CBS and the PU. The weighting vector  $\mathbf{w}_i$  (of size  $M \times 1$ ) denotes the beamforming weight vector for the  $i$ -th selected secondary user. The beamforming weight matrix, of size  $M \times N_s$ , is denoted as  $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{N_s}]$ .

Let  $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \dots, h_{k,M}]$ , where  $h_{k,t}$  is the channel tap gain between the  $t$ -th transmit antenna of the CBS and the  $k$ -th secondary user, for  $1 \leq t \leq M$  and  $1 \leq k \leq K$ . Let  $\mathbf{g} = [g_1, g_2, \dots, g_K]$ , where  $g_k$  denotes the channel tap gain between the transmit antenna at the PBS and the  $k$ -th cognitive user receive antenna. The entries of channel vectors  $\mathbf{h}_k$ ,  $\mathbf{h}_{pu}$  and  $\mathbf{g}$  are independent and identically distributed (i.i.d.) complex Gaussian samples of a random variable with zero mean and unit variance. We assume that the channels are constant during the transmission of a burst of  $T$  symbols and vary independently from burst to burst. The received signal at the  $k$ -th cognitive user, for  $1 \leq k \leq K$ , can be written as:

$$y_k = \sqrt{P_s}\mathbf{h}_k \sum_{i \in \mathbb{S}} \mathbf{w}_i x_i + \sqrt{P_{pu}}g_k x_{pu} + n_k, \quad (2)$$

where  $n_k$  denotes the noise at the  $k$ -th cognitive user which is a zero-mean Gaussian random variable with variance  $\sigma_k^2$ .

In our work, we consider the two steps scheduling method proposed in [6]. We propose to further reduce the feedback amount and hence the wasted energy by using multiple thresholds on the SINR.

In the next section, we recall the scheduling method of [6] and the analytical expression of the SINR distribution.

## 3. TWO STEPS SCHEDULING METHOD

In the first step, the CBS sends pilot symbols to the PU. The latter estimates the interference channel  $\mathbf{h}_{pu}$  and sends

back the channel estimate  $\hat{\mathbf{h}}_{pu}$  to the CBS. Then, based on the received channel estimate, the CBS generates appropriate beamforming weights in order to minimize interference to the PU.

In the second step, using the generated beamforming matrix  $\mathbf{W}$ , the cognitive base station selects a set  $\mathbb{S}$  of  $N_s$  secondary users by applying the opportunistic beamforming approach proposed in [5]. Thus, the cognitive base station transmits the beams of matrix  $\mathbf{W}$  to all SUs. Then, by using (2), each SU  $k$  calculates the following  $N_s$  SINRs by assuming that  $x_j$ ,  $1 \leq j \leq N_s$ , is the desired signal and the other  $x_i$ ,  $i \neq j$ ,  $1 \leq i \leq N_s$ , are interfering signals as:

$$SINR_{k,j} = \frac{|\mathbf{h}_k \mathbf{w}_j|^2 P_s}{\sum_{i=1, i \neq j}^{N_s} |\mathbf{h}_k \mathbf{w}_i|^2 P_s + |g_k|^2 P_{pu} + \sigma_k^2}. \quad (3)$$

We assume that each secondary receiver  $k$  knows  $\mathbf{h}_k \mathbf{w}_i$ , for  $1 \leq i \leq N_s$  (this can be readily arranged by training). Therefore, the  $k$ -th receiver can perfectly compute the SINRs according to (3). Classically in opportunistic beamforming [5], each user feeds back its maximum SINR and the index of the corresponding beam to the CBS. The CBS schedules transmissions to the  $N_s$  users with the highest SINRs and assigns to each of these users the beam corresponding to the highest SINR.

Notice that in [6], we derived the expression of the probability density function (pdf),  $f_X(x)$ , and the cumulative density function (cdf),  $F_X(x)$ , of  $SINR_{k,j}$ , as:

$$f_X(x) = \frac{a \exp(-x/\rho)}{(\alpha x + 1)^2} \left( \frac{\alpha x + 1}{\rho} + \alpha \right) + \sum_{l=1}^{N_s-1} \frac{b_l \exp(-x/\rho)}{(x+1)^{l+1}} \left( \frac{x+1}{\rho} + l \right) \quad (4)$$

$$F_X(x) = 1 - \left( \frac{a \exp(-x/\rho)}{(\alpha x + 1)} + \sum_{l=1}^{N_s-1} \frac{b_l \exp(-x/\rho)}{(x+1)^l} \right) \quad (5)$$

where:

- $\alpha = \frac{P_{pu}}{P_s}$ ,
- $\rho = \frac{P_s}{\sigma_k^2}$ ,
- $a, b_l$  (for  $1 \leq l \leq N_s - 1$ ) are constants.

In this paper, we propose a novel scheduling scheme which employs multiple feedback thresholds in order to further reduce the amount of feedback.

The proposed multi-threshold algorithm is explained in the following section.

#### 4. MULTI-THRESHOLD ALGORITHM

In this section, we describe the proposed multi-threshold feedback scheme. We first derive the scheduling outage probability. Then, we explain how to choose the multiple thresholds. The thresholds are denoted as  $S_1 < S_2 < \dots < S_n$

as shown in Figure 2. To initiate the feedback process, the CBS requests feedback from SUs whose SINR is above  $S_n$ . If the system is in scheduling outage, the threshold is successively lowered to  $S_{n-1}$ ,  $S_{n-2}$  down to  $S_1$ . Consequently, the thresholds are sequentially lowered and the CBS requests again feedback until a successful scheduling is obtained (the system is not in outage).



Fig. 2. Multi-threshold model

#### 4.1. Scheduling Outage Probability

For a predetermined threshold  $S_t$ , a scheduling outage event occurs if at least one transmit beam has no feedback information (all users' SINRs for a given beam are below  $S_t$ ). Let  $P_{out}(S_t)$  denotes the probability of scheduling outage for the threshold  $S_t$ . Using (5), the outage probability can be written as follows:

$$P_{out}(S_t) = 1 - \left( 1 - (F_X(S_t))^K \right)^{N_s} \quad (6)$$

In the following, we propose to validate (6) by simulations.

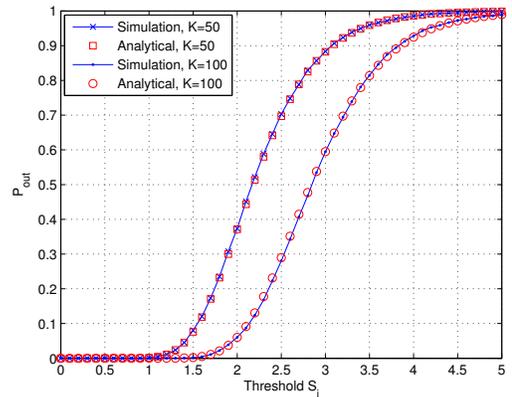


Fig. 3. Probability of scheduling outage versus the threshold  $S_t$  for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33dBm$  and different values of  $K$

Figure 3 shows the probability of scheduling outage as a function of the threshold  $S_t$  for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33dBm$  and  $K \in \{50, 100\}$ . In figure 3, the analytical results in (6) are compared to the simulation results. The figure shows that the analytical curves are inline with the curves obtained by simulations. In addition, it can be seen from the figure that when  $S_t$  is high and/or  $K$  is low, the scheduling outage probability increases.

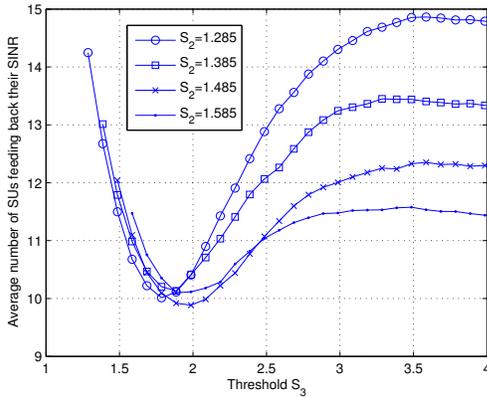
## 4.2. Multiple Thresholds Choice

The lowest threshold  $S_1$  is chosen to insure a predefined allowed scheduling outage probability  $P_{out}(S_1)$  and is determined from (6). The other thresholds must be chosen in order to reduce the number of SUs feeding back their maximum SINR (and hence the wasted energy) and the delay due to the number of attempts. Notice that it is difficult to calculate analytically the number of SUs feeding back their maximum SINR since it requires to derive the expression of the cdf  $F_X(x)$  conditionally to the fact that there is an outage or not for a certain threshold which depends on the channels quality.

In this paper, we propose to choose the thresholds  $S_t$  for  $2 \leq t \leq n$  via simulations. For a given number  $n$  of thresholds, assuming that the system allows a certain quantity of wasted energy, we determine via simulations the thresholds leading to the minimum number of attempts (minimum delays).

## 5. SIMULATION RESULTS

In this section, we present simulation results showing how to design the multi-threshold feedback scheme. We assume that the variances  $\sigma_{pu}^2$  and  $\sigma_k^2$  (for  $1 \leq k \leq K$ ) are equal to  $\sigma^2 = 0 \text{ dBm}$ .

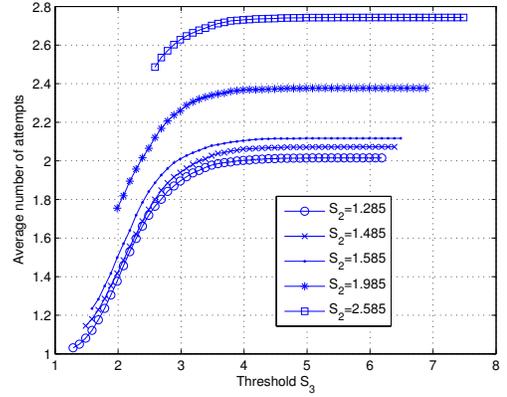


**Fig. 4.** Average number of SUs feeding back their SINR versus the threshold  $S_3$  for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33 \text{ dBm}$ ,  $K = 50$ ,  $P_{out}(S_1) = 1\%$  and different values of  $S_2$

In figures 4,5 and 6, we consider the case where three thresholds are used ( $n = 3$ ),  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33 \text{ dBm}$  and  $K = 50$ . The lowest threshold  $S_1$  is chosen equal to 1.185 in order to insure an outage probability  $P_{out}(S_1) = 1\%$ .

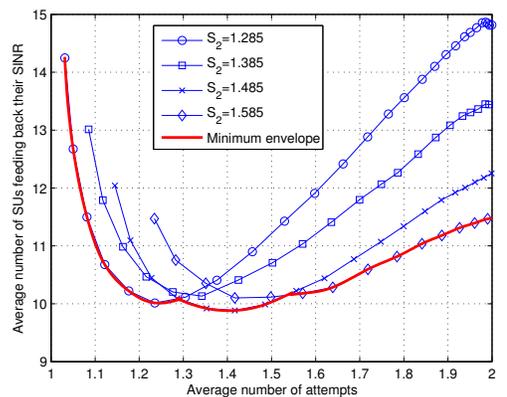
Figure 4 shows the average number of SUs feeding back their SINR with respect to the third threshold  $S_3$ . The threshold  $S_2$  is in the set  $\{1.285, 1.385, 1.485, 1.585\}$ . We notice that the number of SUs feeding back their SINR has a minimum value reached for  $S_3 = S_3^*$  (for example  $S_3^* \simeq 1.98$  when  $S_2 = 1.485$ ). In addition, it can be seen from the figure

that for a given number of SUs feeding back their maximum SINR, there may be many corresponding values of the threshold  $S_3$ . Indeed, when  $S_3$  increases from  $S_2$  to  $S_3^*$ , the number of SUs feeding back their maximum SINR decreases, because the number of users having their SINR below  $S_3$  decreases. However, when  $S_3$  increases beyond  $S_3^*$ , the probability of scheduling outage increases. Therefore, the threshold is lowered to  $S_2$  down to  $S_1$ . In this case, the number of SUs feeding back their SINR increases.



**Fig. 5.** Average number of attempts versus the threshold  $S_3$  for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33 \text{ dBm}$ ,  $K = 50$ ,  $P_{out}(S_1) = 1\%$  and different values of  $S_2$

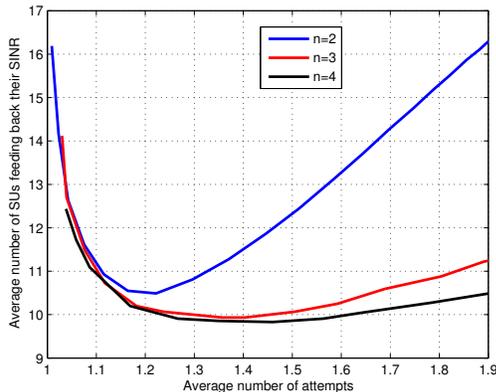
Figure 5 shows the average number of attempts with respect to the third threshold  $S_3$ . The threshold  $S_2$  is in the set  $\{1.285, 1.485, 1.585, 1.985, 2.585\}$ . It can be seen from the figure that the average number of attempts increases with the thresholds ( $S_2$  and  $S_3$ ) as expected, since the probability of outage  $P_{out}$  increases as explained in section 4.1.



**Fig. 6.** Average number of SUs feeding back their SINR  $K_F$  versus the average number of attempts for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33 \text{ dBm}$ ,  $K = 50$ ,  $P_{out}(S_1) = 1\%$  and different values of  $S_2$

Figure 6 shows the average number of SUs feeding back

their SINR versus the average number of attempts. The threshold  $S_2$  is in the set  $\{1.285, 1.385, 1.485, 1.585\}$ . We notice that for a given number of SUs  $K_F$ , there may be many corresponding values of the average number of attempts. For example, if the system tolerates a wasted energy equal to 10 users feeding back their SINR in average, the value of  $S_2 = 1.285$  and  $S_3 = 1.786$ , leading to 1.2 attempts in average, must be used. Thus, the value of  $S_2$  and  $S_3$  must correspond to the minimum envelope of the curves shown in red in figure 6. For a given value of the average wasted energy, the value of the thresholds must correspond to the minimum average of attempts.



**Fig. 7.** Minimum average number of SUs feeding back their SINR during the feedback process (minimum envelope) versus the average number of attempts for  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33dBm$ ,  $K = 50$ ,  $P_{out}(S_1) = 1\%$  and different values of  $n$

Figure 7 shows the minimum average number of SUs feeding back their SINR during the feedback process (minimum envelope) versus the average number of attempts when two, three and four thresholds are used ( $n = 2$ ,  $n = 3$  and  $n = 4$ ),  $M = 4$ ,  $N_s = 3$ ,  $P_s = 3.33dBm$  and  $K = 50$ . The lowest threshold  $S_1$  is chosen equal to 1.185 in order to insure an outage probability  $P_{out}(S_1) = 1\%$ .

Notice that when only one threshold is used ( $S_1 = 1.185$ ) the average number of SUs is about 16, we deduce that there is a gain in terms of wasted energy when the number of thresholds  $n$  increases. Most of the gain is obtained when  $n$  increases from one to two thresholds.

## 6. CONCLUSIONS

In this paper, we considered a cognitive radio network with a single CBS, equipped with multiple transmitting antennas in order to schedule a large number of secondary users. The cognitive network coexists with a primary network. We assumed that the CBS does not have the full channel state information from SUs while it has an imperfect CSI from the PU.

We considered a two steps scheduling method based on opportunistic beamforming. We proposed to further reduce the feedback amount and hence the wasted energy by using multiple thresholds on the SINR. The lowest threshold is chosen to insure a predefined allowed scheduling outage probability. The other thresholds are chosen in order to reduce the number of SUs feeding back their maximum SINR (and hence the wasted energy) and the delay due to the number of attempts. Simulations showed that a significant gain in terms of wasted energy is obtained when the number of thresholds increases.

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