ROBUST AND RAPID ESTIMATION OF THE PARAMETERS OF HARMONIC SIGNALS IN THREE PHASE POWER SYSTEMS

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ABSTRACT

We present a novel algorithm for rapid and efficient estimation of the fundamental frequency, phase and amplitude of harmonically distorted signals in balanced three-phase (3PH) power systems. The proposed algorithm exploits the harmonic structure of the signal to enhance the quality of the parameter estimates. It operates in the frequency domain, employing an efficient iterative interpolation procedure on the Fourier coefficients. The estimator has a low computational complexity, being of the same order as the fast Fourier transform (FFT) algorithm. Yet, it outperforms state-of-art high resolution parameter estimators for 3PH power system signals, especially when the available data points are limited and/or the signal to noise ratio is poor.

***Index Terms*—** Fundamental frequency estimation, smart grid, Fourier interpolation, three-phase power system.

1. INTRODUCTION

The estimation of the parameters of the voltage in a three-phase (3PH) power system is of significant importance in order to ensure the balance between energy generation and consumption, as it allows the system to deploy the power flows between the main grid and micro-girds and hence optimize the power delivery [1]. However, this task can be complicated by variations in the fundamental frequency and harmonic distortion [2] arising from factors such as the distributed nature of power sources, duality between loads and supplies, and frequent demand-supply mismatch [3]. As a result, research to develop robust estimation algorithms, especially those that can accurately track the change of frequency and are capable of tackling harmonic distortion, is urgent and necessary.

State-of-the-art frequency tracking methods for 3PH signal include the Augmented Complex Least Mean Square (A-CLMS) method, the Kalman filter (KF) method [4], and the zero-crossing method [5]. They can achieve reliable tracking of the frequency with reasonable accuracy when the 3PH signal model simply contains only the fundamental frequency component. Therefore, they exhibit undesirable performance for harmonically distorted signals.

High resolution parameter estimators for exponential signals, such as MUltiple SIgnal Classification (MUSIC) [6] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [7], are popular methods for parametric estimation of 3PH signals that are capable of producing unbiased estimates for harmonic distorted signals. Clarke's α,β transformation is first used to convert the 3PH voltage to a complex multi-tone signal before applying these algorithms. In the case of harmonically distorted signals, the Weighted Least Squares (WLS) approach exploits the harmonic structure to obtain further improvement in the estimation accuracy, [8] [2]. However, these methods approach the Cramer-Rao Lower Bound (CRLB) only when all the harmonics have relatively high signal to noise ratio (SNR) and are plagued by a high computational cost which is of order N^3 where N is the number of time samples.

In this paper, we present a novel efficient and accurate parameter estimator for harmonically distorted 3PH signals in balanced power system which overcomes the limitations of the previously proposed methods. The new algorithm employs the Clarke transform followed by interpolation on the Fourier coefficients of the signal combined with an iterative leakage subtraction step. It exploits the harmonic structure of the signal to yield an excellent estimation performance.

The rest of the paper is organised as follows. We first present the harmonic signal model in Section 2. The new algorithm is then described in Section 3. Simulation results are shown in Section 4 followed by conclusions in Section 5.

2. THE SIGNAL MODEL

The voltage in a balanced 3PH power system follows the model

$$\begin{aligned} v_a(n) &= \sum_{k=1}^{K} V_k \cos\left[k\left(2\pi\frac{f_0}{f_s}n+\phi\right)\right]+w_a(n),\\ v_b(n) &= \sum_{k=1}^{K} V_k \cos\left[k\left(2\pi\frac{f_0}{f_s}n+\phi-\frac{2\pi}{3}\right)\right]+w_b(n),\\ v_c(n) &= \sum_{k=1}^{K} V_k \cos\left[k\left(2\pi\frac{f_0}{f_s}n+\phi+\frac{2\pi}{3}\right)\right]+w_c(n), \quad (1)\end{aligned}$$

where $n = 0, 1 \ldots N-1$ is the sampling time index and f_s is the sampling frequency. K is the number of harmonic components present in the signal and is assumed given. $V_k > 0, k = 1, \ldots, K$ is the amplitude of the k^{th} harmonic component. f_0 is the fundamental frequency and $\phi \in [0, 2\pi)$ is the phase. The noise terms of the three phases $\{w_a(n), w_b(n), w_c(n)\}$ are mutually independent and assumed to be real Gaussian with zero mean and variance σ^2.

The 3PH signal model in Eq. (1) can be mapped to the $\alpha, \beta, 0$ reference frame using an orthogonal transformation matrix known as Clarke's transform:

$$\begin{bmatrix} v_0(n) \\ v_\alpha(n) \\ v_\beta(n) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(n) \\ v_b(n) \\ v_c(n) \end{bmatrix}. \quad (2)$$

Then the direct-axis component $v_\alpha(n)$ and quadrature-axis component $v_\beta(n)$ can be combined into the following complex harmonic exponential signal [2],

$$\begin{aligned} x(n) &= v_\alpha(n) + j v_\beta(n) \\ &= \sum_{k=1}^{K} A_k e^{j l_k (2\pi f n + \phi)} + w(n), \; n = 0, \ldots, N-1, \quad (3) \end{aligned}$$

where $l_k = [(-1)^{k-1}(6k-3)+1]/4$ and $A_k = V_{|l_k|}$. Clearly, $l_1 = 1$. Without loss of generality, we set $f = f_0/f_s \in [-0.5, 0.5]$ as the normalised frequency. The noise terms $w(n)$ are complex Gaussian with zero mean and variance $4\sigma^2/3$ [2]. For practical 3PH signals, we usually have $A_k > A_{k+1}, k = 1, \ldots, K-1$ [9], and we define the SNR of the signal as $\rho = 3A_1^2/4\sigma^2$.

As a result, given observations $v_a(n), v_b(n), v_c(n)$, the original parameter estimation problem in a 3PH power system is converted into the parameter estimation problem of the single-phase complex harmonic exponentials in noise. The novel estimation algorithm presented in the following section aims to estimate f_0, ϕ and A_1 of the transformed exponential signal.

3. THE PROPOSED METHOD

In the rest of this paper, we use $\hat{\lambda}$ to denote the estimate of the parameter λ. Given estimates of the frequencies in Eq. (3), least squares estimates of the amplitudes and phases of the 3PH signal in Eq. (3) can be obtained [10]. Consequently, the proposed method aims to obtain highly accurate and robust estimates of the frequencies. The frequency estimator at the heart of our approach consists of two steps, an initial coarse estimator followed by a fine estimation step. The coarse estimate of the fundamental frequency is obtained by a peak search of the periodogram of the signal [11],

$$\hat{f}_{\text{initial}} = \frac{\hat{m}_0}{N}, \quad \text{where} \quad \hat{m}_0 = \arg\max_m |X(m)|^2, \quad (4)$$

and $X(m)$ is the N-point FFT of $x(n)$.

Assuming that only the fundamental component $x_1(n) = A_1 e^{j\phi} e^{j2\pi f n}$ is present in the signal, then its frequency estimate obtained from the coarse step can be further refined using any efficient single-tone estimator such as Quinn's algorithm [12] or the A&M estimator [13, 14]. These interpolate on a small number of Fourier coefficients either side of the maximum bin $\hat{m}_0$. Let X_p denote the noiseless Fourier coefficients of $x_1(n)$ at frequencies $(\hat{m}_0 \pm p)/N$ where $|p| \leq 1$. Then we have

$$\begin{aligned} X_p &= \sum_{n=0}^{N-1} A_1 e^{j\phi} e^{j2\pi f n} e^{-j2\pi(\hat{f} + \frac{p}{N})n} \\ &= A_1 e^{j\phi} \frac{1 - e^{j2\pi(\delta - p)}}{1 - e^{j\frac{2\pi}{N}(\delta - p)}}, \quad (5) \end{aligned}$$

where $\delta = N(f - \hat{f}_{\text{initial}}) \in [-0.5, 0.5]$ is the frequency residual.

Defining $z = e^{j2\pi\delta/N} = e^{j2\pi(f - \hat{f}_{\text{initial}})}$, the estimate of f can be refined by

$$\hat{f} = \frac{\Im\{\ln(\hat{z})\}}{2\pi} + \hat{f}_{\text{initial}}, \quad (6)$$

where $\Im\{\bullet\}$ is the imaginary part of $\bullet$. In Quinn's method, z is estimated by

$$\hat{z} = \frac{1 - v}{1 - v e^{-j2\pi\frac{p}{N}}}, \quad (7)$$

where

$$v = \frac{X_0}{X_p}, \quad p = \text{sgn}(|X_1| - |X_{-1}|), \quad (8)$$

and sgn($\bullet$) signifies the sign of $\bullet$. In the A&M algorithm, on the other hand, $\hat{z}$ is given by

$$\hat{z} = \left[\cos\left(\frac{\pi}{N}\right) - j\frac{X_{0.5} + X_{-0.5}}{X_{0.5} - X_{-0.5}} \sin\left(\frac{\pi}{N}\right)\right]^{-1}. \quad (9)$$

For a complex signal with harmonic distortion as Eq. (3), however, the actual noiseless Fourier interpolated coefficients are the sum of X_p and the spectral leakage terms introduced by the $K-1$ harmonic components, which we denote as $\{X_{p,k}\}_{k=2}^{K}$. It is then necessary to subtract this leakage before refining the normalized frequency f by exploiting the harmonic structure.

Using $\tilde{X}_p$ to denote the actual interpolated Fourier coefficients of the harmonic signal, we have

$$\begin{aligned} \tilde{X}_p &= \sum_{n=0}^{N-1} x(n) e^{-j2\pi n(\hat{f} + \frac{p}{N})} \\ &= A_1 e^{j\phi} \frac{1 + e^{j2\pi N(f - \hat{f})}}{1 - e^{j2\pi(f - \hat{f})} e^{-j2\pi\frac{p}{N}}} \\ &\quad + \sum_{k=2}^{K} A_k e^{j l_k \phi} \frac{1 + e^{j2\pi N(l_k f - \hat{f})}}{1 - e^{j2\pi(l_k f - \hat{f})} e^{-j2\pi\frac{p}{N}}} + W_p \\ &= X_p + \sum_{k=2}^{K} X_{p,k} + W_p, \quad (10) \end{aligned}$$

where $W_\pm$ are the Fourier coefficients of the noise at the interpolation locations. Due to the relationship between the

Table 1. The proposed algorithm

Given	A length-N complex harmonic signal $x(n)$;
Calculate	$X(m) = \text{FFT}\{x(n)\}$, $m = 0, 1, \ldots, N-1$;
Find	$\hat{m}_0 = \arg\max_m \lvert X(m)\rvert^2$;
Initialise	$\hat{f} = \frac{\hat{m}_0}{N}$, $\{\hat{A}_k\}_{k=1}^{K} = \hat{\phi} = 0$;
Do	For $q = 1$ to Q, loop:
	(1) $\tilde{X}_{\pm} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n(\hat{f} \pm \frac{0.5}{N})}$;
	(2) $\hat{X}_{\pm,k} = \hat{A}_k e^{jl_k\hat{\phi}} \frac{1+e^{j2\pi N\hat{f}(l_k-1)}}{1-e^{j2\pi\hat{f}(l_k-1)}e^{\mp j\frac{\pi}{N}}}$;
	(3) $\hat{X}_{\pm} = \tilde{X}_{\pm} - \sum_{k=2}^{K} \hat{X}_{\pm,k}$;
	(4) $\hat{z} = \left[\cos\left(\frac{\pi}{N}\right) - j\frac{\hat{X}_{+}+\hat{X}_{-}}{\hat{X}_{+}-\hat{X}_{-}}\sin\left(\frac{\pi}{N}\right)\right]^{-1}$;
	(5) $\hat{f} = \frac{\Im\{\ln(\hat{z})\}}{2\pi} + \hat{f}$;
	(6) $\hat{\mathbf{a}} = [\mathbf{Z}^H(\hat{f})\mathbf{Z}(\hat{f})]^{-1}\mathbf{Z}^H(\hat{f})\mathbf{x}$;
Finally	$\hat{f}_0 = \hat{f} f_s$, $\hat{A}_1 = \lvert\hat{\mathbf{a}}(1)\rvert$ and $\hat{\phi} = \angle\hat{\mathbf{a}}(1)$.

harmonic frequencies, the leakage terms $X_{p,k}$ and hence $\tilde{X}_p$ are functions of f. Therefore, assuming that the estimates of $\{A_k\}_{k=2}^{K}$ and ϕ are available, the reconstructed leakage terms $\{X_{p,k}\}_{k=2}^{K}$ become

$$\hat{X}_{p,k} = \hat{A}_k e^{jl_k\hat{\phi}} \frac{1 + e^{j2\pi N(l_k-1)\hat{f}}}{1 - e^{j2\pi(l_k-1)\hat{f}} e^{-j2\pi\frac{p}{N}}},\ k = 2, \ldots, K, \quad (11)$$

and the leakage-free Fourier Coefficients are recovered as

$$\hat{X}_p = \tilde{X}_p - \sum_{k=2}^{K} \hat{X}_{p,k}. \quad (12)$$

We can now use these leakage-free coefficients to interpolate for the fundamental frequency. Given the frequency estimate, the amplitudes and phases of the fundamental and harmonics are obtained as follows. Writing the signal in vector notation, we have that

$$\mathbf{x} = \mathbf{Z}(\hat{f})\mathbf{a} + \mathbf{w}, \quad (13)$$

where $\mathbf{x} = [x(0), \ldots, x(N-1)]^T$, $\mathbf{w} = [w(0), \ldots, w(N-1)]^T$, $\mathbf{a} = [A_1 e^{j\phi}, \ldots, A_K e^{jl_K\phi}]^T$, $\mathbf{Z}(\hat{f}) = [\mathbf{z}_1, \ldots, \mathbf{z}_K]$, and $\mathbf{z}_k = [1, e^{j2\pi l_k\hat{f}}, \ldots, e^{j2\pi(N-1)l_k\hat{f}}]^T$. Solving the LS problem yields the complex amplitude vector $\mathbf{a}$,

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} \|\mathbf{x} - \mathbf{Z}(\hat{f})\mathbf{a}\| = [\mathbf{Z}^H(\hat{f})\mathbf{Z}(\hat{f})]^{-1}\mathbf{Z}^H(\hat{f})\mathbf{x}. \quad (14)$$

Equivalently, each entry in $\hat{\mathbf{a}}$ can also be calculated using DFT [10], which costs $O(N)$. The estimated values of amplitude and phase vectors are given by $|\mathbf{a}|$ and $\angle\mathbf{a}$ respectively.

Table 1 summarises the proposed algorithm. The fine estimation step for the frequency and LS amplitude estimator are implemented for $Q (Q \geq 2)$ iterations. We refer to the new method the Harmonic A&M (HAM) algorithm and the reason that the A&M algorithm is considered here is that it works in an iterative fashion and converges to the minimum variance at $\delta = 0$ [13]. Quinn's algorithm, on the other hand, is not used as it cannot be implemented iteratively due to the fact that its estimation variance is maximum at $\delta = 0$ [12]. It is also worth noting that HAM can also handle the estimation of singe-phase complex harmonic exponentials. From the procedure we can see that the overall computational cost is $O(N \log_2 N)$ and unlike the state-of-art high resolution estimators such as MUSIC and ESPRIT, no singular value decomposition (SVD) or eigenvalue decomposition (EVD) is required.

4. SIMULATION RESULTS

Simulation results are presented in this section to verify the performance of the proposed estimator. We consider a 3PH signal based on the Australian standard AS/NZS 61000.2.2 [9]. Specifically, the fundamental frequency f_0 is 50Hz and the phase ϕ is 10°. After performing the Clarke's transform Eq. (2), the signal comprises the six harmonic components shown in Table 2. The sampling frequency f_s is chosen as 4,000Hz, which means that there are 80 samples in a single cycle. We compare the performance of the HAM algorithm with ESPRIT [7], ESPRIT-WLS [8] and the Multi-tone A&M (MAM) algorithm [10]. We also include the CRLB [2] for reference. To generate each graph, 5,000 Monte Carlo runs were used.

Figs. 1 to 3 show the root mean square error (RMSE) of $\hat{f}_0$, $\hat{\phi}$ and $\hat{A}_1$ versus SNR obtained by various methods when $N = 64$. To implement ESPRIT and ESPRIT-WLS, we set the degree of freedom $L = \lceil N/3 \rceil$ where $\lceil\bullet\rceil$ is the ceiling operation. As neither HAM and MAM shows any improvement for $Q > 4$, we run HAM for $Q = 2$ and 4 iterations, and MAM for $Q = 4$ iterations only. Observe that when $Q = 4$, the RMSE of HAM is just slightly above the CRLB at SNR $>$ 0dB. The other methods, on the other hand, exhibit high SNR thresholds below which the estimates are not reliable. This is because the performance of MAM and ESPRIT methods is dependent on the SNR of all harmonic components, while HAM performs estimation by only utilising the Fourier coefficients close to the fundamental frequency. Therefore, for 3PH signals where the SNR of the harmonic components are small compared to that of the fundamental component, HAM is the most robust method.

In practical application, a good tracking performance of the algorithm is always expected. This requires the parameter estimator to be capable of achieving accurate estimation when the data record is short. In Figs. 4 to 6, we compare the performance of the methods when the available data points are within one cycle. In this test, the SNR is fixed to 60dB. HAM is implemented using $Q = 4$ and 6. We find that HAM outperforms other methods as the number of iterations increases and $Q = 6$ brings the RMSE of HAM down to the CRLB when only 0.3 cycles, i.e. 24 samples, are available.

Table 2. The orders of harmonic components and the corresponding amplitudes of the simulated power signal

Order	1	5	7	11	13	17
Amplitude	1	0.06	0.05	0.032	0.03	0.018

For further demonstration of HAM's performance, we apply the algorithm to the following harmonic signal with a varying number of harmonic components K,

$$x(n) = \sum_{k=1}^{K} \frac{e^{j2\pi kfn}}{2^{(k-1)}} + w(n), \; n = 0, \ldots, 63. \qquad (15)$$

To generate the signal, f is chosen randomly in $[3/N, 4/N]$ and SNR is set to 10dB. The RMSE of $\hat{f}$ is shown in Fig. 7 where we vary K from 2 to 10. The estimation results of phase and amplitudes are similar as $\hat{f}$ and therefore not shown here. We see that when the harmonic order K is small such that all harmonics have high SNR, WLS can perform better than HAM. However, as the number of harmonic components increases, HAM starts to outperform the other methods. The reason is that as the number of harmonics in the signal increases, their SNR decreases and the frequency estimation of ESPRIT and MAM becomes less reliable. When the estimation of the harmonics is inaccurate, WLS does not lead to any performance improvement as the estimation error of all components will be accumulated. The performance of HAM, on the other hand, is not affected as it is independent of the estimation accuracy of the harmonic components.

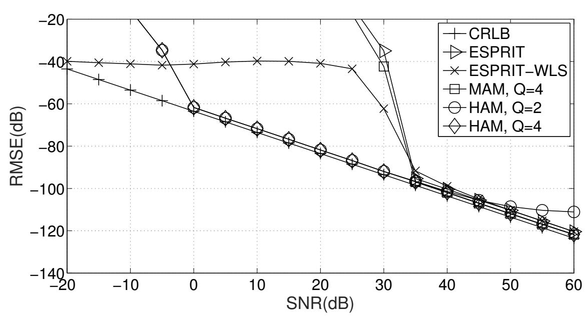

Fig. 1. RMSE of $\hat{f}_0$ versus SNR when $N = 64$.

5. CONCLUSION

We proposed to estimate the fundamental frequency, phase and amplitude of harmonic distorted signals in 3PH power systems. The application of Clarke's transform converts the 3PH signal to a complex harmonic exponential signal in noise. The idea of this novel algorithm is to iteratively perform interpolation on Fourier coefficients while eliminating the spectral leakage introduced by the embedded harmonic structure. We demonstrated through simulations that the proposed estimator can obtain RMSE values that are close to CRLB under limited sampling data and low SNR conditions.

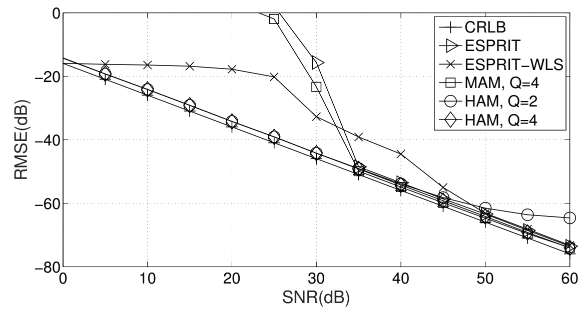

Fig. 2. RMSE of $\hat{\phi}$ versus SNR when $N = 64$.

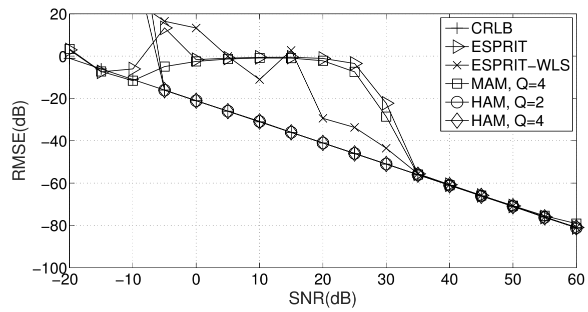

Fig. 3. RMSE of $\hat{A}_1$ versus SNR when $N = 64$.

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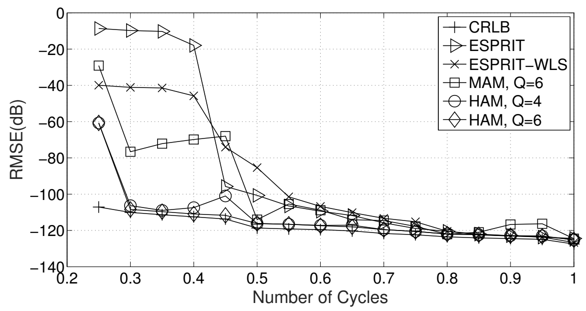

Fig. 4. RMSE of $\hat{f}_0$ versus the number of cycles when SNR = 60dB.

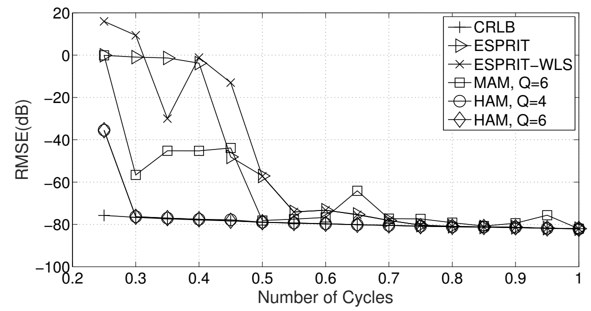

Fig. 6. RMSE of $\hat{A}_1$ versus the number of cycles when SNR = 60dB.

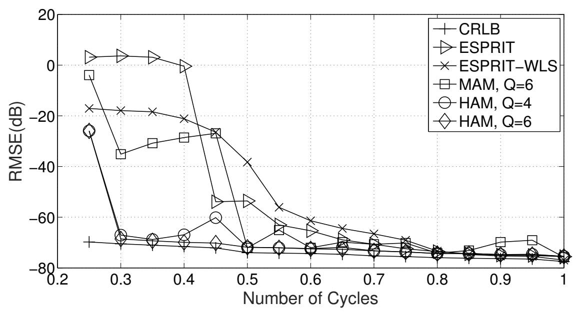

Fig. 5. RMSE of $\hat{\phi}$ versus the number of cycles when SNR = 60dB.

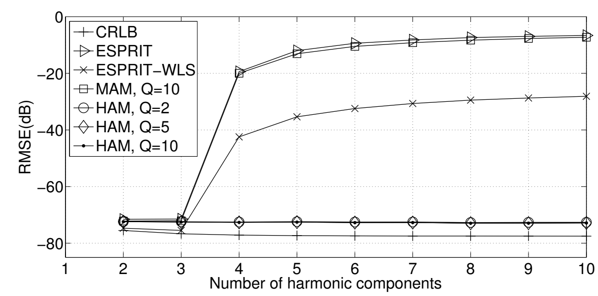

Fig. 7. RMSE of $\hat{f}$ versus the number of harmonic components.

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