Improving Physical Layer Security in DF Relay Networks via Two–Stage Cooperative Jamming

Nicholas Kolokotronis*† and Manos Athanasakos*

*Department of Informatics and Telecommunications
University of Peloponnese, 22100 Tripolis, Greece
Emails: {nkolok, mathan}@uop.gr

†Computer Technology Institute and Press
Diophantus (CTI), Patras, Greece

***Abstract*—The design of a cooperative protocol relying on both cooperative relaying and jamming in order to provide security at the physical layer of wireless communications is considered in this paper. We suppose that pair of nodes is assisted by a number of helpers in their communication, which either relay information or cause harmful interference to an eavesdropper, at both stages of the relaying protocol. Instead of maximizing the secrecy capacity, a signal–to–noise ratio based approach is taken. Solutions for the optimal weights used at each protocol and stage are sought along with the optimal power distribution. To solve this problem, tools from semi–definite and geometric programming are utilized, and an iterative algorithm is proposed. Simulations show noticeable gains (up to 50dB) compared to the non–cooperative case.**

***Index Terms*—Physical layer security, cooperative transmission protocols, cooperative jamming, optimization, wireless networks.**

I. Introduction

Physical layer security has received considerable attention in the past few years. The goal is to exploit the characteristics of the wireless medium to enable legitimate nodes communicate securely in the presence of eavesdroppers that could intercept transmissions due to the broadcast nature of wireless networks. The main metric of interest is the so–called *secrecy capacity* [1], which is defined as the maximum achievable secrecy rate, that is, the rate at which information may be transmitted with perfect secrecy from the source to the destination. Therefore, the secrecy rate controls the communication rate of a source–destination pair so that their transmissions are perfectly secure. In single antenna systems, the secrecy capacity is positive only when the source–eavesdropper's channel is worse; otherwise, it is zero. The use of cooperative protocols, like *decode–and–forward* (DF), *cooperative jamming* (CJ), and *amplify–and–forward* (AF) help to overcome such a limitation [5], [14]. In most cases, the availability of global *channel state information* (CSI) is assumed [9], [14], [17]. Deriving the optimal helper weights in closed form for a single eavesdropper is in general not easy and becomes quite hard to solve when the number of eavesdroppers increases, or when the cooperative protocols are coupled with other schemes, like partner selection [17].

In this paper we consider a cooperative protocol that jointly employs the DF and CJ schemes, referred to as DFCJ. There exists a large number of works to study DFCJ–based protocols, which could be classified according to whether they perform jamming at both stages of cooperative relaying [6], [9], [10], [16], [17], or a single stage [3], [8], [11], [12], [19], [20] where the vast majority of the works is considering the second stage ([3] being the exception). Our work assumes jamming at both stages of the DF protocol, but unlike [6], [9], [10], [16], [17] it assumes the existence of a direct link between the source and the destination; another difference with all the aforementioned works is that we aim at maximizing the *signal–to–noise* (SNR) difference between the source and the eavesdropper, as this can be proved to be more practical [13], e.g. when a closed–form expression for the capacity of a particular channel is unknown or quite complex to work with. Techniques from *semi–definite programming* (SDP) relaxation are used to prove the optimal jamming weights, subject to power, and nulling constraints at the destination. An iterative algorithm, that relies on *geometric programming* (GP), is designed to compute the optimal relay weights and power allocation. The simulation results illustrate the superiority of the proposed scheme.

The paper is organized as follows. Section II introduces the notation, provides the background, and defines the cooperative scheme that is investigated. The main results are presented in Section III, where the relay weights and the powers to allocate for each protocol are determined either in closed–form, or by the proposed algorithm. The simulation results and concluding remarks are given in Sections IV and V respectively.

II. System model

Let us consider a wireless network where a pair of nodes, the source S and the destination D, need to communicate in a secure manner in the presence of a passive adversary E that is simply eavesdropping the information exchanged by these two nodes. We further assume the presence of m helping nodes that are assisting the source by either causing severe interference to the eavesdropper, or by relaying messages to their destination. We next assume that the number of jammers J_j, $j = 1, \ldots, l$, and relays R_i, $i = 1, \ldots, n$, are fixed $(m = n + l)$, whereas the helpers and the source are cooperating by means of the DF and CJ protocols. The number m of helpers, the role of each node, and the cooperative protocols being used are assumed to be public information. All nodes are operating in half–duplex mode and are equipped with a single omni–directional antenna. The above setup is illustrated in Fig. 1.

Global CSI is assumed to be available at the trusted nodes so that to allow for efficient cooperation [5], [14]; this implies that not only are the channel gains of the links between the trusted nodes considered to be known, but also those corresponding to the eavesdropper. This assumption is common in practice, and can be used to model *honest–but–curious* nodes (e.g. untrusted

This work was supported by the research project HANDiCAMS, which acknowledges the financial support of the Future and Emerging Technologies (FET) programme, within the 7th Framework Programme for Research of the European Commission, under FET–Open grant no. 323944.

978-0-9928-6265-7/16/$31.00 ©2016 IEEE

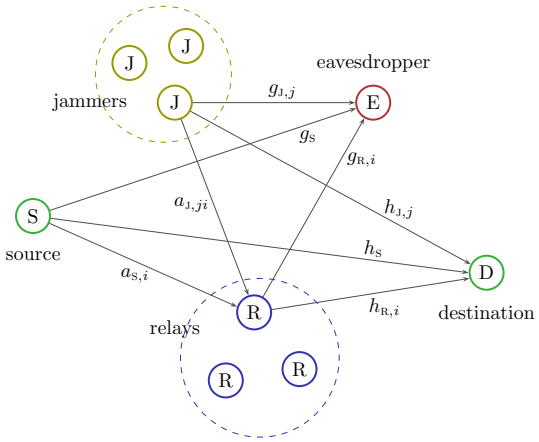

Fig. 1. The network model; the source is assisted by trusted nodes in order to communicate securely with the destination in the presence of an eavesdropper.

relays, etc.) [1]. Next, we let h_{S}^*, $h_{\mathsf{R},i}^*$ and $h_{\mathsf{J},j}^*$ (resp. g_{S}^*, $g_{\mathsf{R},i}^*$ and $g_{\mathsf{J},j}^*$) be the baseband complex channel gains between the source, the ith relay, the jth jammer and the destination (resp. eavesdropper), for $i = 1, \ldots, n$ and $j = 1, \ldots, l$; likewise, we use $a_{\mathsf{S},i}^*$ or $a_{\mathsf{J},ji}^*$ for the channel gain between the source or the jth jammer and the ith relay. Furthermore, all the channels are assumed to undergo flat fading.

Suppose that P equals the total power budget available for transmitting a symbol x, with $\mathbb{E}\big[|x|^2\big] = 1$, from the source to the destination. If the source transmits x with power $P_{\mathsf{S}} = P$, the signal at the destination and the eavesdropper is

$$y_{\mathsf{D}} = \sqrt{P_{\mathsf{S}}}\, h_{\mathsf{S}}^* x + \eta_{\mathsf{D}} \tag{1a}$$

$$y_{\mathsf{E}} = \sqrt{P_{\mathsf{S}}}\, g_{\mathsf{S}}^* x + \eta_{\mathsf{E}} \tag{1b}$$

where $\eta_{\mathsf{D}}, \eta_{\mathsf{E}}$ represent the noise at the receiver and follow the circularly symmetric complex Gaussian distribution $\mathcal{N}(0, \sigma^2)$, with mean 0 and variance σ^2. The signal received at a relay is similarly given by $y_{\mathsf{R},i} = \sqrt{P_{\mathsf{S}}}\, a_{\mathsf{S},i}^* x + \eta_{\mathsf{R},i}$. These expressions correspond to the case of *direct transmission* (DT).

The cooperative scheme. In the sequel, letters in boldface are column vectors $\boldsymbol{x}$ if lowercase, or matrices $\boldsymbol{X}$ otherwise. The conjugate and conjugate transpose are denoted as $\boldsymbol{x}^*$ and $\boldsymbol{x}^\dagger$, whereas $\|\boldsymbol{x}\|^2 = \boldsymbol{x}^\dagger \boldsymbol{x}$. The notation $\boldsymbol{X} \succ 0$ (resp. $\boldsymbol{X} \succcurlyeq 0$) is used for positive definite (resp. semi–definite) matrices and $\boldsymbol{I}_n$ for the identity matrix of order n.

The helpers cooperate with the source using a combination of the DF and the CJ protocols; hence, the cooperative scheme described next is divided in two phases. During the first phase, source node S broadcasts the signal x by using power P_{S}; since this transmission could be intercepted by the eavesdropper, the friendly jammers simultaneously broadcast a weighted version of a jamming signal $z^{(1)}$; to be more precise, the jth jammer J_j transmits $u_j z^{(1)}$, $u_j \in \mathbb{C}$. Let the power used by the jammers during the first phase be equal to $P_{\mathsf{J},1}$. The jamming signal $z^{(1)}$ is assumed to be known among the jammers, and independent from the transmitted signal and the channel's noise; likewise, we assume that $\mathbb{E}\big[|z^{(1)}|^2\big] = 1$. Let $\boldsymbol{y}_{\mathsf{R}} = (y_{\mathsf{R},1} \ \cdots \ y_{\mathsf{R},n})^T$; the signals at the receiving nodes are given by

$$y_{\mathsf{D}}^{(1)} = \sqrt{P_{\mathsf{S}}}\, h_{\mathsf{S}}^* x + \sqrt{P_{\mathsf{J},1}}\, \boldsymbol{h}_{\mathsf{J}}^\dagger \boldsymbol{u}\, z^{(1)} + \eta_{\mathsf{D}}^{(1)} \tag{2a}$$

$$y_{\mathsf{E}}^{(1)} = \sqrt{P_{\mathsf{S}}}\, g_{\mathsf{S}}^* x + \sqrt{P_{\mathsf{J},1}}\, \boldsymbol{g}_{\mathsf{J}}^\dagger \boldsymbol{u}\, z^{(1)} + \eta_{\mathsf{E}}^{(1)} \tag{2b}$$

$$\boldsymbol{y}_{\mathsf{R}} = \sqrt{P_{\mathsf{S}}}\, \boldsymbol{a}_{\mathsf{S}}^* x + \sqrt{P_{\mathsf{J},1}}\, \boldsymbol{A}_{\mathsf{J}}^\dagger \boldsymbol{u}\, z^{(1)} + \boldsymbol{\eta}_{\mathsf{R}} \tag{2c}$$

where $\boldsymbol{A}_{\mathsf{J}}$ is the $l \times n$ matrix $(a_{\mathsf{J},ji})_{j,i}$ and $\boldsymbol{u} = (u_1 \ \cdots \ u_l)^T$ satisfies $\|\boldsymbol{u}\| = 1$. Furthermore, the column vectors $\boldsymbol{a}_{\mathsf{S}}$ and $\boldsymbol{\eta}_{\mathsf{R}}$ of length n are comprised of the channel gains of the source–relays' links and the errors at the relays; the vectors $\boldsymbol{h}_{\mathsf{J}}, \boldsymbol{g}_{\mathsf{J}}$ of length l are similarly defined. The SNR at the ith relay is

$$\gamma_{\mathsf{R},i} = \frac{P_{\mathsf{S}}\, |a_{\mathsf{S},i}|^2}{\sigma^2 + P_{\mathsf{J},1}\, \boldsymbol{u}^\dagger \boldsymbol{R}_{\mathsf{JR},i} \boldsymbol{u}} \tag{3}$$

where $\boldsymbol{R}_{\mathsf{JR},i} \succcurlyeq 0$ is the $l \times l$ matrix $\boldsymbol{a}_{\mathsf{J},i} \boldsymbol{a}_{\mathsf{J},i}^\dagger$ whose rank equals one, and $\boldsymbol{a}_{\mathsf{J},i} = (a_{\mathsf{J},1i} \ \cdots \ a_{\mathsf{J},li})^T$.

Both relays and jammers participate in the second phase; to be more precise, only the relays that have successfully decoded the signal x participate in the cooperative scheme. Each relay re–encodes x and transmits the weighted version $w_i x$ (for the ith relay), $w_i \in \mathbb{C}$, to the destination. Just like the first phase, the set of jammers simultaneously sends the weighted version of a signal $z^{(2)}$ that is independent of $z^{(1)}$ and $\eta_{\mathsf{D}}^{(1)}, \eta_{\mathsf{E}}^{(1)}, \eta_{\mathsf{D}}^{(2)}, \eta_{\mathsf{E}}^{(2)}$, with the jth jammer transmitting $v_j z^{(2)}$, $v_j \in \mathbb{C}$. The vectors $\boldsymbol{w} = (w_1 \ \cdots \ w_n)^T$ and $\boldsymbol{v} = (v_1 \ \cdots \ v_l)^T$ are also assumed to have unit norm. If $P_{\mathsf{R}}, P_{\mathsf{J},2}$ are the powers used for relaying and jamming in this phase respectively, then the signal at the destination and the eavesdropper is

$$y_{\mathsf{D}}^{(2)} = \sqrt{P_{\mathsf{R}}}\, \boldsymbol{h}_{\mathsf{R}}^\dagger \boldsymbol{w} x + \sqrt{P_{\mathsf{J},2}}\, \boldsymbol{h}_{\mathsf{J}}^\dagger \boldsymbol{v}\, z^{(2)} + \eta_{\mathsf{D}}^{(2)} \tag{4a}$$

$$y_{\mathsf{E}}^{(2)} = \sqrt{P_{\mathsf{R}}}\, \boldsymbol{g}_{\mathsf{R}}^\dagger \boldsymbol{w} x + \sqrt{P_{\mathsf{J},2}}\, \boldsymbol{g}_{\mathsf{J}}^\dagger \boldsymbol{v}\, z^{(2)} + \eta_{\mathsf{E}}^{(2)} \tag{4b}$$

where the sum of the powers used in both phases should not exceed the total power budget P; the power that is devoted to jamming in the above cooperation scheme is $P_{\mathsf{J}} = P_{\mathsf{J},1} + P_{\mathsf{J},2}$. As the destination and the eavesdropper receive transmissions of x in both phases of the protocol, by using a strategy known as *maximal ratio combining* (MRC) [7], they may achieve the following SNR values

$$\gamma_{\mathsf{D}} = \frac{P_{\mathsf{S}}\, |h_{\mathsf{S}}|^2}{\sigma^2 + P_{\mathsf{J},1}\, \boldsymbol{u}^\dagger \boldsymbol{R}_{\mathsf{JD}} \boldsymbol{u}} + \frac{P_{\mathsf{R}}\, \boldsymbol{w}^\dagger \boldsymbol{R}_{\mathsf{RD}} \boldsymbol{w}}{\sigma^2 + P_{\mathsf{J},2}\, \boldsymbol{v}^\dagger \boldsymbol{R}_{\mathsf{JD}} \boldsymbol{v}} \tag{5a}$$

$$\gamma_{\mathsf{E}} = \frac{P_{\mathsf{S}}\, |g_{\mathsf{S}}|^2}{\sigma^2 + P_{\mathsf{J},1}\, \boldsymbol{u}^\dagger \boldsymbol{R}_{\mathsf{JE}} \boldsymbol{u}} + \frac{P_{\mathsf{R}}\, \boldsymbol{w}^\dagger \boldsymbol{R}_{\mathsf{RE}} \boldsymbol{w}}{\sigma^2 + P_{\mathsf{J},2}\, \boldsymbol{v}^\dagger \boldsymbol{R}_{\mathsf{JE}} \boldsymbol{v}} \tag{5b}$$

where the Hermitian matrices $\boldsymbol{R}_{\mathsf{RD}} = \boldsymbol{h}_{\mathsf{R}} \boldsymbol{h}_{\mathsf{R}}^\dagger$ and $\boldsymbol{R}_{\mathsf{JD}} = \boldsymbol{h}_{\mathsf{J}} \boldsymbol{h}_{\mathsf{J}}^\dagger$ (those of the eavesdropper $\boldsymbol{R}_{\mathsf{RE}}$ and $\boldsymbol{R}_{\mathsf{JE}}$ are similarly defined) are positive semi–definite of rank one. Moreover, for the shake of simplicity, we assume $\eta_{\mathsf{D}}^{(1)}, \eta_{\mathsf{E}}^{(1)}, \eta_{\mathsf{D}}^{(2)}, \eta_{\mathsf{E}}^{(2)} \sim \mathcal{N}(0, \sigma^2)$.

Instead of maximizing the secrecy capacity, our objective is to maximize the difference $\Delta\Gamma = \Gamma_{\mathsf{D}} - \Gamma_{\mathsf{E}}$, which is referred to as *security gap* [13], in the SNRs between the destination and the eavesdropper; the terms $\Gamma_{\mathsf{D}}, \Gamma_{\mathsf{E}}$ are equal to the values of $\gamma_{\mathsf{D}}, \gamma_{\mathsf{E}}$ in dB. Due to the use of the DF protocol in the above scheme, we need to assure that the relays can correctly decode the signals in (2c) they receive during the first phase. This is achieved if the rate at each relay is no less than the rate at the destination [5]; this in turn implies that the secrecy rate does not exceed the minimum rate at the relays [14]. Hence, from the above, we get the optimization problem

$$\delta^\star = \max_{\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{u}, \boldsymbol{v}} \quad \gamma_{\mathsf{D}} \big/ \gamma_{\mathsf{E}} \tag{P1}$$

$$\text{s.t.} \quad P_{\mathsf{S}} + P_{\mathsf{R}} + P_{\mathsf{J},1} + P_{\mathsf{J},2} = P \tag{C1}$$

$$\boldsymbol{w}^{\dagger}\boldsymbol{w} = 1,\ \boldsymbol{u}^{\dagger}\boldsymbol{u} = 1,\ \boldsymbol{v}^{\dagger}\boldsymbol{v} = 1 \tag{C2}$$

$$\gamma_{\mathsf{R},i} \geqslant \gamma_{\mathsf{D}},\ \forall i = 1, \ldots, n \tag{C3}$$

with $\boldsymbol{p} = (P_{\mathsf{S}}\ P_{\mathsf{R}}\ P_{\mathsf{J},1}\ P_{\mathsf{J},2})^T$. In its full generality, (P1) is not easy to solve as it is highly nonconvex and, due to the use of the DF protocol, the decoding constraints are coupled with the objective function.

III. Security gap maximization strategy

In order to get either closed–form, or efficiently computable, solutions of the optimization problem (P1), we introduce new constraints; these concern the cooperative jamming protocol in both phases of the scheme. In particular, we next require that the interference caused by the jammers does not decrease the value of the SNR γ_{D} at the destination. According to (5a), this is achieved by adding the constraints

$$\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{u} = 0,\ \boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{v} = 0 \tag{C4}$$

to the problem (P1). The advantage of doing so is that we can directly determine the optimal solution for the jamming vector used during the second phase of the scheme.

Proposition 1. *The optimal vector $\boldsymbol{v}^{\star}$ of the problem* (P1)*, s.t.* (C1)–(C4)*, is given by*

$$\boldsymbol{v}^{\star} = \frac{\|\boldsymbol{h}_{\mathsf{J}}\|^2 \boldsymbol{g}_{\mathsf{J}} - (\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{g}_{\mathsf{J}})\boldsymbol{h}_{\mathsf{J}}}{\|\boldsymbol{h}_{\mathsf{J}}\|\sqrt{\|\boldsymbol{h}_{\mathsf{J}}\|^2\|\boldsymbol{g}_{\mathsf{J}}\|^2 - |\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{g}_{\mathsf{J}}|^2}} \tag{6}$$

and let $\beta = |\boldsymbol{g}_{\mathsf{J}}^{\dagger}\boldsymbol{v}^{\star}|^2$.

Proof: Application of the nulling constraints (C4) to the objective function of (P1) yields that the optimal value of (P1), s.t. (C1)–(C4), is equivalently determined via

$$\max_{\boldsymbol{p},\boldsymbol{w},\boldsymbol{u}} \frac{P_{\mathsf{S}}|h_{\mathsf{S}}|^2 + P_{\mathsf{R}}\boldsymbol{w}^{\dagger}\boldsymbol{R}_{\mathsf{RD}}\boldsymbol{w}}{\dfrac{P_{\mathsf{S}}|g_{\mathsf{S}}|^2}{1 + \frac{P_{\mathsf{J},1}}{\sigma^2}\boldsymbol{u}^{\dagger}\boldsymbol{R}_{\mathsf{JE}}\boldsymbol{u}} + \dfrac{P_{\mathsf{R}}\boldsymbol{w}^{\dagger}\boldsymbol{R}_{\mathsf{RE}}\boldsymbol{w}}{1 + \frac{P_{\mathsf{J},2}}{\sigma^2}\beta}} \tag{P2}$$

according to (5); the value of $\beta \in \mathbb{R}^+$ is nonzero and it equals $\beta = \max_{\boldsymbol{v}} \boldsymbol{v}^{\dagger}\boldsymbol{R}_{\mathsf{JE}}\boldsymbol{v}$, s.t. $\boldsymbol{v}^{\dagger}\boldsymbol{v} = 1$ and $\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{v} = 0$. The solution to the latter problem is the unit–norm vector corresponding to the vector $\widetilde{\boldsymbol{v}} = \|\boldsymbol{h}_{\mathsf{J}}\|^2\boldsymbol{g}_{\mathsf{J}} - (\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{g}_{\mathsf{J}})\boldsymbol{h}_{\mathsf{J}}$ [5]. The fact that the norm of $\widetilde{\boldsymbol{v}}$ equals the denominator of (6) completes the proof. ∎

The optimization variable $\boldsymbol{u}$ not only exists in the objective function of (P2), but also in the decoding constraints (C3) due to (3). As a consequence, the problem (P2) does not allow to independently compute the optimal value of $\boldsymbol{u}$ as well. In order to overcome this problem we define the non–negative constant $c \in \mathbb{R}^+$ and impose the constraints

$$\boldsymbol{u}^{\dagger}\boldsymbol{R}_{\mathsf{JR},i}\boldsymbol{u} \leqslant c^2,\ \forall i = 1, \ldots, n \tag{C5}$$

to the problem (P2) that bound the interference caused to the relays during the first phase of the scheme. If $b = \min_i |a_{\mathsf{S},i}|$, then from (3), (5a), (C4), (C5) we see that the SNR at the ith relay satisfies the inequality $\gamma_{\mathsf{R},i} \geqslant \widetilde{\gamma}_{\mathsf{R}} = P_{\mathsf{S}}b^2/(\sigma^2 + P_{\mathsf{J},1}c^2)$. Similarly, due to (C2), we have that the SNR at the destination satisfies $\gamma_{\mathsf{D}} \leqslant \widetilde{\gamma}_{\mathsf{D}} = (P_{\mathsf{S}}|h_{\mathsf{S}}|^2 + P_{\mathsf{R}}\|\boldsymbol{h}_{\mathsf{R}}\|^2)/\sigma^2$. Hence, instead of (C3) we subsequently impose the constraint $\widetilde{\gamma}_{\mathsf{R}} \geqslant \widetilde{\gamma}_{\mathsf{D}}$ so that correct decoding at the relays is achieved. It is straightforward to verify that the latter inequality leads to

$$\theta_1 P_{\mathsf{J},1} + \theta_2 P_{\mathsf{R}} P_{\mathsf{S}}^{-1} + P_{\mathsf{R}} P_{\mathsf{J},1} P_{\mathsf{S}}^{-1} \leqslant \theta_3 \tag{C3$'$}$$

where the coefficients are $\theta_1 = |h_{\mathsf{S}}|^2/\|\boldsymbol{h}_{\mathsf{R}}\|^2$, $\theta_2 = \sigma^2/c^2$, and $\theta_3 = \theta_1\theta_2\,(b^2/|h_{\mathsf{S}}|^2 - 1)$, which are all positive when $b > |h_{\mathsf{S}}|$ i.e. if the relays are closer to the source than the destination; we also define the vector $\boldsymbol{\theta} = (\theta_1\ \theta_2\ \theta_3)$. The above allows us to determine the optimal jamming vector that is used during the first phase of the scheme, under certain conditions.

Proposition 2. *Let $l > n + 1$. Then, the optimal vector $\boldsymbol{u}^{\star}$ of the problem* (P1)*, s.t.* (C1)–(C5)*, is the rank–one solution that is obtained from the SDP problem*

$$\begin{aligned} \max_{\boldsymbol{U} \succcurlyeq 0} \quad & \operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JE}}\boldsymbol{U}\big) \\ \text{s.t.} \quad & \operatorname{tr}\big(\boldsymbol{U}\big) = 1 \\ & \operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JR},i}\boldsymbol{U}\big) \leqslant c^2,\ \forall i = 1, \ldots, n \\ & \operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JD}}\boldsymbol{U}\big) = 0 \end{aligned} \tag{7}$$

where $\operatorname{tr}(\cdot)$ is the trace function; moreover, let $\alpha = |\boldsymbol{g}_{\mathsf{J}}^{\dagger}\boldsymbol{u}^{\star}|^2$.

Proof: Applying all the constraints to (P1) we get that its optimal value, s.t. (C1)–(C5), is equivalently determined via

$$\max_{\boldsymbol{p},\boldsymbol{w}} \frac{P_{\mathsf{S}}|h_{\mathsf{S}}|^2 + P_{\mathsf{R}}\boldsymbol{w}^{\dagger}\boldsymbol{R}_{\mathsf{RD}}\boldsymbol{w}}{\dfrac{P_{\mathsf{S}}|g_{\mathsf{S}}|^2}{1 + \frac{P_{\mathsf{J},1}}{\sigma^2}\alpha} + \dfrac{P_{\mathsf{R}}\boldsymbol{w}^{\dagger}\boldsymbol{R}_{\mathsf{RE}}\boldsymbol{w}}{1 + \frac{P_{\mathsf{J},2}}{\sigma^2}\beta}} \tag{P3}$$

according to (5), where β is given by Proposition 1; the value of $\alpha \in \mathbb{R}^+$ is nonzero and is equal to $\alpha = \max_{\boldsymbol{u}} \boldsymbol{u}^{\dagger}\boldsymbol{R}_{\mathsf{JE}}\boldsymbol{u}$, s.t. $\boldsymbol{u}^{\dagger}\boldsymbol{u} = 1$, $\boldsymbol{h}_{\mathsf{J}}^{\dagger}\boldsymbol{u} = 0$, and (C5). By defining the rank–one matrix $\boldsymbol{U} = \boldsymbol{u}\boldsymbol{u}^{\dagger}$, we transform the latter problem into its equivalent relaxed SDP form (7) by dropping the $\operatorname{rank}(\boldsymbol{U}) = 1$ constraint [18]. In general, the optimal solution $\boldsymbol{U}^{\star}$ of (7) does not have rank one; however, we next prove (using an approach similar to [15]) that if $l > n + 1$, we necessarily have $\operatorname{rank}(\boldsymbol{U}^{\star}) = 1$. The Lagrangian of the SDP problem is

$$\begin{aligned} \mathcal{L}(\boldsymbol{U}, \boldsymbol{V}, \boldsymbol{\lambda}) = & -\operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JE}}\boldsymbol{U}\big) - \operatorname{tr}\big(\boldsymbol{V}\boldsymbol{U}\big) + \lambda_0\Big(\operatorname{tr}\big(\boldsymbol{U}\big) - 1\Big) \\ & + \sum_{i=1}^{n} \lambda_i\Big(\operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JR},i}\boldsymbol{U}\big) - c^2\Big) + \lambda_{n+1}\operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JD}}\boldsymbol{U}\big) \end{aligned}$$

where $\boldsymbol{V} \succcurlyeq 0$ and $\boldsymbol{\lambda} = (\lambda_0\ \cdots\ \lambda_{n+1})$ are the dual variables. As optimal solutions should satisfy the *Karush–Kuhn–Tucker* (KKT) conditions [2], the equation $\partial\mathcal{L}/\partial\boldsymbol{U} = \boldsymbol{0}$ gives

$$\boldsymbol{V} = -\boldsymbol{R}_{\mathsf{JE}} + \lambda_0\boldsymbol{I}_l + \sum_{i=1}^{n} \lambda_i\boldsymbol{R}_{\mathsf{JR},i} + \lambda_{n+1}\boldsymbol{R}_{\mathsf{JD}} \tag{8}$$

where $\lambda_0, \lambda_{n+1} \neq 0$ and $\lambda_1, \ldots, \lambda_n \geqslant 0$. Let us first assume that $\lambda_0 < 0$; then, $\lambda_0\boldsymbol{I}_l - \boldsymbol{R}_{\mathsf{JE}}$ has full rank, and in particular we have $\lambda_0\boldsymbol{I}_l - \boldsymbol{R}_{\mathsf{JE}} \prec 0$. The fact that $l > n + 1$ implies that the Lagrange multipliers must satisfy $\boldsymbol{V} \preccurlyeq 0$ ($\boldsymbol{V} \neq \boldsymbol{0}$) by (8); this however contradicts $\boldsymbol{V} \succcurlyeq 0$, and hence it must be $\lambda_0 > 0$. Furthermore, it should also be $\lambda_{n+1} > 0$; indeed, the SDP (7) is equivalent to the one obtained if the last constraint changes to $\operatorname{tr}\big(\boldsymbol{R}_{\mathsf{JD}}\boldsymbol{U}\big) \leqslant 0$ since $\boldsymbol{R}_{\mathsf{JD}}, \boldsymbol{U} \succcurlyeq 0$. The above imply that

$$l = \operatorname{rank}\bigg(\lambda_0\boldsymbol{I}_l + \sum_{i=1}^{n} \lambda_i\boldsymbol{R}_{\mathsf{JR},i} + \lambda_{n+1}\boldsymbol{R}_{\mathsf{JD}}\bigg)$$

$$= \operatorname{rank}\big(\boldsymbol{V} + \boldsymbol{R}_{\mathsf{JE}}\big) \leqslant \operatorname{rank}\big(\boldsymbol{V}\big) + \operatorname{rank}\big(\boldsymbol{R}_{\mathsf{JE}}\big)$$
$$= \operatorname{rank}\big(\boldsymbol{V}\big) + 1$$

and hence, the rank of $\boldsymbol{V}$ is either l or $l-1$. From the KKT conditions, any optimal solution should also satisfy $\boldsymbol{V}\boldsymbol{U} = \boldsymbol{0}$; thus, $\operatorname{rank}\big(\boldsymbol{V}\big) \neq l$ since otherwise $\boldsymbol{U} = \boldsymbol{0}$ would be obtained as the optimal solution. By the fact that $\operatorname{rank}\big(\boldsymbol{V}\big) = l-1$ and that $\boldsymbol{U}$ lies in the null space of $\boldsymbol{V}$, we eventually obtain the desired result $\operatorname{rank}\big(\boldsymbol{U}\big) = 1$. ∎

Corollary 1. *With the notation of Proposition 2, assume that $c = 0$; then, the optimal vector $\boldsymbol{u}^\star$ is given by*

$$\boldsymbol{u}^\star = \frac{\big(\boldsymbol{I}_l - \boldsymbol{B}_{\mathsf{J}}\boldsymbol{B}_{\mathsf{J}}^{\#}\big)\,\boldsymbol{g}_{\mathsf{J}}}{\big\|\big(\boldsymbol{I}_l - \boldsymbol{B}_{\mathsf{J}}\boldsymbol{B}_{\mathsf{J}}^{\#}\big)\,\boldsymbol{g}_{\mathsf{J}}\big\|} \tag{9}$$

where $\boldsymbol{B}_{\mathsf{J}} = (\boldsymbol{h}_{\mathsf{J}}\ \boldsymbol{A}_{\mathsf{J}})$ and $\boldsymbol{B}_{\mathsf{J}}^{\#} = (\boldsymbol{B}_{\mathsf{J}}^\dagger\boldsymbol{B}_{\mathsf{J}})^{-1}\boldsymbol{B}_{\mathsf{J}}^\dagger$ is the Moore–Penrose pseudoinverse of $\boldsymbol{B}_{\mathsf{J}}$.

Proof: If $c = 0$, then (C4), (C5) suggest that it should be $\boldsymbol{B}_{\mathsf{J}}^\dagger\boldsymbol{u} = \boldsymbol{0}$, for the $l \times n+1$ matrix $\boldsymbol{B}_{\mathsf{J}} = (\boldsymbol{h}_{\mathsf{J}}\ \boldsymbol{A}_{\mathsf{J}})$. Then, (9) is known to be the optimal solution —*see* e.g. [5], [13]. ∎

In order to prove the results of Proposition 2 and Corollary 1 we implicitly assumed that $\operatorname{rank}(\boldsymbol{B}_{\mathsf{J}}) = n+1$ (recall that we had $l > n+1$), or that all the decoding constraints in (C5) do contribute in shaping the feasibility set of (P1) —i.e. they do not trivially hold. As an example, if we take $c = \max_i \|\boldsymbol{a}_{\mathsf{J},i}\|$ none of the constraints in (C5) contributes, and then we obtain $\boldsymbol{u}^\star = \boldsymbol{v}^\star$. The relays being far away from the jammers are not affected to the same extent as those being close enough (in the former case the magnitude of $\|\boldsymbol{a}_{\mathsf{J},i}\|$ is close to zero). Hence, if $l \leqslant n+1$, we can set $k = n-l+2$ and take c as the kth largest norm amongst the columns of $\boldsymbol{A}_{\mathsf{J}}$; this is next denoted as $c = \max_i^{(k)} \|\boldsymbol{a}_{\mathsf{J},i}\|$. We can then apply Proposition 2, but with $n' = n-k$ instead of n.

As a result of Propositions 1, 2, after having computed the values of α, β, we end–up with the optimization problem (P3), where the remaining constraints are (C1), (C3′), and $\boldsymbol{w}^\dagger\boldsymbol{w} = 1$ from (C2). An efficient way to solve this problem is to design an iterative algorithm computing the relay weights and power allocation independently and in an alternating fashion [5], [13], [14] —*see* Alg. 1. These steps are further studied below.

Relay weights design. For a given power allocation vector $\boldsymbol{p}$, (P3), s.t. $\boldsymbol{w}^\dagger\boldsymbol{w} = 1$, becomes a generalized Rayleigh quotient

$$\max_{\boldsymbol{w}\neq\boldsymbol{0}} \frac{\boldsymbol{w}^\dagger\widetilde{\boldsymbol{R}}_{\mathsf{RD}}\boldsymbol{w}}{\boldsymbol{w}^\dagger\widetilde{\boldsymbol{R}}_{\mathsf{RE}}\boldsymbol{w}} \quad \text{s.t.} \quad \boldsymbol{w}^\dagger\boldsymbol{w} = 1 \tag{P4a}$$

where $\widetilde{\boldsymbol{R}}_{\mathsf{RD}}, \widetilde{\boldsymbol{R}}_{\mathsf{RE}} \succ 0$, with $\widetilde{\boldsymbol{R}}_{\mathsf{RD}} = P_{\mathsf{S}}\,|h_{\mathsf{S}}|^2\,\boldsymbol{I}_n + P_{\mathsf{R}}\,\boldsymbol{R}_{\mathsf{RD}}$ and

$$\widetilde{\boldsymbol{R}}_{\mathsf{RE}} = \frac{P_{\mathsf{S}}\,|g_{\mathsf{S}}|^2}{1 + \frac{P_{\mathsf{J},1}}{\sigma^2}\,\alpha}\,\boldsymbol{I}_n + \frac{P_{\mathsf{R}}}{1 + \frac{P_{\mathsf{J},2}}{\sigma^2}\,\beta}\,\boldsymbol{R}_{\mathsf{RE}}\,.$$

Note that we can drop the norm constraint in (P4a), since the objective function's value is independent of $\|\boldsymbol{w}\|$. It is known that the optimal value of (P4a) is the largest eigenvalue of the matrix $\widetilde{\boldsymbol{R}}_{\mathsf{RE}}^{-1}\widetilde{\boldsymbol{R}}_{\mathsf{RD}}$. Furthermore, the optimal solution $\boldsymbol{w}^\star$ is the unit–norm vector associated with the largest eigenvalue, which is next denoted as $\lambda_{\mathsf{max}}\big(\widetilde{\boldsymbol{R}}_{\mathsf{RE}}^{-1}\widetilde{\boldsymbol{R}}_{\mathsf{RD}}\big)$. The values $\mu = |\boldsymbol{h}_{\mathsf{R}}^\dagger\boldsymbol{w}^\star|^2$ and $\nu = |\boldsymbol{g}_{\mathsf{R}}^\dagger\boldsymbol{w}^\star|^2$ are used below.

ALG. 1 The function DFCJmax( )

input: $h_{\mathsf{S}}, g_{\mathsf{S}}, \boldsymbol{R}_{\mathsf{RD}}, \boldsymbol{R}_{\mathsf{RE}}, \alpha, \beta$
initialization: $\boldsymbol{p}^{(0)} \leftarrow \boldsymbol{p}_0$, $\{k, \mu, \nu, \delta^{(0)}\} \leftarrow 0$, $\delta^{(-1)} \leftarrow 2\varepsilon$

while $|\delta^{(k)} - \delta^{(k-1)}| > \varepsilon$ **do** ▸ tolerance ε
  $k \leftarrow k+1$
  update $\boldsymbol{\zeta}, \boldsymbol{\xi}$ ▸ using μ, ν
  $\boldsymbol{p}^{(k)} \leftarrow \mathsf{CGPsolve}\big(\boldsymbol{p}^{(k-1)}; \boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\theta}\big)$
  update $\widetilde{\boldsymbol{R}}_{\mathsf{RD}}, \widetilde{\boldsymbol{R}}_{\mathsf{RE}}$ ▸ using $\boldsymbol{p}^{(k)}$
  $\boldsymbol{w}^{(k)} \leftarrow \mathsf{GRQsolve}\big(\widetilde{\boldsymbol{R}}_{\mathsf{RD}}, \widetilde{\boldsymbol{R}}_{\mathsf{RE}}\big)$
  update μ, ν ▸ using $\boldsymbol{w}^{(k)}$
  $\delta^{(k)} \leftarrow \gamma_{\mathsf{D}}^{(k)} \big/ \gamma_{\mathsf{E}}^{(k)}$ ▸ from (5)
end

output: $\boldsymbol{p}^\star \leftarrow \boldsymbol{p}^{(k)}$, $\boldsymbol{w}^\star \leftarrow \boldsymbol{w}^{(k)}$, $\delta^\star \leftarrow \delta^{(k)}$

Power allocation. For a given relay weight vector $\boldsymbol{w}$, and the values μ, ν, the problem (P3), s.t. (C1), (C3′), is re–written as the ratio $\zeta(\boldsymbol{p})\big/\xi(\boldsymbol{p})$ of the posynomials $\zeta, \xi : \mathbb{R}^4 \to \mathbb{R}$ [2]

$$\max_{\boldsymbol{p}} \quad \sum_{\boldsymbol{e}\in F^4} \zeta_{\boldsymbol{e}}\,\boldsymbol{p}^{\boldsymbol{e}} \Big/ \sum_{\boldsymbol{e}\in F^4} \xi_{\boldsymbol{e}}\,\boldsymbol{p}^{\boldsymbol{e}} \tag{P4b}$$
$$\text{s.t.} \quad \tfrac{\theta_1}{\theta_3}\,P_{\mathsf{J},1} + \tfrac{\theta_2}{\theta_3}\,P_{\mathsf{R}}P_{\mathsf{S}}^{-1} + \tfrac{1}{\theta_3}\,P_{\mathsf{R}}P_{\mathsf{J},1}P_{\mathsf{S}}^{-1} \leqslant 1$$
$$\tfrac{1}{P}\,P_{\mathsf{S}} + \tfrac{1}{P}\,P_{\mathsf{R}} + \tfrac{1}{P}\,P_{\mathsf{J},1} + \tfrac{1}{P}\,P_{\mathsf{J},2} \leqslant 1$$

where $F = \{0,1\}$, and we define $\boldsymbol{p}^{\boldsymbol{e}} = P_{\mathsf{S}}^{e_1}P_{\mathsf{R}}^{e_2}P_{\mathsf{J},1}^{e_3}P_{\mathsf{J},2}^{e_4}$. The nonzero coefficients of the posynomials are given by

$$\zeta_{1000} = |h_{\mathsf{S}}|^2\sigma^2 \qquad \zeta_{0100} = \mu\sigma^2 \qquad \xi_{1000} = |g_{\mathsf{S}}|^2\sigma^2$$
$$\zeta_{1010} = |h_{\mathsf{S}}|^2\alpha \qquad \zeta_{0110} = \mu\alpha \qquad \xi_{0100} = \nu\sigma^2$$
$$\zeta_{1001} = |h_{\mathsf{S}}|^2\beta \qquad \zeta_{0101} = \mu\beta \qquad \xi_{1001} = |g_{\mathsf{S}}|^2\beta$$
$$\zeta_{1011} = |h_{\mathsf{S}}|^2\alpha\beta\sigma^{-2} \qquad \zeta_{0111} = \mu\alpha\beta\sigma^{-2} \qquad \xi_{0110} = \nu\alpha$$

and they are all positive. It is known that maximizing a ratio of posynomials belongs to the truly nonconvex class of problems referred to as *complementary GP*, which are NP–hard [4]. The problem formulation of (P4b) belongs to this class of problems. A method to transform a complementary GP into a GP (which is convex) has been proposed in [4], and is called the *single condensation method for GP*. The main idea for transforming the problem into GP is to approximate the denominator of the ratio of posynomials with a proper monomial, but leaving the numerator as a posynomial. The following result provides the details of the approximation.

Lemma 1 ([4])**.** *Let $f(\boldsymbol{x}) = \sum_i f_i(\boldsymbol{x})$ be a posynomial. Then*

$$f(\boldsymbol{x}) \geqslant \widetilde{f}(\boldsymbol{x}) = \prod_i \left(\frac{f_i(\boldsymbol{x})}{q_i}\right)^{q_i} \tag{10}$$

If, in addition, $q_i = f_i(\boldsymbol{x}_0)/f(\boldsymbol{x}_0)\ \forall i$, for any fixed positive $\boldsymbol{x}_0$, then $\widetilde{f}(\boldsymbol{x}_0) = f(\boldsymbol{x}_0)$ and $\widetilde{f}(\boldsymbol{x}_0)$ is the best local monomial approximation to $f(\boldsymbol{x}_0)$ near $\boldsymbol{x}_0$ in the sense of the first order Taylor approximation.

Thus, in order to apply Lemma 1, we should solve (P4b) in its equivalent form $\min_{\boldsymbol{p}} \xi(\boldsymbol{p})\big/\zeta(\boldsymbol{p})$, s.t. (C1), (C3′). The above lead to Alg. 1, where $\boldsymbol{p}_0$ is an initial value in the feasibility set. The function GRQsolve computes the optimal relay weights for a given power allocation that defines $\widetilde{\boldsymbol{R}}_{\mathsf{RD}}, \widetilde{\boldsymbol{R}}_{\mathsf{RE}}$, and solves the problem (P4a), whereas the function CGPsolve computes the

optimal power allocation by solving (P4b) using Lemma 1; its details are provided in [4, §IV.C]. After computing the value of the denominator $\zeta(\boldsymbol{p}^{(k-1)})$ and the values of the monomials for which $\zeta_{\boldsymbol{e}} \neq 0$, the terms $q_{\boldsymbol{e}}$ along with the approximation $\widetilde{\zeta}(\boldsymbol{p}^{(k-1)})$ are determined by Lemma 1. The resulting problem $\min_{\boldsymbol{p}} \xi(\boldsymbol{p})/\widetilde{\zeta}(\boldsymbol{p})$, s.t. (C1), (C3′), is then solved with an interior point method.

IV. Simulation results

Throughout the simulations, helping nodes are assumed to be randomly distributed in an 18m–radius disk with the source at its center. The destination is fixed at 30m from the source, and the eavesdropper is moving along the source–destination line. A simple line–of–sight model is considered $h = d^{-\frac{\tau}{2}} e^{j\varphi}$, where d is the receiver's distance, φ the phase offset, whereas $\tau = 4$ is the path loss exponent. The noise variance σ^2 is equal to -40dBm. The performance of the joint DF–CJ scheme is evaluated based on Alg. 1, with a varying number of helpers, distributions of relays/jammers, and an 20dBm power budget. Monte–Carlo simulations are performed, where each setup has been repeated 10^2 times to get average results.

The DF–CJ protocol was compared against the DF and CJ schemes, where all the helpers are relaying (resp. jamming) in the null space of E (resp. D), and was shown to have superior performance. Fig. 2 depicts the result for 16 helpers (4 relays, 12 jammers), where the figure at the bottom shows the power allocated to the source. It can be seen that the DF–CJ protocol gives a higher SNR gap $\Delta\Gamma$ (up to 50dB) than the rest of the protocols while decreasing the helpers' load. Indeed, when the eavesdropper is close to the source, the DF protocol requires almost all the power be allocated to the relays. This drawback is eliminated with the DF–CJ scheme. Simulations have shown that most of the power that is allocated to jamming is utilized in the first phase of DF–CJ, in contrast to [8], [11], [12], [19], [20], with the special case of DF–CJ with $P_{\mathrm{J},2} = 0$ being close enough to the general DF–CJ in terms of performance.

V. Conclusions

The use of the DF and CJ cooperative protocols for securing wireless communications at the physical layer in the presence of an eavesdropper was studied in this paper. The CJ protocol was applied in both stages. Due to the inherent difficulty of the problem, suboptimal solutions are derived regarding the relay weights, as well as the distribution of power amongst relaying and jamming. The initial problem is transformed into a power allocation problem that can be efficiently solved via geometric programming. Ongoing work seeks to yield results by relaxing the assumptions on jamming, and to couple the scheme with a partner selection mechanism.

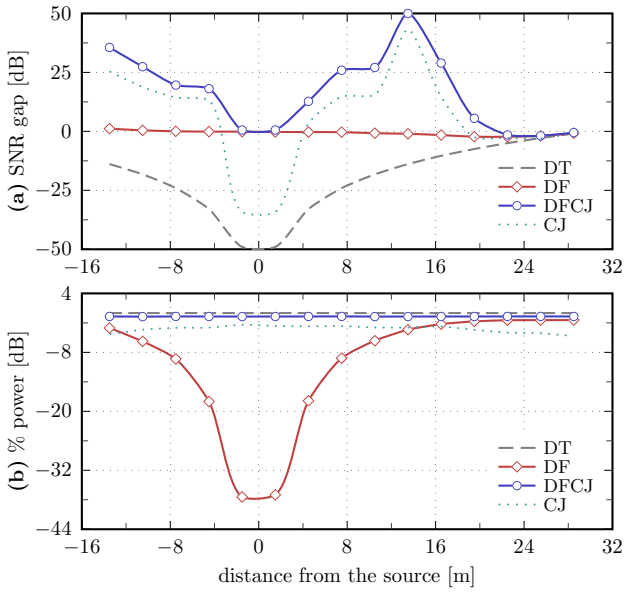

Fig. 2. Comparison of the suboptimal solutions for the DF, CJ, and DFCJ protocols, with 16 helpers (4 relays, 12 jammers) and 20dBm power budget.

References

[1] R. Bassily, E. Ekrem, *et al.*, "Cooperative security at the physical layer," *IEEE Signal Process. Mag.*, vol. 30, no. 5, pp. 16–28, 2013.

[2] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge Univ. Press, 2004.

[3] T. Chen, "Improving physical layer security of cooperative relay networks via destination jamming," *J. Comput. Inform. Syst.*, vol. 9, no. 11, pp. 4231–4238, 2013.

[4] M. Chiang, C.-W. Tan, D. P. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640–2651, 2007.

[5] L. Dong, Z. Han, A. Petropulu, and H. V. Poor, "Improving wireless physical layer security via cooperating relays," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1875–1888, 2010.

[6] L. Dong, H. Yousefi'zadeh, and H. Jafarkhani, "Cooperative jamming and power allocation for wireless relay networks in presence of eavesdropper," in proc. *IEEE ICC '11*, pp. 1–5, 2011.

[7] A. Goldsmith, *Wireless Communications*. Cambridge Univ. Press, 2005.

[8] X. Guan, Y. Cai, Y. Wang, and W. Yang, "Increasing secrecy capacity via joint design of cooperative beamforming and jamming," *KSII Trans. Internet Inform. Systems*, vol. 6, no. 4, pp. 1041–1062, 2012.

[9] J. Huang and A. L. Swindlehurst, "Secure communications via cooperative jamming in two–hop relay systems," in proc. *IEEE GLOBECOM '10*, pp. 1–5, 2010.

[10] J. Huang and A. L. Swindlehurst, "Cooperative jamming for secure communications in MIMO relay networks," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 4871–4884, 2011.

[11] S. Huang, J. Wei, Y. Cao, and C. Liu, "Joint decode–and–forward and cooperative jamming for secure wireless communications," in proc. *7th IEEE WiCOM '11*, pp. 1–4, 2011.

[12] S. Huang, and J. Tan, "Decode–and–forward plus cooperative jamming based cooperation for wireless physical layer security," in proc. *CCIS '13 — Wksp Cloud Comput. Inform. Security*, pp. 290–293, 2013.

[13] N. Kolokotronis, *et al.*, "Cooperation for secure wireless communications with resource–bounded eavesdroppers," in proc. *IEEE GLOBECOM '14 — Wksp Physical Layer Security*, pp. 1483–1488, 2014.

[14] J. Li, A. Petropulu, and S. Weber, "On cooperative relaying schemes for wireless physical layer security," *IEEE Trans. Signal Process.*, vol. 59, pp. 4985–4997, 2011.

[15] Q. Li, Q. Zhang, and J. Qin, "A special class of fractional QCQP and its applications on cognitive collaborative beamforming," *IEEE Trans. Signal Process.*, vol. 62, no. 8, pp. 2151–2164, 2014.

[16] Y. Liu, J. Li, and A. Petropulu, "Destination assisted cooperative jamming for wireless physical–layer security," *IEEE Trans. Inf. Forensics Security*, vol. 8, no. 4, pp. 682–694, 2013.

[17] W. Liu, D. Tan, and G. Xu, "Low complexity power allocation and joint relay–jammer selection in cooperative jamming DF relay wireless secure networks," in proc. *IEEE ASID '13*, pp. 1–5, 2013.

[18] Z.-Q. Luo, W.-K. Ma, A. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, 2010.

[19] L. Tang, X. Gong, J. Wu, and J. Zhang, "Secure wireless communications via cooperative relaying and jamming," in proc. *IEEE GLOBECOM '11 — Wksp Physical Layer Security*, pp. 849–853, 2011.

[20] Y.-C. Yang, H. Zhao, *et al.*, "Joint power allocation and relay selection for decode–and–forward cooperative relay in secure communication," *J. China Univ. Posts Telecom.*, vol. 20 no. 2, 79–85. 2013.