Sparse Channel Estimation Based on a Reweighted Least-Mean Mixed-Norm Adaptive Filter Algorithm

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***Abstract*—A sparsity-aware least-mean mixed-norm (LMMN) adaptive filter algorithm is proposed for sparse channel estimation applications. The proposed algorithm is realized by incorporating a sum-log function constraint into the cost function of a LMMN which is a mixed norm controlled by a scalar-mixing parameter. As a result, a shrinkage is given to enhance the performance of the LMMN algorithm when the majority of the channel taps are zeros or near-zeros. The channel estimation behaviors of the proposed reweighted sparse LMMN algorithm is investigated and discussed in comparison with those of the standard LMS and the least-mean square/fourth (LMS/F) and previously sparse LMS/F algorithms. The simulation results show that the proposed reweighted sparse LMMN algorithm is superior to aforementioned algorithms with respect to the convergence speed and steady-state error floor.**

Keywords:least-mean mixed-norm, LMS, LMS/F, sparse channel estimation, sparse adaptive filtering

I. Introduction

With the increasing of the wireless communication technologies, broadband transmission has been becoming an important method to obtain high date rate and wide bandwidth for modern wireless communication systems, such as mobile communications [1], [2], [3]. Coherent detection at the receiver side needs accurate state information of the channel, which is usually implemented by using adaptive filter algorithms [4], such as least-mean square (LMS) [5], affine projection algorithm [6], least-mean sauqre/fourth (LMS/F) [7], [8], [9] and least-mean mixed-norm (LMMN) algorithms [10]. Furthermore, the measured broadband multi-path wireless channel is always sparse, which means that most of the channel taps are zeros or near-zeros, while only a few channel taps are dominant taps whose magnitudes are non-zeros [11]. Unluckily, these conventional adaptive filter algorithms cannot utilize the inherent sparsity properties of the sparse broadband multi-path channel.

For these reasons, zero-attracting LMS (ZA-LMS) algorithms have been proposed by introducing a l_1-norm penalty into the cost function of the conventional LMS and variable step-size LMS algorithms to speed up the convergence and to reduce the channel estimation bias [12], [13]. After that, the zero-attracting technique has been used to develop sparse LMS algorithm by using l_p-norm for channel estimation [14]. However, the LMS algorithms are sensitive to the scaling of the input signal and the noise in the low signal-to-noise environment and colored input signals. To improve the performance of the LMS-based channel estimation algorithms, zero-technologies are introduced into the affine projection algorithms [15], [16], [17], [18], [19] and Set-membership normalized least-mean-square (SN-NLMS) algorithm [20]. However, the affine projection algorithms have high computational complexity. By compromising the complexity and estimation performance, a LMS/F algorithm has been presented by the combination of the LMS and least-mean fourth (LMF) algorithms and its sparse form has been proposed and investigated over a sparse wireless channel in low signal-to-noise environment [21], [22], [23]. However, the performance are affected by the LMF because the convergence of the LMF algorithms are sensitive to the proximity of the adaptive weights to the optimal Wiener solution. What's more, a LMS/F algorithm and its sparse forms (zero-attracting LMS/F (ZA-LMS/F) and reweighted ZA-LMS/F (RZA-LMS/F)) have been proposed to improve both the LMS and LMF algorithm for channel estimation applications [7], [23]. Unfortunately, the computational complexity is increased by using a logarithm in its cost function.

Recently, a LMMN algorithm has been reported to overcome the sensitivity and to improve the channel estimation bias behavior [10]. Although the LMMN algorithm can improve the estimation performance, it cannot utilize the sparse structure of the pre-known channel state information. In this paper, a sparsity-aware LMMN algorithm is proposed by using a linear combination of the l_2 and l_4 norms of the estimation error and a sum-log constraint on the estimation channel vector to exploit the sparsity properties of the channel and to improve the estimation behaviors of the LMMN algorithm, which is denoted as reweighted sparse LMMN (RS-LMMN) algorithm. The simulation results obtained from a sparse channel are given to verify that the proposed RS-LMMN algorithm is superior to those of the conventional LMS, LMS/F, LMMN and previously proposed sparse LMS/F algorithms.

The paper is constructed as follows. Section 2 reviews the traditional LMMN algorithm. In Section 3, we present the proposed sparse-aware LMMN algorithm. Section 4 gives the performance of the proposed sparse LMMN algorithm. Finally, this paper is concluded in Section 5.

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II. Conventional LMMN algorithm

The LMMN algorithm is used to estimate a sparse multi-path fading channel. We assume the unknown sparse finite impulse response (FIR) channel vector is $\mathbf{h} = [h_0, h_1, \cdots, h_{N-1}]^T$ whose length is N. The LMMN-based channel estimation is to use the input signal $\mathbf{x}(n)$, the output of the FIR channel $y(n)$ and the instantaneous error $e(n)$ to estimate the sparse channel $\mathbf{h}$, and hence the desired signal at the receiver side is given by [10]

$$d(n) = \mathbf{h}^T\mathbf{x}(n) + v(n), \tag{1}$$

where $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-N-1)]^T$ and $v(n)$ is the additive white Gaussian noise (AWGN) which is independent with input signal $\mathbf{x}(n)$. Then, the instantaneous error $e(n)$ is defined as

$$e(n) = d(n) - \hat{\mathbf{h}}^T(n)\mathbf{x}(n), \tag{2}$$

where $\hat{\mathbf{h}}(n)$ is the estimated channel vector at the iteration n. The cost function that is minimized in the LMMN-based channel estimation is a linear combination of $J_2(n) \triangleq E\{e^2(n)\}$ and $J_4(n) \triangleq {}^1\!/_4 E\{e^4(n)\}$, which is given by

$$J(n) = \frac{\delta}{2}J_2(n) + \frac{1-\delta}{4}J_4(n), \tag{3}$$

where $0 \le \delta \le 1$ controls the mixture. The gradient minimization technique is used to find a solution of the optimization problem and the update equation of the LMMN algorithm is written as

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu_{\mathrm{LMMN}} e(n)\{\delta + (1-\delta)e^2(n)\}\mathbf{x}(n), \tag{4}$$

where μ_{LMMN} is the step-size. It is worth noting that the LMMN algorithm is a LMS algorithm when $\delta = 1$, while the LMMN algorithm is a LMF algorithm when δ is set to 0. Thus, the advantages of both the LMS and LMF algorithms can be utilized with intermediate values of δ.

III. Proposed reweighted sparse LMMN algorithm

On the basis of the LMMN algorithm and previously zero-attracting LMS and LMS/F algorithms [12], [13], [14], [21], [23], [24], we propose a RS-LMMN algorithm. The proposed algorithm is realized by incorporating a sum-log function constraint of the channel coefficient vector into the cost function of the LMMN algorithm, which is based on the compressed sensing (CS), sparse enhancement and zero-attracting concepts [12], [25], [26]. Thereby, the new cost function is defined as

$$J_{\mathrm{p}}(n) = \frac{\delta}{2}J_2(n) + \frac{1-\delta}{4}J_4(n) + \gamma' \sum_{i=1}^{N} \log\left(1 + \frac{\left|\hat{h}_i(n)\right|}{\varepsilon'}\right), \tag{5}$$

where $\gamma' > 0$ and $\varepsilon' > 0$. The gradient vector which defines the search direction is

$$\begin{aligned}\nabla J_{\mathrm{p}}(n) \triangleq \frac{\partial J_p(n)}{\partial \hat{\mathbf{h}}(n)} &= -\mathrm{E}\{e(n)\{\delta + (1-\delta)e^2(n)\}\mathbf{x}(n)\} \\ &\quad + \gamma' \frac{\partial \sum_{i=1}^{N}\left(1 + \frac{\left|\hat{h}_i(n)\right|}{\varepsilon'}\right)}{\partial \hat{\mathbf{h}}(n)}\end{aligned}. \tag{6}$$

A stochastic gradient algorithm is defined based on an instantaneous estimation of $\nabla J_{\mathrm{p}}(n)$. In terms of channel estimation vector $\hat{\mathbf{h}}(n)$ and equation (6), the update equation of the proposed RS-LMMN algorithm is

$$\begin{aligned}\hat{h}_i(n+1) = \hat{h}_i(n) &+ \mu_{\mathrm{RS}} e(n)\{\delta + (1-\delta)e^2(n)\}x_i(n) \\ &- \rho\frac{\mathrm{sgn}[\hat{h}_i(n)]}{1 + \varepsilon\left|\hat{h}_i(n)\right|}\end{aligned}. \tag{7}$$

or equivalently, in vector form

$$\begin{aligned}\hat{\mathbf{h}}(n+1) = &\underbrace{\hat{\mathbf{h}}(n) + \mu_{\mathrm{RS}} e(n)\{\delta + (1-\delta)e^2(n)\}\mathbf{x}(n)}_{\text{LMMN algorithm}} \\ &\underbrace{-\rho\frac{\mathrm{sgn}[\hat{\mathbf{h}}(n)]}{\mathbf{1} + \varepsilon\left|\hat{\mathbf{h}}(n)\right|}}_{\text{Sparse penalty}}\end{aligned}. \tag{8}$$

where $\rho = \mu_{\mathrm{RS}}\gamma'\varepsilon$ is a regularization parameter which is used for balancing the estimation error and sparsity strength and $\varepsilon = 1/\varepsilon'$ is to control the reweighting factor. $\mathrm{sgn}(\cdot)$ is a component-wise function which is defined as

$$\mathrm{sgn}(\hat{\mathbf{h}}) = \begin{cases} \dfrac{\hat{h}_i}{\left|\hat{h}_i\right|}, & \hat{h}_i \neq 0 \\ 0, & \hat{h}_i = 0 \end{cases}. \tag{9}$$

Comparing (4) and (8), we note that the proposed RS-LMMN algorithm has an additional sparse penalty term denoted as zero attractor, which always attracts the channel taps to zero. The zero attractor is controlled by the parameter ρ. Therefore, the proposed RS-LMMN algorithm can speed up the convergence when most of the channel taps are zeros or near-zeros, which means the channel is sparse. From the zero attractor of the proposed RS-LMMN algorithm, it is found that there is a strong zero attracting on the channel taps whose magnitudes are compared to ε'. On the contrary, the shrinkage exerted on the channel taps with magnitudes larger than ε' is weak. Thus, the proposed RS-LMMN algorithm can selectively assign different zero attracting to the channel taps. Here, we can see that the sum-log function is close to the l_0-norm, which results in a different weight assignment to the zero-attracting term to obtain a fast convergence.

IV. Results and discussions

In this section, 100 independent Monte Carlo runs are used to obtain a point for all the adaptive filter algorithms. The channel estimation performance is evaluated by using mean square error (MSE) which is defined as

$$\mathrm{MSE}\{\hat{\mathbf{h}}(n)\} = \mathrm{E}\left\{\left\|\mathbf{h} - \hat{\mathbf{h}}(n)\right\|_2^2\right\}, \tag{10}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator, $\mathbf{h}$ is the actual channel and $\hat{\mathbf{h}}(n)$ is the estimated channel, respectively. In this paper, a sparse channel with length $N = 16$ is employed and the non-zero channel taps are set as $K \in \{1, 2, 4\}$, which

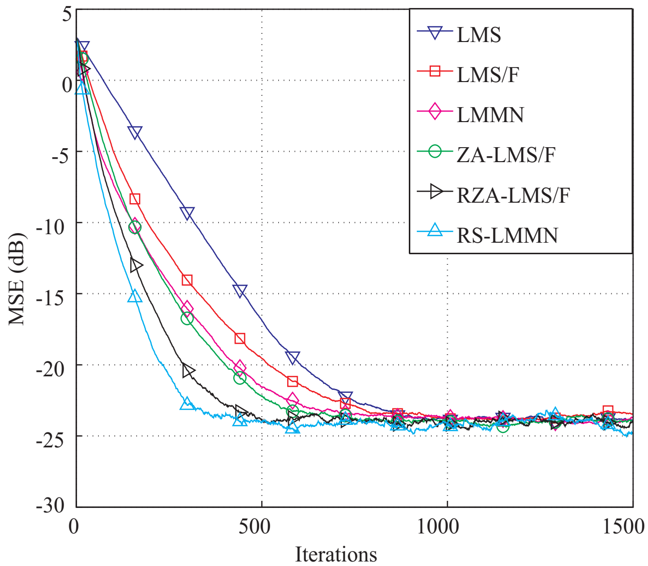

Fig. 1. Convergence of the proposed RS-LMMN algorithm for $K = 1$.

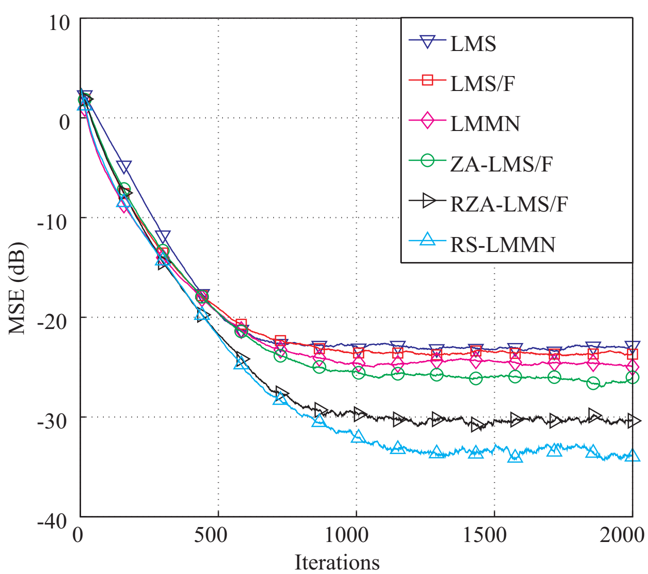

Fig. 2. Steady-state behavior of the proposed RS-LMMN algorithm for $K = 1$.

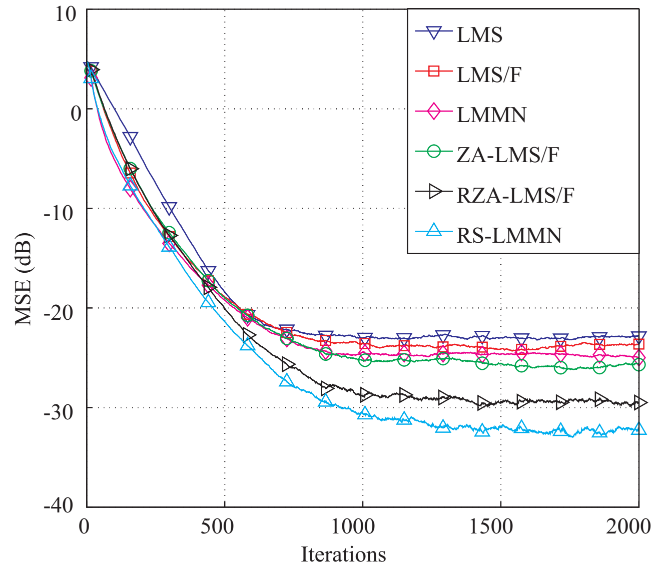

Fig. 3. Steady-state behavior of the proposed RS-LMMN algorithm for $K = 2$.

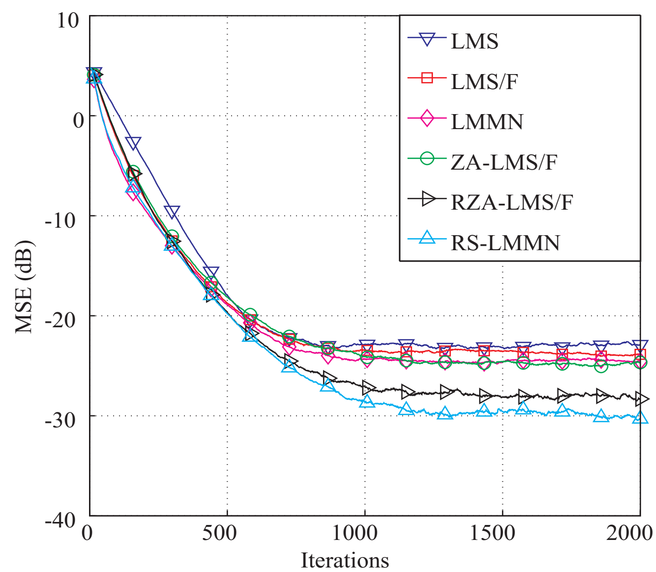

Fig. 4. Steady-state behavior of the proposed RS-LMMN algorithm for $K = 4$

is similar to the previous researches in [12], [13], [14], [21], [23]. The K non-zero taps are Gaussian distribution and their positions are randomly distributed within the length of the channel and $E\left\{\|h\|_2^2\right\} = 1$. The transmitted signal power is 1, while the noise power is 10^{-1}. Thus, the signal-to-noise (SNR) is 10 dB.

The convergence of the proposed RS-LMMN algorithm is investigated and compared with conventional LMS, LMS/F, LMMN, ZA-LMS/F and RZA-LMS/F algorithms. All the simulation parameters are optimized to obtain the same estimation error floor, which are listed as follows: $\mu_{\mathrm{LMS}} = 0.005$, $\mu_{\mathrm{LMS/F}} = 0.015$, $\mu_{\mathrm{LMMN}} = 0.008$, $\mu_{\mathrm{ZA-LMS/F}} = 0.02$, $\mu_{\mathrm{RZA-LMS/F}} = 0.027$, $\mu_{\mathrm{RS}} = 0.015$, $\rho_{\mathrm{ZA-LMS/F}} = 5\times10^{-5}$, $\rho_{\mathrm{RZA-LMS/F}} = 2\times10^{-4}$, $\rho = 3\times10^{-4}$, $\varepsilon = 20$, $\delta = 0.4$, $\lambda = 0.8$, where μ_{LMS}, $\mu_{\mathrm{LMS/F}}$, $\mu_{\mathrm{ZA-LMS/F}}$, $\mu_{\mathrm{RZA-LMS/F}}$ are the step-sizes of the LMS, LMS/F, ZA-LMS/F and RZA-LMS/F algorithms, respectively, while $\rho_{\mathrm{ZA-LMS/F}}$ and $\rho_{\mathrm{RZA-LMS/F}}$ are the regularization parameters of the ZA-LMS/F and RZA-LMS/F algorithms, respectively. λ is the positive constant of LMS/F algorithm which is to balance the convergence and the steady-state performance. We can see from Fig. 1 that our proposed RS-LMMN algorithm has the fastest convergence speed in comparison with the conventional LMS, LMS/F, LMMN, ZA-LMS/F and RZA-LMS/F algorithms. This is because the proposed RS-LMMN algorithm not only use the benefits of the LMS and LMF algorithms but also can provide a zero attractor to attract the zero and near-zero taps to zero quickly. Fig. 2 shows the steady-state performance of the proposed RS-

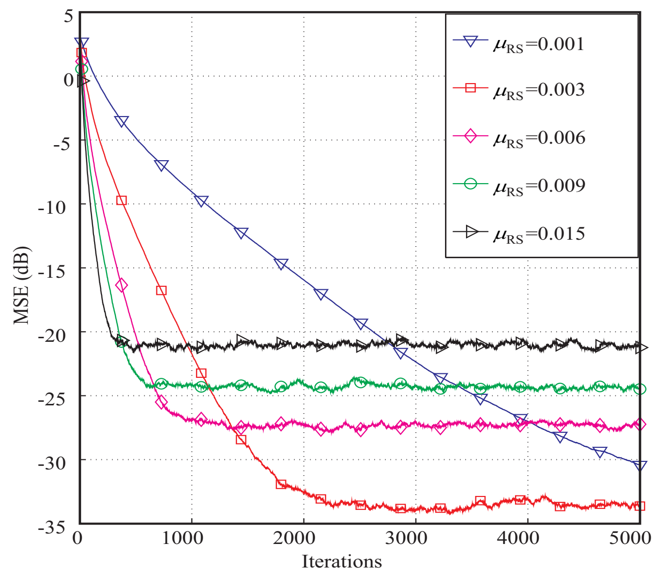

Fig. 5. Effects of different step-sizes $\mu_{\rm RS}$ on the proposed RS-LMMN algorithm.

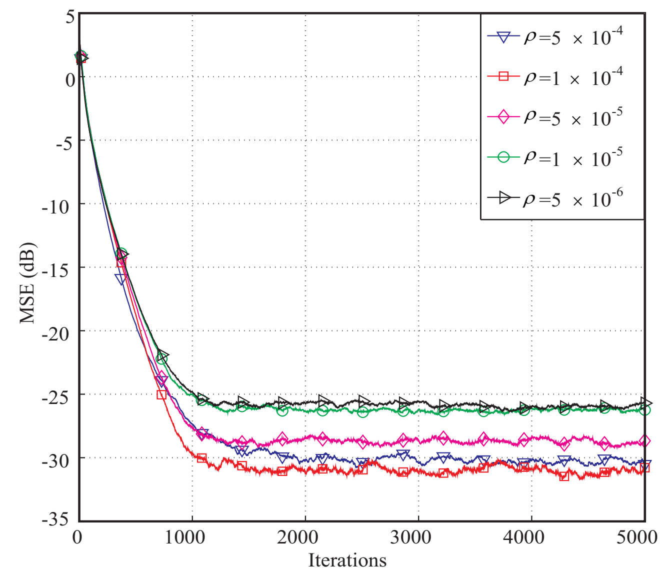

Fig. 6. Effects of different regularization parameter ρ on the proposed RS-LMMN algorithm.

LMMN algorithm compared with conventional LMS, LMS/F, LMMN, ZA-LMS/F and RZA-LMS/F algorithms for $K = 1$ and the simulation parameter are to obtain the same convergence speed, where $\mu_{\rm LMS} = 0.006$, $\mu_{\rm LMS/F} = 0.0125$, $\mu_{\rm LMMN} = 0.0065$, $\mu_{\rm ZA-LMS/F} = \mu_{\rm RZA-LMS/F} = 0.012$, $\mu_{\rm RS} = 0.006$. It is found that the proposed RS-LMMN algorithm has the lowest steady-state error floor with the same convergence speed as that of the other algorithms. This is because the proposed RS-LMMN algorithm is benefited from hybrid power optimization criterion. The steady-state performance of the proposed algorithm for $K = 2$ and $K = 4$ are shown in Figs. 3 and 4, respectively. We note that the steady-state error floor is deteriorated with an increment of the sparse level K. However, our proposed RS-LMMN algorithm still has the lowest steady-state error floor for $K = 2$ and $K = 4$.

Form the sparse channel estimation aforementioned, we can see that our proposed RS-LMMN algorithm always outperforms the LMS, LMS/F and LMMN algorithms and the sparse LMS/F algorithms. However, the parameters of the proposed RS-LMMN algorithm have important effects on the estimation performance. Therefore, $\mu_{\rm RS}$, ρ and δ are investigated in detail. Form Fig. 5, we can see that the step-size $\mu_{\rm RS}$ of the proposed RS-LMMN algorithm gives important effects on both the convergence speed and steady-state error. With an increment of the parameter $\mu_{\rm RS}$, the convergence speed increased while the steady-state error is getting worse. Thus, we can select proper parameter $\mu_{\rm RS}$ to balance the convergence and the steady-state behavior. When $\mu_{\rm RS}$ is small, the convergence speed is too slow. On the contrary, the convergence speed is improved by sacrificing the mean-square-error (MSE) for large $\mu_{\rm RS}$.

Since the proposed RS-LMMN algorithm introduces a zero attractor in its iterations which can be controlled by the regularization parameter ρ, parameter ρ is selected to evaluate

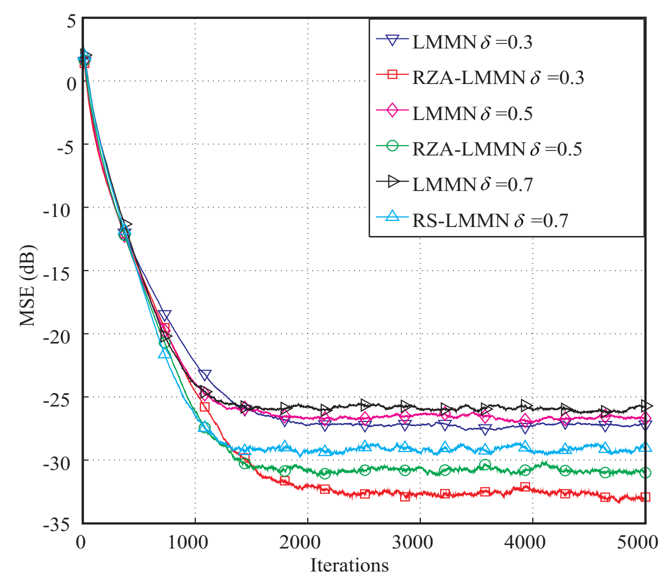

Fig. 7. Steady-state behavior of the proposed RS-LMMN algorithm with different δ.

the effects on both the convergence and steady-state error floor. Its effects are illustrated in Fig. 6. It is found that the parameter ρ mainly affect the steady-state performance of the proposed RS-LMMN algorithm. As ρ increases from $\rho = 5 \times 10^{-6}$ to $\rho = 1 \times 10^{-4}$, the steady-state error floor is reduced, which means that the proposed RS-LMMN algorithm can achieve good estimation performance. Unluckily, the convergence speed becomes slow and the steady-state error floor increases for $\rho = 5 \times 10^{-4}$ because the small ρ exerts a weak zero-attracting on the zero-attractor, while the large ρ gives strong zero-attracting on the channel coefficients.

Moreover, the inherent parameter δ can affect the performance of the LMMN algorithm as well as the proposed RS-LMMN algorithm. Thereby, the effect of the parameter δ was investigated herein and the simulated results are shown in Fig.7. We can see that the proposed RS-LMMN algorithm still converges faster than that of the conventional LMMN algorithm. When δ increases from 0.3 to 0.7, the steady-state error floor is becoming worse. Additionally, the proposed RS-LMMN algorithm is more sensitive to the δ according to the MSE. In a word, the proposed RS-LMMN algorithm outperforms the conventional LMS, LMS/F, LMMN, ZA-LMS/F and RZA-LMS/F algorithms by properly selecting the parameters mentioned above.

V. Conclusion

In this paper, a reweighted sparse LMMN has been proposed for sparse channel estimation applications. The derivation of the proposed RS-LMMN algorithm has been analyzed by exerting a sum-log penalty on the cost function of the conventional LMMN algorithm. The convergence and channel estimation performance of the proposed RS-LMMN algorithm are investigated over a sparse channel. The simulation results obtained from the sparse channel estimation were given to show that the proposed RS-LMMN algorithm has fastest convergence and lowest steady-state error floor and achieves about 6.5 dB gain compared to the conventional LMMN algorithm when the channel is sparse. In the presentation, we will compare the channel estimation in comparison with conventional LMS, LMF, LMS/F and their sparse-aware forms.

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