DESIGN OF DYNAMIC LINEAR-IN-THE-PARAMETERS NONLINEAR FILTERS FOR ACTIVE NOISE CONTROL

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ABSTRACT

Traditional active noise control (ANC) systems, which uses a fixed tap length adaptive filter as the controller may lead to non optimal noise mitigation. In addition, the conventional filtered-x least mean square algorithm based ANC schemes fail to effectively perform noise cancellation in the presence of nonlinearities in the ANC environment. In order to overcome these limitations of traditional ANC techniques, in this paper, we propose a class of dynamic nonlinear ANC systems, which adapts itself to the noise cancellation scenario. The dynamic behaviour has been achieved by developing a variable tap length and variable learning rate adaptive algorithms for functional link artificial neural network (FLANN) and generalized FLANN (GFLANN) based ANC systems. The proposed ANC schemes have been shown through a simulation study to provide an optimal convergence behaviour. This improvement has been achieved by providing a balance between the number of filter coefficients and the mean square error.

***Index Terms*—** Active noise control, functional link artificial neural network, GFLANN.

1. INTRODUCTION

Active noise control (ANC), which is based on the destructive superposition principle, has gained considerable attention in the recent past in comparison with traditional passive noise mitigation techniques, when applied for low frequency noise cancellation. The basic single channel feed-forward ANC system consists of a reference microphone which senses the noise to be cancelled, a loudspeaker which generates the anti-noise and an error microphone which measures the level of noise cancellation achieved. The anti-noise generated by the loudspeaker is governed using an adaptive controller, which is conventionally an adaptive filter trained using a suitable learning algorithm. The filtered-x least mean square (FxLMS) algorithm is the most common adaptive algorithm employed in an ANC system. FxLMS algorithm is used when the controller is a finite impulse response (FIR) filter [1].

This work was supported by the Department of Science and Technology, Government of India under the Fast Track Scheme for Young Scientists (SERB/ET−0018/2013).

The conventional FxLMS algorithm based ANC systems have been designed to mitigate noise assuming a linear ANC scenario. In a practical ANC system, nonlinearities exist at different stages including the secondary and primary paths. FxLMS algorithm based ANC systems fail to effectively tackle noise in those scenarios and a few nonlinear ANC systems have been designed in the recent past to overcome this limitation. Das and Panda [2] has reported a functional link artificial neural network (FLANN) based ANC system, which uses a filtered-s least mean square (FsLMS) algorithm as the learning rule. Recently, a generalized FLANN (GFLANN) filter has been employed for nonlinear ANC [3]. Variants of these nonlinear ANC schemes, including convex combination of FLANN and Volterra filters have been attempted lately to improve the noise mitigation capability of the ANC system [4].

In all the ANC scenarios discussed so far, the length of the adaptive controller needs to be fixed *aprori* and the controller length is a critical parameter that can determine the effectiveness of the ANC system. Selecting a controller length which is smaller than optimal will result in an underestimation scenario and will lead to ineffective noise cancellation. On the other hand, selecting a longer controller length will lead to an ANC situation which has slow convergence and can significantly increase the computational load. In an endeavour to avoid the above discussed limitations of ANC systems, this paper proposes a class of variable tap/coefficient length adaptive algorithms for nonlinear ANC. The proposed ANC schemes are designed to adapt the tap length of linear-in-the parameters nonlinear filters according to the ANC scenario to which the controller is applied and thus can provide optimal noise cancellation.

A few works have been reported in the literature on variable tap length schemes for adaptive filters. A segmented filter (SF) approach has been presented in [5], where in the filter length is increased or decreased by one segment based on an error norm before and after the tap length change. A gradient descent (GD) variable tap length scheme, in which the tap length is updated in a smooth manner has been reported in [6]. One of the most prominent disadvantage of the GD approach is that the filter length may be considered to be more than the optimal tap length [7]. By introducing the concept of pseudo

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fractional tap length, this limitation of the GD approach has been solved in [7], resulting in a fractional tap (FT) algorithm. The FT algorithm is computationally efficient and has been shown to provide improved convergence in comparison with other popular variable tap length approaches.

A variable tap-length and variable step-size algorithm for an FxLMS based ANC system has been recently reported [8]. As discussed previously, FxLMS algorithm is not effective under nonlinear ANC situations, which are frequently encountered in practical implementations. In contrast to a variable tap length algorithm in a linear ANC system, a change in tap length will affect the length of the controller in a different fashion in nonlinear ANC schemes, depending upon the nature of the nonlinear controller employed. In order to achieve the dynamic behaviour in a nonlinear ANC system, this paper proposes a class of nonlinear ANC algorithms, which can adapt the structure of the controller as well as its weights in accordance with the noise control situation. This dynamic nature of the proposed schemes have been further enhanced by introducing the concept of variable learning rates in nonlinear ANC systems. In this paper, we have considered FLANN and GFLANN as the candidates for the nonlinear controllers and the corresponding variable tap/coefficient length, variable learning rate update rules have been developed.

The rest of the paper is organized as follows. A set of variable coefficient length nonlinear ANC schemes, which are based on popular nonlinear controllers like FLANN and GFLANN are designed in Section 2. An extensive simulation study has been carried out to understand the behaviour of the new techniques in Section 3 and the concluding remarks are made in Section 4.

2. DESIGN OF DYNAMIC NONLINEAR ANC SYSTEMS

A class of dynamic nonlinear ANC systems, which are based on popular nonlinear controller structures employed in ANC scenarios are designed in this section. Most of the nonlinear active noise controllers consist of a linear as well as a nonlinear sub-block, the weights of which are updated using a suitable gradient descent algorithm. In addition to the weights, the number of coefficients as well as the learning rates are considered as variables in this study. The two controllers considered in this work are the ones based on FLANN [9] and GFLANN.

2.1. FLANN based ANC

In the proposed FLANN based variable tap-length, variable learning rate ANC scheme, the tap delayed input signal vector $\boldsymbol{u}(n) = [x(n), x(n-1), \cdots, x(n-L(n)+1)]^T$ of variable length $L(n)$ is functionally expanded to $\Lambda(n) = L(n)(2P+1)$ terms, where P is the order of the functional expansion and $\Lambda(n)$ is the steady state coefficient count after functional expansion. Even though several functional expansion mechanisms have been reported in the literature [2, 3], a trigonometric expansion is considered in this paper. The functionally expanded reference signal vector is given by

$$\begin{aligned}\boldsymbol{X}_{\Lambda(n)}(n) = \{&x(n), x(n-1), \cdots, x(n-L(n)+1),\\ &\sin[\pi x(n)], \sin[\pi x(n-1)], \cdots, \sin[\pi x(n-L(n)+1)],\\ &\cdots, \sin[P\pi x(n)], \cdots, \sin[P\pi x(n-L(n)+1)], \cos[\pi x(n)],\\ &\cos[\pi x(n-1)], \cdots, \cos[\pi x(n-L(n)+1)], \cos[P\pi x(n)],\\ &\cos[P\pi x(n-1)], \cdots, \cos[P\pi x(n-L(n)+1)]\}^T \end{aligned} \quad (1)$$

The segmented steady state residual noise is given by

$$e^{\Lambda(n)}_{\Lambda(n)}(n) = d(n) - S_N(n) * \left(\boldsymbol{X}^{\mathrm{T}}_{\Lambda(n)} \boldsymbol{W}_{\Lambda(n)}\right) \quad (2)$$

where $d(n)$ is the primary noise signal observed at the cancellation point, $S_N(n)$ is the impulse response of the secondary path, $*$ represents the linear convolution operation and $\boldsymbol{W}_{\Lambda(n)} = [\boldsymbol{w}^T_{L,L(n)}(n), \boldsymbol{w}^T_{s,L(n)}(n), \boldsymbol{w}^T_{c,L(n)}(n)]^T$ represents the weight vector with $\boldsymbol{w}_{L,L(n)}(n)$, $\boldsymbol{w}_{s,L(n)}(n)$ and $\boldsymbol{w}_{c,L(n)}(n)$ denoting the weights corresponding to the linear, sine terms and the cosine terms respectively. It is convenient to represent the weight vector $\boldsymbol{W}_{\Lambda(n)}$ as a vector of weights corresponding to the linear $(\boldsymbol{W}_{L,L(n)}(n))$ and nonlinear $(\boldsymbol{W}_{N,L(n)}(n))$ parts. i.e. $\boldsymbol{W}_{\Lambda(n)} = [\boldsymbol{W}^T_{L,L(n)}(n), \boldsymbol{W}^T_{N,L(n)}(n)]^T$. Weights of the controller are updated using

$$\boldsymbol{W}_{\Lambda(n)}(n) = \boldsymbol{W}_{\Lambda(n)}(n-1) + \boldsymbol{\mu}(n)\boldsymbol{X}'_{\Lambda(n)}(n)e^{\Lambda(n)}_{\Lambda(n)}(n) \quad (3)$$

which is here after called the variable coefficient variable learning rate FsLMS (VCLFsLMS) algorithm. In (3), $\boldsymbol{X}'_{\Lambda(n)}(n)$ is $\boldsymbol{X}_{\Lambda(n)}(n)$ filtered through a model of the secondary path and the learning rate $\boldsymbol{\mu(n)}$ is a diagonal matrix given by

$$\boldsymbol{\mu}(n) = \begin{bmatrix}\mu_L(n) & 0\\ 0 & \mu_N(n)\end{bmatrix}_{\Lambda(n)\times\Lambda(n)}. \quad (4)$$

with $\mu_L(n)$ and $\mu_N(n)$ denoting the learning rates corresponding to the linear and nonlinear blocks. As the tap length $L(n)$ as well as the number of coefficients $\Lambda(n)$ are restricted to be natural numbers, the update of both the terms cannot be achieved using a gradient descent approach. In order to achieve this task, we have used a pseudo fractional tap-length approach [7]. The fractional tap length $l_f(n)$ is updated as

$$l_f(n+1) = [l_f(n) - \alpha] - \beta\{[e^{\Lambda(n)}_{\Lambda(n)}(n)]^2 - [e^{\Lambda(n)}_{\Xi(n)}(n)]^2\} \quad (5)$$

with $\Xi(n)$ as the length of the expanded vector for a tap length of $L(n) - \Delta$, where $1 \le \Delta \le L(n)$ is a positive integer. In (5), α is a positive leakage factor, β is the step-size with $0 < \alpha \ll \beta$ as the condition for ensuring stability of the update algorithm [6–8]. The update rule for the tap length $L(n)$ may be written as

$$L(n+1) = \begin{cases}\lfloor l_f(n) \rfloor, & \text{if } |L(n) - l_f(n)| \ge \delta\\ L(n), & \text{otherwise}\end{cases} \quad (6)$$

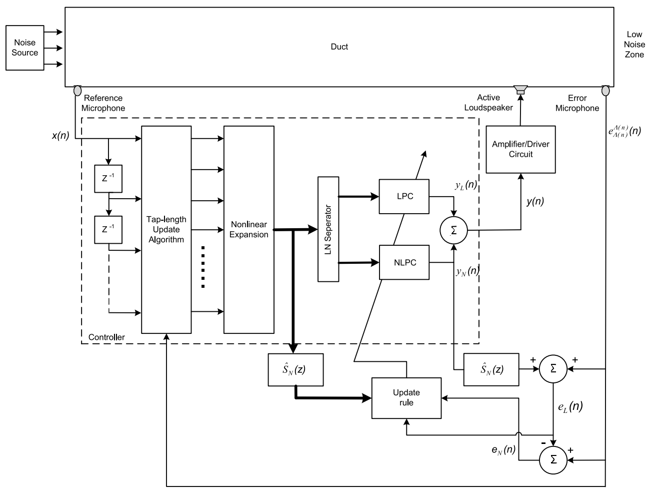

Fig. 1. Schematic diagram of the proposed variable coefficient length variable learning rate nonlinear ANC system.

where $\lfloor \cdot \rfloor$ is the floor operator and δ is a small integer. Fig. 1 shows the schematic diagram of the proposed ANC scheme. It may be noted that the change in coefficient count is achieved by padding or truncating R taps to the linear, sine and cosine blocks, where $R = |L(n+1) - L(n)|$.

2.2. GFLANN based ANC

One of the recently proposed nonlinear controller for ANC is a generalized FLANN (GFLANN), which contains cross terms in addition to the functionally expanded terms of a trigonometric FLANN [3]. In the proposed variable tap length variable learning rate GFLANN based ANC scheme, the tap delayed input signal vector $\boldsymbol{u}(n)$ of length $L(n)$ is expanded to

$$\Lambda(n) = L(n)(2P+1) + 2P \sum_{k=0}^{N_d - 1} [L(n) - 1 + k] \quad (7)$$

terms, with P as the order of the functional expansion and N_d as the cross term parameter as defined in [3]. Considering a first order expansion, the expanded reference signal vector in a GFLANN may be written as

$$\begin{aligned} \boldsymbol{X}_{\Lambda(n)}(n) = \{&x(n), x(n-1), \cdots, x(n-L(n)+1), \\ &\sin[\pi x(n)], \sin[\pi x(n-1)], \cdots, \sin[\pi x(n-L(n)+1)], \\ &\cos[\pi x(n)], \cos[\pi x(n-1)], \cdots, \cos[\pi x(n-L(n)+1)], \\ &x(n-1)\sin[\pi x(n)], \cdots, x(n-L(n)+1)\sin[\pi x(n-L(n)], \\ &x(n-N_d)\sin[\pi x(n-L(n)+1-N_d)], x(n-1)\cos[\pi x(n)], \cdots, \\ &x(n-L(n)+1)\cos[\pi x(n-L(n))]], \cdots, x(n-N_d) \\ &\cos[\pi x(n-L(n)+1-N_d)]\}^T, \end{aligned} \quad (8)$$

which may also be written as

$$\boldsymbol{X}_{\Lambda(n)}(n) = [\boldsymbol{X}^T_{L,L(n)}, \boldsymbol{X}^T_{N,L(n)}, \boldsymbol{X}^T_{C,L(n)}]^T \quad (9)$$

where $\boldsymbol{X}_{L,L(n)}$, $\boldsymbol{X}_{N,L(n)}$ and $\boldsymbol{X}_{C,L(n)}$ represents the linear, nonlinear and cross terms of $\boldsymbol{X}_{\Lambda(n)}(n)$ in (8). The weight vector $\boldsymbol{W}_\Lambda(n) = [\boldsymbol{W}^T_{L,L(n)}, \boldsymbol{W}^T_{N,L(n)}, \boldsymbol{W}^T_{C,L(n)}]^T$ is updated using the proposed variable coefficient variable learning rate generalized FsLMS (VCLGFsLMS) algorithm, which is similar to the update rule given in (3), with

$$\boldsymbol{\mu}(n) = \begin{bmatrix} \mu_L(n) & 0 & 0 \\ 0 & \mu_N(n) & 0 \\ 0 & 0 & \mu_C(n) \end{bmatrix}_{\Lambda(n)\times\Lambda(n)} \quad (10)$$

where $\mu_L(n)$, $\mu_N(n)$ and $\mu_C(n)$ are the variable learning rates for the linear, nonlinear and the cross terms respectively.

In order to incorporate variable step size into the proposed ANC schemes, we divide the total segmented residual noise signal $e^{\Lambda(n)}_{\Lambda(n)}(n)$ into two parts, one due to the contribution of the linear section of the controller and the other because of the nonlinear portion. The error signal may be written as

$$e^{\Lambda(n)}_{\Lambda(n)}(n) = e_L(n) + e_N(n) \quad (11)$$

$$e_L(n) = e^{\Lambda(n)}_{\Lambda(n)}(n) + S_N(n) * y_N(n) \quad (12)$$

$$e_N(n) = e^{\Lambda(n)}_{\Lambda(n)}(n) - e_L(n). \quad (13)$$

The separation of the error signal has also been shown in the schematic diagram of the ANC scheme shown in Fig. 1, where $y_N(n)$ is the output from nonlinear part of controller. Using a variable learning rate approach [8], the learning rate for the linear block of the nonlinear controller may be updated as

$$\mu_L(n) = \rho_L(n)\mu_{Lmax} + [1 - \rho_L(n)]\mu_{Lmin} \quad (14)$$

where $\mu_{Lmax} = \frac{2\gamma_L}{||X'_L||^2}$ with $0 < \gamma_L \leq 1$ as a constant and $\mu_{Lmin} = \zeta_L \mu_{Lmax}$ with $0 < \zeta_L \leq 0.2$ as another constant. In (14), $\rho_L(n)$ is a weighting factor, with $0 < \rho_L(n) \leq 1$ and is given by

$$\rho_L(n) = \frac{A_{e_L}(n) - A_{e_L,min}(n)}{A_{e_L,max}(n) - A_{e_L,min}(n)} \quad (15)$$

where

$$A_{e_L}(n) = \frac{1}{T} \sum_{k=n-T+1}^{n} e^2_L(k) \quad (16)$$

with T as the averaging constant, which has been taken as 100 in this study. $A_{e_L,min}(n)$ and $A_{e_L,max}(n)$ are the minimum and maximum average powers of $e_L(n)$, computed using the average of the first and last 50 samples of $A_{e_L}(n)$. Similarly, the learning rate for the nonlinear portion, $\mu_N(n)$ is updated as

$$\mu_N(n) = \eta_N(n)\mu_{Nmax} + [1 - \eta_N(n)]\mu_{Nmin} \quad (17)$$

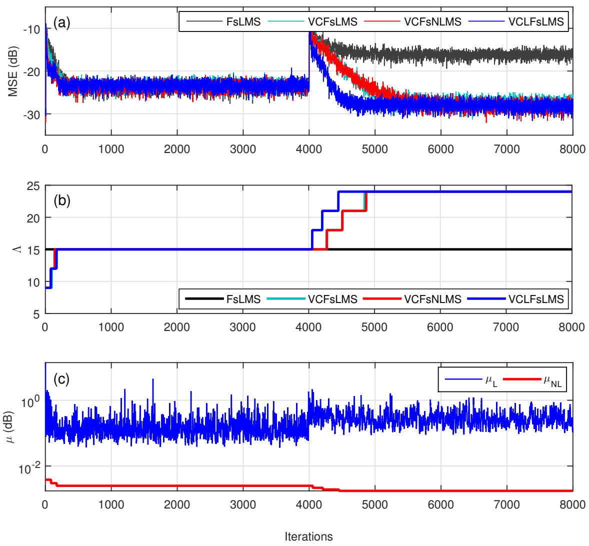

Fig. 2. Case A: The variation of (a) MSE, (b) Λ, and (c) μ with respect to iterations for a the algorithms compared for FLANN based ANC system, considering a uniformly distributed random signal as the primary noise $x(n)$.

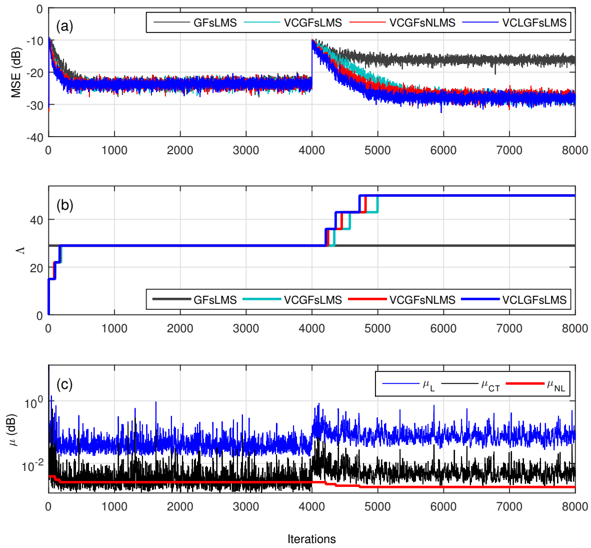

Fig. 3. Case B: The variation of (a) MSE, (b) Λ, and (c) μ with respect to iterations for a the algorithms compared for a GFLANN based ANC system, considering a uniformly distributed random signal as the primary noise $x(n)$.

with $\eta_N(n)$ as the weighting factor. In (17) μ_{Nmax} and μ_{Nmin} are the maximum and the minimum possible values of the learning rates, which have been computed earlier in this section. A similar approach has been used for updating the cross terms in GFLANN.

3. SIMULATION STUDY

In an endeavour to evaluate the performance of the proposed ANC mechanisms, a set of simulation studies have been carried out in a MATLAB environment. The mean square error (MSE), defined by $\xi = 10\log_{10}\left\{\mathrm{E}\left[e^2(n)\right]\right\}$ has been employed in this study as the metric for comparison, where $E[\cdot]$ is the expectation operation. A measurement noise, with a signal to noise ratio of 30 dB has been used and we have assumed perfect modeling of the secondary path (even in cases where the secondary path changes) in this study. The primary noise, at the cancellation point is given by

$$d(n) = u(n-2) + 0.08u^2(n-2) - 0.04u^3(n-1), \quad (18)$$

in all the experiments. In (18), $u(n) = x(n) * q(n)$ with $q(n)$ as the impulse response of the transfer function $Q(z) = z^{-3} - 0.3z^{-4} + 0.2z^{-5}$ and $*$ denotes the linear convolution operator [2]. In all the variable tap-length scenarios the initial tap-length $L(0) = \Delta + \delta = 2+1 = 3$, $\alpha = 0.001$ and $\beta = 0.2$ has been considered. A non-minimum phase secondary path, with a transfer function given by $S_{N1}(z) = z^{-2} + 1.5z^{-3} - z^{-4}$, has been considered for the first 4000 samples of each experiment and a minimum phase secondary path given by $S_{N2}(z) = z^{-2} + 0.5z^{-3}$ has been assumed for the rest of the samples. All the graphs plotted in this section are an average of 50 independent iterations.

3.1. Case A: FLANN based ANC

The primary noise employed is a random signal, which has been uniformly distributed within the range $[-0.5, 0.5]$. The noise cancellation obtained using the proposed VCLFsLMS algorithm has been compared with the results obtained using the conventional FsLMS algorithm as well as a variable coefficient FsLMS (VCFsLMS) algorithm. VCFsLMS algorithm has been designed by considering a fixed value for the learning rate matrix in (3). A comparison has also been made with the noise mitigation achieved using a variable coefficient normalized FsLMS (VCFsNLMS) algorithm. In a VCFsNLMS, the learning rates for the linear as well as the non-linear portions are given by $\mu_L(n) = \frac{\kappa_L}{||X'_L||^2}$ and $\mu_N(n) = \frac{\kappa_N}{||X'_N||^2}$, where κ_L and κ_N are constants. The variation of MSE with respect to iterations in this case is shown in Fig. 2 (a). The improved convergence behaviour of the proposed algorithms over FsLMS algorithm is clear from the simulation results. The change in the number of coefficients (Λ) as well as the learning rate (μ) for all the situations considered has also been shown in the Fig. 2 for better understanding of the algorithm behaviour. The various parameters used in the experiment for the different algorithms are: FsLMS ($P = 1$, Tap length $= 5$, $\mu_L = 2 \times 10^{-2}$ and $\mu_N = 5 \times 10^{-3}$), VCFsLMS

Table 1. Comparison of Number of Coefficients and MSE for ANC with input as uniform random noise

	Number of Coefficients		MSE (dB)	
Secondary Path	$S_{N1}(z)$	$S_{N2}(z)$	$S_{N1}(z)$	$S_{N2}(z)$
ANC Schemes				
FsLMS	15	15	-23.22	-16.25
VCFsLMS	15	24	-22.98	-27.48
VCFsNLMS	15	24	-23.33	-28.14
VCLFsLMS	15	24	-22.97	-28.25
GFsLMS	29	29	-23.95	-16.65
VCGFsLMS	29	50	-23.71	-28.58
VCGFsNLMS	29	50	-23.85	-29.25
VCLGFsLMS	29	50	-23.76	-29.18

($P = 1$, $\mu_L = 4 \times 10^{-2}$ and $\mu_N = 8 \times 10^{-3}$), VCFsNLMS ($P = 1$, $\kappa_L = 1 \times 10^{-2}$ and $\kappa_N = 5 \times 10^{-3}$), VCLFsLMS ($P = 1$, $\gamma_L = 2 \times 10^{-2}$, $\eta_L = 1 \times 10^{-3}$, $\gamma_N = 1 \times 10^{-2}$, $\eta_N = 5 \times 10^{-4}$). Number of coefficients that needs to be updated in each iteration and the corresponding steady state MSE (dB) is shown in Table 1. The ability of the proposed variable tap as well as variable learning rate ANC algorithms, to adapt to the changing ANC scenarios is evident from the experiment.

3.2. Case B: GFLANN based ANC

We have designed a VCLGFsLMS algorithm for updating the coefficients of a GFLANN in a dynamic manner. The convergence characteristics obtained has been compared with that obtained using a variable coefficient normalized GFsLMS (VCGFsNLMS) algorithm and a variable coefficient GFsLMS (VCGFsLMS) algorithm in addition to the GFsLMS algorithm proposed in [3]. The variation of MSE as well as other parameters with respect to iterations is shown in Fig. 3 for an uniformly distributed $x(n)$. The improved and dynamic nature of the proposed algorithms can be seen from the simulation results. The parameters considered for the GFLANN based experiment using a filter order $P = 1$ and $N_d = 2$ are: GFsLMS (Tap length = 5, $\mu_L = 1 \times 10^{-2}$, $\mu_N = 5 \times 10^{-3}$ and $\mu_C = 2 \times 10^{-2}$), VCGFsLMS ($\mu_L = 3 \times 10^{-2}$, $\mu_N = 5 \times 10^{-2}$ and $\mu_C = 2 \times 10^{-3}$), VCGFsNLMS ($\kappa_L = 2 \times 10^{-2}$, $\kappa_N = 1 \times 10^{-3}$ and $\kappa_C = 5 \times 10^{-3}$), VCLGFsLMS ($\gamma_L = 5 \times 10^{-2}$, $\eta_L = 1 \times 10^{-3}$, $\gamma_N = 1 \times 10^{-2}$, $\eta_N = 1 \times 10^{-3}, \gamma_C = 5 \times 10^{-3}, \eta_C = 1 \times 10^{-4}$). Fig. 3 (b) and (c) shows the learning curve of tap-lengths and step sizes (μ_L, μ_N and μ_C) respectively for uniform random noise.

4. CONCLUSION

This paper developed a class of nonlinear ANC schemes, which can adapt the filter weights as well as the structure of the nonlinear controller in a dynamic manner, depending on the ANC scenario. This dynamic behaviour has been achieved by designing variable coefficient length and variable learning rate adaptive algorithms for nonlinear ANC. The proposed schemes have been shown to effectively mitigate noise with optimal resources in comparison with traditional nonlinear ANC methods.

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