MIMO Radar Waveform Design for Transmit Beampattern Synthesis

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***Abstract*—In this paper, the waveform design for transmit beampattern synthesis is studied in MIMO radar systems with colocated antennas. The waveform transmitted at each antenna is defined as a weighted sum of a set of discrete prolate spheroidal (DPS) sequences which have good orthogonal and band-limited properties. Assume that different transmit antennas use the same set of DPS sequences, while the weighting factors are variable which allows the correlation between different waveforms to be flexible and varied with the weighting factors. Optimum waveforms are designed to achieve a desired transmit beampattern under the constraint of a fixed total transmit energy. Unlike a traditional process, in which the waveform covariance matrix is designed in the first step and then the optimal waveforms are synthesised in the second step based on the designed waveform covariance matrix, it is shown in this work that the waveforms constructed by the DPS sequences need only one step to design the optimum waveforms. In this paper, we propose a new waveform design method for transmit beampattern synthesis which can efficiently match a desired transmit beampattern and control the power distributed at each beam in beampattern simultaneously. The choice of the number of the DPS sequences is also analyzed. Numerical simulations are provided to compare the performance of proposed method with that of the traditional methods.**

***Index Terms*—Multiple-input multiple-output (MIMO) radar, waveform design, DPS sequences, transmit beampattern.**

I. Introduction

In the last decade, the advantages of multiple-input multiple-output (MIMO) radar have drawn considerable attention [1]–[10]. For a traditional phased-array radar systems, the space between the adjacent transmit/receive antennas is restrained and the waveforms transmitted by each of the transmitters are identical, while for a MIMO radar system, the transmit/receive antennas can be arbitrary spaced and the waveforms can be different among each of the transmitters. Since the flexibility on the antenna position and waveforms, the MIMO radar system can offer better resolution, higher detection performance for slowly moving target, and higher estimation performance, compared with a phased-array radar systems.

Waveform design is a key issue in radar signal processing. The transmit waveforms of MIMO radar are usually optimized for specific goals, such as improving the signal-to-clutter-plus-noise ratio (SCNR) [11], increasing the resolution in the spatial and temporal domains, enhancing the detection performance, maximizing the mutual information (MI) between the random target impulse response and the reflected waveforms [12], or achieving flexible transmit beampatterns [2]–[5], [13], [14]. In this work, the waveforms of MIMO radar are designed to achieve a desired transmit beampattern.

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The transmit beampattern, which is a function of the spatial angle θ, denotes the power of the signal at each direction θ resulting from all of the transmit waveforms [4]. For a MIMO radar with collocated antennas, the waveforms transmitted from different antennas can be different. By choosing different transmit waveforms, the correlation between each two waveforms can vary from full correlation to mutual orthogonality, which offers the capability of designing the transmit beampattern according to the target scenarios.

In a traditional process of waveform design for transmit beampattern synthesis, there are two steps which lead to two optimization problems. In the first step, the covariance matrix of waveforms $\mathbf{R}$ need to be chosen to obtain some desirable features of transmit beampattern. Several methods for designing $\mathbf{R}$ have been studied in the existing researches, such as maximum power design [2], beampattern matching design [2], and minimum sidelobe beampattern design [2], [15]. The main idea of these methods is to distribute the transmit power to the spatial sector of interest based on the desired transmit beampattern. In the second step, the waveforms are synthesized to satisfy the chosen $\mathbf{R}$ [3], [13]. Considering that the process of synthesizing waveforms with a given $\mathbf{R}$ in the second step is not easy to achieve and the computational burden caused by the optimization problems in the two steps is large, in [16] and [17], a weighting matrix is introduced and imposed on several orthogonal signals to reduce the waveform design problem from two steps to one step. The waveforms in [16] are designed under the constraint of equal elemental power. Although some requirements, including the equal transmit power from each antenna and the constant-envelope of each waveform, are useful for hardware implementation, the waveforms which designed without such requirements have higher diversity and can achieve a better performance in theory. In [17], the aim of the beampattern design is focusing on the control of the ripple levels and transition bandwidth.

The discrete prolate spheroidal (DPS) sequences [18], which are orthogonal and have good band limited and time limited properties, have been widely applied in communication and radar systems [19]–[23]. In [19], DPS sequences are utilized

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for dimension reduction in beamspace for array processing. Prolate spheroidal wave functions are proposed for MIMO radar space-time adaptive processing in [20]. The generation principle of DPS sequences is employed in [21] to maximize the portion of the energy radiated within the desired spatial section so that the signal-to-noise ratio gain at the receive antennas can be improved. In [22], [23], the DPS sequences are used to construct the transmit waveforms for a MIMO-OTH radar system to meet the limited frequency band of MIMO-OTH radar.

In this paper, the waveforms emitted by each of the transmit antennas in MIMO radar systems are formed as a weighted sum of a group of DPS sequences. With this construction for waveforms, it can be shown that the MIMO radar waveform design for transmit beampattern synthesis is reduced from two steps to one step. We model the expression of desired transmit beampattern, based on which the optimization problem is formulated, aiming at maximizing the power at the spatial sector of interest and controlling the power distributed at each individual subinterval within the spatial sector of interest. Except the constraint of the total transmit energy, the waveforms are not limited to any other requirements to promise the best diversity. The method for selecting the number of the DPS sequences is also analyzed to further improve the performance.

Notation: Throughout this paper, the superscripts $(\cdot)^H$, $(\cdot)^*$, and $(\cdot)^T$ denote the complex conjugate transpose, conjugate, and transpose of a matrix, respectively. The $\mathbb{E}\{\cdot\}$ stands for the expectation with respect to all the random variables within the brackets. The symbol $tr(\cdot)$ denotes the trace of a matrix. The $\|\cdot\|$ indicates the Euclidean norm of a vector.

II. Problem Formulation

Consider a MIMO radar equipped with M colocated transmit antennas and L colocated receive antennas. Each of the transmitted waveforms is assumed to be narrow-band signal. Denote the waveform transmitted by the m-th transmit antenna by $s_m(n)$ for $0 \le n \le N-1$, where N is the number of samples within the time duration. Let θ denote the azimuth angle of a potential target. Then the signal, which is radiated towards a potential target located at a direction θ, can be written as

$$z(\theta, n) = \sum\nolimits_{m=1}^{M} s_m(n)\, e^{-j(m-1)\phi_t} = \mathbf{a}^H(\theta)\,\mathbf{s}(n) \tag{1}$$

where $\mathbf{s}(n) = [s_1(n), \cdots, s_M(n)]^T$ is a $M \times 1$ vector, $\mathbf{a}(\theta) = [1, e^{j\phi_t}, \cdots, e^{j(M-1)\phi_t}]^T$ is the steering vector, $\phi_t = 2\pi d_t \sin\theta/\lambda$ and $d_t = \lambda/2$ represent the spatial phase difference and the distance between adjacent transmitters, and λ is the wavelength of each waveform. Then the power of the transmit signal, namely the transmit beampattern, radiated towards the direction θ is given by

$$p(\theta) = \mathbb{E}\left\{z(\theta, n)\, z^H(\theta, n)\right\} = \mathbf{a}^H(\theta)\,\mathbf{R}\mathbf{a}(\theta) \tag{2}$$

where

$$\mathbf{R} = \mathbb{E}\left\{\mathbf{s}(n)\,\mathbf{s}^H(n)\right\} \tag{3}$$

is the covariance matrix of waveforms which satisfies the positive semi-definite property, such that $\mathbf{R} \geq 0$. Stacking the waveforms at N time indices into a column, the $MN \times 1$ waveform vector can be expressed as $\mathbf{s} = [\mathbf{s}^T(0), \cdots, \mathbf{s}^T(N-1)]^T$. As a function of the transmit signal $\mathbf{s}$, the transmit beampattern $p(\theta)$ can also be expressed as $p(\theta; \mathbf{s})$. Under the constraint of the total transmit energy, the waveform optimization problem for transmit beampattern synthesis can be described as

$$\max_{\mathbf{s}} F_1[p(\theta; \mathbf{s})] \quad \|\mathbf{s}\|^2 \le E_0 \tag{4}$$

or

$$\min_{\mathbf{s}} F_2[p(\theta; \mathbf{s})] \quad \|\mathbf{s}\|^2 \le E_0 \tag{5}$$

where $F_1[\cdot]$ and $F_2[\cdot]$ represent two functions of $p(\theta; \mathbf{s})$, E_0 is the total transmit energy. F_1 is usually used to ensure that the transmit energy at desired directions is maximized while F_2 is usually used to suppress the transmit energy outside of the interested directions or the difference between the actual and desired transmit beampattern. By solving the optimization problem in (4) or (5), the optimum waveforms $\mathbf{s}_{\text{opt}}$ for the goals described by the function $F_1[\cdot]$ or $F_2[\cdot]$ can be obtained.

Next we introduce the traditional methods of waveform design for transmit beampattern synthesis, which first design the covariance matrix $\mathbf{R}$ then synthesize the waveforms based on the designed $\mathbf{R}$. In the first step, there are two main methods of designing $\mathbf{R}$ to optimize the beampattern. One method is maximum power design, which maximizes the total power of transmit signals at all directions of interest. Denote the transmit beampattern $p(\theta)$ by $p(\theta; \mathbf{R})$ and assume there are Q directions of interest θ_q, $q = 1, \cdots, Q$, the optimization problem for maximum power design can be described as [2]

$$\max_{\mathbf{R}} \sum\nolimits_{q=1}^{Q} p\left(\theta_q; \mathbf{R}\right) \quad s.t. \quad \text{tr}(\mathbf{R}) \le E_0, \mathbf{R} \ge 0 \tag{6}$$

where $\mathbf{R} \geq 0$ means that $\mathbf{R}$ is a semi-definite matrix. Note that $\text{tr}(\mathbf{R}) = \|\mathbf{s}\|^2$ according to the definition of $\mathbf{R}$ in (3). The other main method of designing $\mathbf{R}$ is beampattern matching design, which minimizes the difference between the actual transmit beampattern and the desired one in a specific sense. Taking least squares (LS) for example, the optimization problem for beampattern matching design in the LS sense can be described as [2]

$$\min_{\mathbf{R},\beta} \sum\nolimits_{l=1}^{L} [\beta p_d(\theta_l) - p(\theta_l; \mathbf{R})]^2 \quad s.t. \quad \text{tr}(\mathbf{R}) \le E_0, \mathbf{R} \ge 0 \tag{7}$$

where θ_l is the l-th direction sampled within an angle range which covers all directions of interest and $p_d(\theta_l)$ is the desired transmit beampattern at θ_l. To meet the further design requirements, some extra constraints and items such as the cross correlations of the beampattern between different directions, can be added to the optimization problem in (7). By solving the problem in (6) or (7), the optimized covariance matrix of waveforms $\mathbf{R}_{\text{opt}}$ can be obtained. Then, in the second step the optimized waveforms $\mathbf{s}_{\text{opt}}$ are usually synthesized based on $\mathbf{R}_{\text{opt}}$ by using some iterative algorithms [3], [13]. In [3], the authors propose a cyclic algorithm to synthesize constant modulus signals. In [13], two algorithms are proposed to design constant modulus waveforms.

The design method in (6) can maximize the total power of transmit signals at all directions of interest. However, the power distributed at each direction can not be controlled very well. Although the design method in (7) can obtain a good approximation of the desired beampattern, it needs to be solved by numerical methods with large computational complexity. Further, both of the design methods in (6) and (7) can only obtain the optimized $\mathbf{R}$, while the optimized waveforms need the other step of waveform synthesis, which leads to a larger computational complexity and a lower accuracy. Next we propose a new waveform design method for transmit beampattern synthesis, which optimizes the waveforms directly. The new method has a good control of transmit signal power at each direction of interest and low computational complexity.

III. DPS Sequence-Based Waveform Design for MIMO radar Transmit Beampattern Synthesis

A. DPS Sequences-Based MIMO Waveforms

To reduce the transmit energy leakage in frequency and time domain, we employ DPS sequences, which are approximately orthogonal and have good band limited and time limited properties, to form the waveforms. Considering DPS sequences with time duration $[0, N-1]$, Slepian [18] pointed out that there are $2NW$ DPS sequences which are approximately band-limited to $[-W, W]$, where $0 < W \le 1/2$. Denote the i-th, $i = 1, ..., 2NW$ normalized DPS sequence time-limited to $[0, N-1]$ and band-limited to $[-W, W]$ by $v_i(n; N, W)$, $n = 0, ..., N-1$, then $\sum_{n=0}^{N-1} v_i(n; N, W) v_{i'}(n; N, W) = \delta_{i,i'}$, $\quad i, i' = 1, ..., 2NW$, where $\delta_{i,i'}$ is the Kronecker delta function, which span a $2NW$-dimensional orthonormal space. Thus, we use the first N_R DPS sequences $v_i(n; N, W)$, $i = 1, \cdots, N_R$, to construct the waveforms, where $N_R \le 2NW$. Then, the m-th ($m = 1, \cdots, M$) waveforms, called DPS sequences-based waveforms in this paper, can be described as

$$s_m(n; \mathbf{d}_m) = \sum_{i=1}^{N_R} d_{m,i} v_i(n; N, W), \quad n = 0, \cdots, N-1. \tag{8}$$

where $d_{m,i}$ is a weighting factor. Then the waveform $\mathbf{s}(n)$ in (1) can be written as

$$\mathbf{s}(n) = \mathbf{D}\mathbf{v}(n) \tag{9}$$

where $\mathbf{v}(n) = [v_1(n; N, W), \cdots, v_{N_R}(n; N, W)]^T$, $\mathbf{D} = [\mathbf{d}_1, \cdots, \mathbf{d}_{N_R}]$, and $\mathbf{d}_i = [d_{1,i}, \cdots, d_{M,i}]^T$. Plugging (9) into (3), the covariance matrix can be rewritten as

$$\mathbf{R} = \mathbb{E}\left\{\mathbf{D}\mathbf{v}(n)\mathbf{v}^T(n)\mathbf{D}^H\right\} = \mathbf{D}\mathbf{D}^H = \sum_{i=1}^{N_R} \mathbf{d}_i\mathbf{d}_i^H, \tag{10}$$

in which the orthogonal property of the DPS sequences has been used, such that $\mathbb{E}\left\{\mathbf{v}(n)\mathbf{v}^T(n)\right\} = \mathbf{I}$. From (10), the covariance matrix of the DPS sequences-based waveforms can be obtained directly from the weighting matrix $\mathbf{D}$, instead of computing the expectation with respect to the transmit signals as per (3). Substituting (10) into (2), the transmit beampattern becomes a function of $\mathbf{D}$, which can be expressed as

$$p(\theta; \mathbf{D}) = \mathbf{a}^H(\theta)\mathbf{D}\mathbf{D}^H\mathbf{a}(\theta) \tag{11}$$

Next we design the waveforms for transmit beampattern synthesis according to the expression of transmit beampattern in (11).

B. DPS Sequence-Based Waveform Design for Transmit Beampattern Synthesis

Due to the transmit beampattern defined in (2) is the power of the transmit signal in spatial domain, the optimization for transmit beampattern is actually a spatial energy distribution optimization problem. Consider a scenario with multiple targets and all of the targets are located within the spatial angle range Θ. Defined $\Theta = \Theta_1 \cup \Theta_2 \cup \cdots \cup \Theta_K$, where Θ_k, $k = 1, \cdots, K$, are K arbitrary non-overlapping subintervals within Θ. The desired transmit beampattern can be expressed as

$$p_d(\theta) = \begin{cases} \alpha_k, & \theta \in \Theta_k, k = 1, \cdots, K \\ 0, & otherwise \end{cases} \tag{12}$$

where α_k denotes the desired transmit power within the subinterval Θ_k. Note that when $K \to \infty$, the Θ can cover the whole spatial domain $[-\pi/2, \pi/2]$, and the expression in (12) can be used to describe a transmit beampattern with arbitrary continuous shape.

In order to choose transmit waveforms under the constraint of the total transmit energy such that the transmit beampattern matches the desired transmit beampattern, a straightforward thought is to maximize the ratio of the transmit power which is distributed within the desired spatial sector Θ to the total transmit power. Further, for each subinterval Θ_k of interest, the transmit power distributed within Θ_k is expected to be proportional to the desired transmit power α_k in (12). Suppose the total transmit energy is upper bounded by E_0 so that the maximum available transmit power is fixed. Assume the spatial ranges for each of the subintervals Θ_k, $k = 1, \cdots, K$, are identical, then the optimization problem can be expressed as

$$\max_{\mathbf{s}} \sum_{k=1}^{K} \int_{\Theta_k} \alpha_k p(\theta; \mathbf{s})\, d\theta \quad \|\mathbf{s}\|^2 \le E_0. \tag{13}$$

When the transmit waveforms are constructed as a weighted sum of a group of DPS sequences, as per (8) and (9), the optimization problem in (13) can be rewrittern as a problem with respect to the weighting matrix $\mathbf{D}$, given by

$$\max_{\mathbf{D}} \sum_{k=1}^{K} \int_{\Theta_k} \alpha_k p(\theta; \mathbf{D})\, d\theta \quad \mathrm{tr}(\mathbf{D}\mathbf{D}^H) \le E_0. \tag{14}$$

The optimization problem in (14) is equivalent to [24]

$$\max_{\mathbf{d}_1, \cdots, \mathbf{d}_{N_R}, N_R} \sum_{i=1}^{N_R} \mathbf{d}_i^H\mathbf{A}\mathbf{d}_i \qquad s.t. \quad \sum_{i=1}^{N_R} \mathbf{d}_i^H\mathbf{d}_i \le E_0 \tag{15}$$

where $\mathbf{A} = \sum_{k=1}^{K} \int_{\Theta_k} \alpha_k \mathbf{a}^H(\theta)\mathbf{a}(\theta)\, d\theta$. Note that in (15), the parameters to be optimized are N_R and $\mathbf{d}_i$ ($i = 1, \cdots, N_R$). The value of N_R denotes the number of DPS sequences which are chosen for constructing the waveforms in (8). Next, we solve the problem in (15) and obtain the optimal solutions of N_R and $\mathbf{d}_i$ ($i = 1, \cdots, N_R$). Then, by substituting the solutions N_R and $\mathbf{d}_i$ ($i = 1, \cdots, N_R$) into (8), the optimal waveforms can be achieved directly.

To solve the problem in (15), we can first solve $\mathbf{d}_i$ ($i = 1, \cdots, N_R$) in terms of N_R. When the value of N_R is fixed, the $\mathbf{d}_i$ can be solved by the Lagrange multiplier method. Define the Lagrangian as

$$\mathcal{L} = \sum_{i=1}^{N_R} \mathbf{d}_i^H \mathbf{A} \mathbf{d}_i + \lambda \sum_{i=1}^{N_R} \mathbf{d}_i^H \mathbf{d}_i. \tag{16}$$

Taking a derivative of $\mathcal{L}$ with respect to $\mathbf{d}_i$ ($i = 1, \cdots, N_R$), and letting the derivative be zero, we have

$$\frac{\partial \mathcal{L}}{\partial \mathbf{d}_i} = 2\mathbf{A}\mathbf{d}_i + 2\lambda \mathbf{d}_i = 0 \tag{17}$$

By solving (17), it is easy to get that for any given N_R, the value of the objective function in (15) reaches the maximum when

$$\mathbf{d}_1 = \cdots = \mathbf{d}_{N_R} = \sqrt{E_0/N_R}\mathbf{q}_{\mathbf{A},1}, \tag{18}$$

where $\mathbf{q}_{\mathbf{A},1}$ is the normalized eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{A}$. However, when the equation in (18) holds, it can be shown that the m-th transmit waveform in (8) becomes

$$\begin{aligned} s_m(n) &= \sum_{i=1}^{N_R} d_{m,i} v_i(n; N, W) = d_{m,1} \sum_{i=1}^{N_R} v_i(n; N, W) \\ &= \sqrt{E_0/N_R} q_{\mathbf{A},1,m} \sum_{i=1}^{N_R} v_i(n; N, W) \end{aligned} \tag{19}$$

where $q_{\mathbf{A},1,m}$ is the m-th entry of $\mathbf{q}_{\mathbf{A},1}$. It is seen from (19) that each of transmit antennas transmits the same waveform $\sum_{i=1}^{N_R} v_i(n; N, W)$ with just a different coefficient $d_{m,1} = \sqrt{E_0/N_R} q_{\mathbf{A},1,m}$, which means the waveform diversity is not obtained. It is known that a MIMO radar system with colocated antennas can obtain the waveform diversity by transmitting different waveforms with different antennas. To produce waveform diversity, next we impose an orthogonality constraint, $\mathbf{d}_i^H \mathbf{d}_{i'} = 0$ when $i \neq i'$, on the problem in (15). Under this constraint, the solution of $\mathbf{d}_i$ ($i = 1, \cdots, N_R$) for a fixed N_R can be given by

$$\mathbf{d}_{\text{opt},i} = \left\|\mathbf{d}_{\text{opt},i}\right\| \mathbf{q}_{\mathbf{A},i} \tag{20}$$

where

$$\left\|\mathbf{d}_{\text{opt},i}\right\| = \sqrt{\lambda_{\mathbf{A},i}^2 E_0 / \sum_{j=1}^{N_R} \lambda_{\mathbf{A},j}^2}, \tag{21}$$

the $\lambda_{\mathbf{A},i}$ is the i-th largest eigenvalue of the matrix $\mathbf{A}$, and $\mathbf{q}_{\mathbf{A},i}$ is the normalized eigenvector corresponding to $\lambda_{\mathbf{A},i}$.

Next, we analyze how to choose the number of the DPS sequences N_R. On one hand, when the value of N_R is increased, the diversity of weight vector $\mathbf{d}$ will be enhanced which is useful for improving the performance of the systems.

On the other hand, based on Lemma 1 given below, increasing the value of N_R will lead to decreased beamspace energy which is concentrated within the desired spatial sector. From (1) and (8), N_R transmit beams are formed when radiating the N_R DPS sequence $v_i(n; N, W)$, $i = 1, \cdots, N_R$. The signal radiated through the i-th beam can be modeled as $\mathbf{a}^H(\theta)\mathbf{d}_i v_i(n; N, W)$. Then, the radiated energy within the desired spatial sector from the i-th beam can be given by

$$\Gamma_i = \sum_{n=0}^{N-1} \int_{\Theta} |\mathbf{a}^H(\theta)\mathbf{d}_i v_i(n; N, W)|^2 d\theta = \mathbf{d}_i^H \mathbf{A} \mathbf{d}_i \tag{22}$$

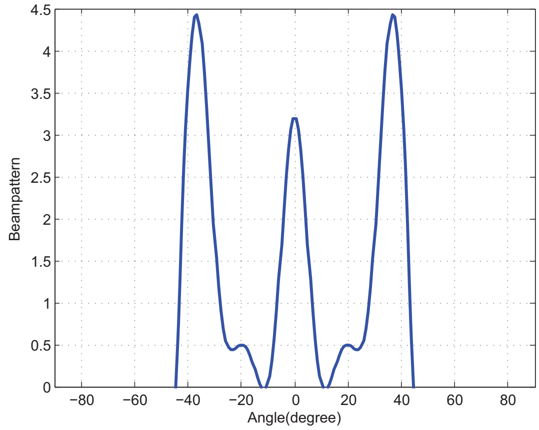

Fig. 1. The beampattern obtained by the maximum power design.

Lemma 1: Let $\Gamma_{\text{opt},i} = \mathbf{d}_{\text{opt},i}^H \mathbf{A} \mathbf{d}_{\text{opt},i}$, where $\mathbf{A} = \sum_{k=1}^{K} \int_{\Theta_k} \alpha_k \mathbf{a}^H(\theta) \mathbf{a}(\theta) d\theta$ as per (15), $\mathbf{d}_{\text{opt},i} = \left\|\mathbf{d}_{\text{opt},i}\right\| \mathbf{q}_{\mathbf{A},i}$ and $\left\|\mathbf{d}_{\text{opt},i}\right\| = \sqrt{\lambda_{\mathbf{A},i}^2 / \sum_{i=1}^{N_R} \lambda_{\mathbf{A},i}^2 E_0}$ as per (20) and (21). The $\lambda_{\mathbf{A},i}$ is the i-th largest eigenvalue of the matrix $\mathbf{A}$, and $\mathbf{q}_{\mathbf{A},i}$ is the normalized eigenvector corresponding to $\lambda_{\mathbf{A},i}$. For any $i > j (i, j = 1, \cdots, M)$, we have $\Gamma_i < \Gamma_j$.

Proof of Lemma 1: See [24].

So, there is a trade off between the diversity of the weight vector and the energy concentration when choosing the value of N_R. Considering N_R is an integer which satisfies $1 \leq N_R \leq 2NW$ such that the set of the available values for N_R is small, so we choose the value of N_R by using the exhaustion method. For each optional value of N_R, we compute the solutions of $\mathbf{d}_i$ ($i = 1, \cdots, N_R$) based on (16)-(21). Then, we substitute the values of N_R and $\mathbf{d}_{\text{opt},i}$ ($i = 1, \cdots, N_R$) into the objective function of the optimization problem in (15). By comparing the values of the objective function corresponding to each of optional values of N_R, the value of N_R, which causes the maximum values of objective function, can be chosen.

IV. Simulations

In this section, some numerical results are presented. Consider a MIMO radar system has $M = 10$ closely and linearly spaced transmit antennas. The total transmit energy is upper bounded by $E_0 = 1$. Assume there are three targets located at directions $-40°$, $0°$, and $40°$, then the number of subintervals K in (12) is $K = 3$ and the subintervals Θ_k, $k = 1, 2, 3$, can be chosen as $\Theta_1 = [-50°, -30°]$, $\Theta_2 = [-10°, 10°]$, and $\Theta_3 = [30°, 50°]$. Assume the three targets have the same status, so that the power of the desired beampattern in terms of the three subintervals should be identical. Then, we set α_k in (12) as $\alpha_k = 1$ for $k = 1, 2$ and 3.

In Fig. 1, the transmit beampattern synthesized using the optimized covariance matrix obtained by the method in (6), is shown. It is seen that the power distributed at each beam in the beampattern is not uniform. The result shown in Fig. 1 indicates that although the design in (6) maximizes the power

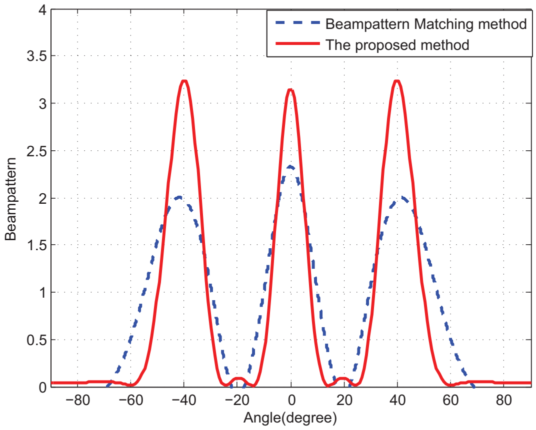

Fig. 2. The beampatterns obtained by the beampattern matching method and the proposed method for $K = 3$.

distributed at the whole spatial sector of interest, the control of the power distributed in each subinterval is not good.

In Fig. 2, the transmit beampattern synthesized using the optimized covariance matrix which is obtained by the beampattern matching method in (7), is plotted using dashed curve. The transmit beampattern corresponding to the designed DPS sequence-based waveforms, which is obtained by the proposed method is also plotted in Fig. 2 using solid curve. By comparing the two curves in Fig. 2 to the curve in Fig. 1, it can be observed that the energy distribution control effection in both of the beampatterns in Fig. 2 are better than that in the beampattern in Fig. 1. The reason is that the design method used in Fig. 1 only maximizes the total power distributed at the whole spatial sector of interest, while the weights for each spatial sector of interest are not considered according to the desired energy distribution. Further, in Fig. 2, note that for the beampattern obtained by our proposed method, the energy distributed at each of three subintervals are almost totally uniform, while for the beampattern obtained by the beampattern matching method, the energy of the middle beam is obviously higher than that of the beams on both sides. It shows the superiority of our proposed method.

V. Conclusions

In this paper, the DPS sequence-based waveform design method for transmit beampattern systhesis was proposed for MIMO radar systems. Before optimization, the waveforms were firstly constructed by a weighted sum of DPS sequences. It was shown that for DPS sequence-based waveform design method, the waveforms can be optimized by solving only one optimization problem instead of designing the waveform covariance matrix first and then synthesising the optimal waveforms from the covariance matrix. The computational burden were decreased though the construction of the DPS sequence-based waveforms. It was found that there is trad off between the diversity and the energy concentration when choosing the number of the DPS sequences. The simulation results showed that the beampattern obtained by the proposed DPS sequence-based waveform design method has a superior performance on energy control to the beampatterns obtained by the traditional methods.

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