Decentralizing Multi-cell Maximum Weighted Sum Rate Precoding via Large System Analysis

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***Abstract*—We propose a decentralized algorithm for weighted sum rate (WSR) maximization via large system analysis. The rate maximization problem is done via weighted sum mean-squared error (WSMSE) minimization. Decentralized processing relies on the exchange via a backhaul link of a low amount of information. The inter-cell interference terms couple the maximization problems at the different base stations (BS)s. Large system approximations are used to replace the inter-cell interference terms and to decouple the problems. We demonstrate that the approximates depend only on the slow fading terms or second order statistics of the channels. Then, each BS computes the transmit precoders to serve its own user equipments (UE)s locally. No feedback channels from the UEs to the serving BSs will be required.**

Keywords*—*Large System analysis, coordinated beamforming, decentralization.

I. Introduction

We consider multiple-input single-output (MISO) system, formed by cellular network comprising multiple multi-antennas BSs, each serving a limited number of UEs. In order to mitigate or avoid the inter-cell interference, the BSs employ coordinated beamforming. This latter requires information of the channels between each BS and all active UEs within the coordination group of cells, it is assured by a centralized controller that collects the CSI or channel matrices from the coordinated cells, calculates all the precoders and then distributes them to all the BSs. This approach requires a relatively large amount of information exchange via low-latency links to avoid delays between the CSI acquisition phase and the data transmission. An example of this coordinated beamforming approach is the one that exploits the relation between the WSMSE minimization and the WSR maximization problems established in [1], [2] and [3]. To relax the latency requirements or if centralized controller is not available, decentralized methods can be applied. In this paper we extend the works in [1], [2] and [3] to propose a decentralized beamforming approach that relies on the slow fading exchange of information between the BSs. Our work is based on large system analysis. Other works on decentralization exist already in the literature, but they rather rely only on optimization techniques to decentralize such as [4], [5] and [6], or on estimation tricks such as [7]. To the best of our knowledge, only one work considers decentralized coordinated beamforming using large system analysis [8], but it is sub-optimal. A work on decentralization techniques for decentralized minimum transmit power beamforming exists in [9], which is different of our WSMSE technique.

II. System Model

In the following, we analyze a cellular downlink interference broadcast channels (IBC) MISO scenario where C cells are presented, c=1...C, each of the C cells consists of one BS associated with a number K of single-antenna receivers. We assume transmission on a single narrow-band carrier. the received signal $y_{c,k}$ at the kth user in cell c reads

$$y_{c,k} = \sum_{m=1}^{C} \sum_{l=1}^{K} h_{m,c,k}^{H} g_{m,l} s_{m,l} + n_{c,k} \qquad (1)$$

where the user symbols are chosen from a Gaussian codebook, i.e, $s_{m,l} \sim \mathcal{NC}(0,1)$, are linearly precoded and form the transmit signal; $g_{m,l} \in \mathbb{C}^{M}$ is the precoding vector of user l of cell m, M is the number of antennas at each BS, $h_{m,c,k}^{H} \in \mathbb{C}^{1\times M}$ is the channel vector from the mth transmitter to the kth user of cell c, and the $n_{c,k}$ are independent complex Gaussian noise terms with zero mean and variance σ^2. Moreover, the precoders are subject to an average power constraint and the channel $h_{i,c,k}^{H}$ is correlated as
$\mathbb{E}\left[h_{i,c,k} h_{i,c,k}^{H}\right] = \Theta_{i,c,k}$ thus

$$h_{i,c,k} = \sqrt{M}\Theta_{i,c,k}^{1/2} z_{i,c,k} \qquad (2)$$

$$tr G_c G_c^H \preceq P_c \; for \; c \in \mathcal{C} \qquad (3)$$

where $z_{i,c,k}$ has i.i.d. complex entries of zero mean and variance $\frac{1}{M}$ and the $\Theta_{i,c,k}^{1/2}$ is the Hermitian square-root of $\Theta_{i,c,k}$. The correlation matrix $\Theta_{i,c,k}$ is non-negative Hermitian and of uniformly bounded spectral norm w.r.t. to M. For notational convenience, we denote $\Theta c, c, k$ as $\Theta c, k$.
$G_c = [g_{c,1}, g_{c,2}, ..., g_{c,K}] \in \mathbb{C}^{M\times K}$ is the precoding matrix and P_c is the total available transmit power of cell c.
Under the assumption of optimal single-user decoding and perfect Channel State Information (CSI) at the transmitters and receivers, the achievable rate of the kth user of cell c is given by

$$R_{c,k} = \log(1 + \gamma_{c,k}); \qquad (4)$$

$$\gamma_{c,k} = \frac{|h_{c,c,k}^H g_{c,k}|^2}{\sum_{(m,l)\neq(c,k)} h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k} + \sigma^2}. \quad (5)$$

where $\gamma_{c,k}$ is the signal-to-interference plus noise ratio (SINR) of the kth user of cell c.

III. WSR MAXIMIZATION VIA WSMSE MINIMIZATION

The precoders maximize the WSR of all users so we are facing an optimization problem which is the following

$$\begin{aligned} G^* =& arg \max_G \sum_{c=1}^{C} \sum_{k=1}^{K} u_{c,k} R_{c,k} \\ & s.t.\ trG_c G_c^H \leq P_c\ for\ c \in \mathcal{C} \end{aligned} \quad (6)$$

where G is the short notation for $\{G_c\}_{c\in\mathcal{C}}$ and where $u_{c,k} \geq 0$ is the weight of the k^{th} user of cell c. The optimization problem in (6) is hard to solve directly, since it is highly non convex in the precoding matrix G. To solve the problem in (6), we consider the virtual linear receive filters $a_{c,k} \in \mathbb{C}$, the error variance $e_{c,k}$ after the linear receive filtering, given in (8), and we introduce additional weighting scalars $w_{c,k}$, so that the utility function (6) can be modified and an equivalent WSMSE optimization problem can be formulated as in [1] and [2] :

$$\begin{aligned} & \{G^*, \{a_{c,k}^*\}, \{w_{c,k}^*\}\} = \\ & arg \min_{G,\{a_{c,k}\},\{w_{c,k}\}} \sum_{(c,k)} w_{c,k} e_{c,k} - u_{c,k} \log(u_{c,k}^{-1} w_{c,k}) \\ & s.t.\ trG_c G_c \leq P_c\ for\ c \in \mathcal{C} \end{aligned} \quad (7)$$

with

$$e_{c,k} = E[(a_{c,k} y_{c,k} - s_{c,k})(a_{c,k} y_{c,k} - s_{c,k})^H]. \quad (8)$$

This latter problem is not jointly convex on all sets of optimization variables ($\{G\}$, $\{a_{c_k}\}$and $\{w_{c_k}\}$), but it turns out to be convex for each set of variables separately and each of them can be derived analytically assuming that the other two sets of variables are fixed. A local optimum of the problem can be found by alternate optimization of the variables.

IV. CENTRALIZED TRANSMIT FILTERS DESIGN

The solution of the alternate optimization can be expressed by the following:

$$a_{c,k}^* = g_{c,k}^H h_{c,c,k} (\sigma^2 + \sum_{m=1}^{C} \sum_{l=1}^{K} h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k})^{-1} \quad (9)$$

$$e_{c,k}^* = (1 + \gamma_{c,k})^{-1} \quad (10)$$

$$w_{c,k}^* = u_{c,k} (e_{c,k}^*)^{-1} \quad (11)$$

$$\widetilde{g}_{c,k}^* = (H_c^H D H_c + \frac{trD_c}{\rho_c} I_M)^{-1} h_{c,c,k} a_{c,k}^H w_{c,k} \quad (12)$$

$\rho_c = \frac{P_c}{\sigma^2}$, the signal-to-noise ratio (SNR) in cell c, where $g_{c,k}^* = \xi_c \widetilde{g}_{c,k}^*$ with $\xi_c = \sqrt{\frac{P_c}{tr\widetilde{G_c}^* \widetilde{G_c}^{*H}}}$. Also we defined $W_c = diag(w_{c,1}^*, ..., w_{c,K}^*)$, $A_c = diag(a_{c,1}^*, ..., a_{c,K}^*)$, $D_c = A_c^H W_c A_c$,and $A = diag(A_1, A_2, ...A_C)$ $D = diag(D_1, D_2, ..., D_C)$, $H_c = [h_{c,1,1}, ..., h_{c,1,K}, h_{c,2,1}, \ldots, h_{c,2,K}, \ldots, h_{c,C,K}]^H \in \mathbb{C}^{KC\times M}$ is the compound channel. For notational convenience, we drop the superscript* in the sequel. Subsequently $a_{c,k}$ and $w_{c,k}$ are computed, which then constitute the new precoder $g_{c,k}$. This process is repeated until convergence to a local optimum. This is a centralized procedure since it requires that all channel matrices from all BSs to all UEs (i.e., $\{H_{m,c,k}\}$, $\forall m, \forall c, \forall k$) are collected in a central processor node.

V. DECENTRALIZED APPROACH FOR LARGE DIMENSION SYSTEM

The idea is to try to identify the quantities that require global knowledge of the channel vectors, the intercell interference $\Upsilon_{inter,c,k}$ and D in our case, and exchange them (or the quantities related to them) between the different BSs in such a way that the maximum WSR problem will decompose into parallel sub-problems (one per BS). However, it is required to limit as possible this exchange in order to be backhaul friendly (efficient). The solution in the last section can be reformulated as the following:

$$a_{c,k} = g_{c,k}^H h_{c,c,k} (\sigma^2 + \Upsilon_{intra,c,k} + \Upsilon_{inter,c,k})^{-1} \quad (13)$$

$$e_{c,k} = (1 + \gamma_{c,k})^{-1} \quad (14)$$

$$w_{c,k} = u_{c,k} (e_{c,k})^{-1} = u_{c,k} (1 - a_{c,k} h_{c,c,k}^H g_{c,k})^{-1} \quad (15)$$

$$\widetilde{g}_{c,k} = (H_c^H D H_c + \frac{trD_c}{\rho_c} I_M)^{-1} h_{c,c,k} a_{c,k}^H w_{c,k} \quad (16)$$

with

$$\Upsilon_{intra,c,k} = \sum_n h_{c,c,k}^H g_{c,n} g_{c,n}^H h_{c,c,k} \quad (17)$$

$$\Upsilon_{inter,c,k} = \sum_{m;m\neq c} \Upsilon_{inter,m,c,k} \quad (18)$$

and

$$\Upsilon_{inter,m,c,k} = \sum_n h_{m,c,k}^H g_{m,n} g_{m,n}^H h_{m,c,k} \quad (19)$$

This solution can be initialized by a random precoder, e.g., a matched filter (MF) precoder and requires in general a central processing node to be implemented because of (18) which depends on global channels knowledge as shown in (19). In the case of absence of this central node, (18), can be detected by each receiver and then fed back using an over-the-air link as in [1]. However, this approach is spectral inefficient. Another way to decentralize consists in that each BS m calculates the quantities in (19), $\Upsilon_{inter,m,c,k}$ considered as the interference leakage from BS m to user k of cell $c \neq m$ for all the users and sends them to the BS c using a backhaul link. This procedure is a bit heavy one, so that it is beneficial to gorge the most the number of iterations. However, at high snr, the solution above requires a lot of iterations to converge, hence, requires an extensive exchange of information using the backhaul link which burdens this latter and makes it practically infeasible.

For a limited number of iterations, the solution becomes very suboptimal. Thus, in the following we present a new initialization method which accelerates the convergence and hence few iterations are no more suboptimal and the backhaul-based decentralization becomes realistic. In this following, performance analysis is conducted for the proposed precoder. The large-system limit is considered, where M and K go to infinity while keeping the ratio K/M finite such that $limsup_M K/M < \infty$ and $liminf_M K/M > 0$. All vectors and matrices should be understood as sequences of vectors and matrices of growing dimensions. In the following we will determine a new expression for the precoders based on large system analysis.

Theorem 1: For a large MISO system, precoders $\tilde{g}_{c,k}$ can be written as the following:

$$\widetilde{g}_{c,k}\widetilde{g}_{c,k}^H - \overline{\widetilde{g}}_{c,k}\overline{\widetilde{g}}_{c,k}^H \xrightarrow{M\to\infty} 0 \tag{20}$$

where

$$\overline{\widetilde{g}}_{c,k} = (H_c^H \overline{D} H_c + \frac{tr\overline{D}_c}{\rho_c} I_M)^{-1} h_{c,c,k} \overline{a}_{c,k}^H \overline{w}_{c,k} \tag{21}$$

Thus, we propose that (21) serves as an initialization for the iterative solution above. It serves it-self as a precoder as well. Further details will be provided in the following sections. The fast-converging iterative algorithm behind (21) and the definitions of its terms are summarized in Algorithm 1. The proof is given in [11]. However, the proof is incomplete, and the proof of the deterministic limit of the intercell interference term (18) is not provided in [11]. We give here the large system approximation of the intercell interference term.

$$\Upsilon_{inter,c,k} = \lim_{j\to\infty} \frac{\overline{\xi}_c^{2,(j)} [\overline{\hat{\Upsilon}}_{c,k}^{(j)}]}{\overline{d}_{c,k}^{(j)} (1+\overline{m}_{c,k}^{(j)})^2}. \tag{22}$$

The proof is given in the Appendix.

VI. SIGNALING

This section summarizes the iterative procedure to design optimal transmit beamformers in a decentralized manner which will be introduced in this section. In other words, it describes how to implement the precoders in a decentralized way. The authors of [1] proposed a decentralized reasoning as well; so we will compare it to ours. They have assumed that local channel information is available at each BS and for each UE; we assume that as well. Moreover, they assume that each UE has an additional channel to feedback information, which is $d_{i,k} = |a_{i,k}|^2 w_{i,k}$, to the BS; however, we relax this assumption and we assume instead the existence of a backhaul link which is a way to save the wireless capacity consumption w.r.t an over-the-air link. It is used as explained in the previous section. However, we would propose three different strategies: a) The intercell interference free strategy where for every iteration of the precoders design each BS c calculates only the quantities $\Upsilon_{intra,c,k}$ using the local channel information and supposes the $\Upsilon_{inter,c,k}$ is null. b) The constant intercell interference strategy where for every iteration of the precoders design each BS c calculates the quantities $\Upsilon_{intra,c,k}$

Algorithm 1 Large System Computation of Dual UL Scalars

Step 1: Set $j = 0$ and calculate

$$\overline{\gamma}_{c,k}^{(0)} = \frac{1}{\frac{1}{\beta_c \rho_c} + \frac{1}{M^2} \sum_{(l,i)\neq(c,k)} tr\Theta_{l,c,k}\Theta_{l,i}}.$$

$$\overline{a}_{c,k}^{(0)} = \frac{1}{\sqrt{\overline{P}_{c,k}^{(0)}}} \frac{\overline{\gamma}_{c,k}^{(0)}}{1+\overline{\gamma}_{c,k}^{(0)}}, \sqrt{\overline{P}_{c,k}^{(0)}} = \sqrt{\frac{P}{\frac{1}{M^2}\sum_{k=1}^K \Theta_{c,c,k}}},$$

$$\overline{w}_{c,k}^{(0)} = u_{c,k}(1+\overline{\gamma}_{c,k}^{(0)}), \overline{d}_{c,k}^{(0)} = \overline{w}_{c,k}^{(0)} \overline{a}_{c,k}^{2,(0)}$$

Step 2: Set $j = j+1$ and calculate the following quantities:

$$\overline{\hat{\Upsilon}}_{c,k}^{(j)} = \frac{1}{M} \sum_{m=1,m\neq c}^{C} \frac{(1+\overline{m}_{c,k}^{(j)})^2}{(1+\overline{m}_{m,c,k}^{(j)})^2} \sum_{l=1}^{K} \frac{\overline{w}_{m,l}^{(j)}}{(1+\overline{m}_{m,l}^{(j)})^2} e_{m,c,k,m,l}^{',(j)};$$

$$\overline{m}_{m,c,k}^{(j)} = \frac{1}{M} tr\overline{\Theta}_{m,c,k}^{(j)} V_m^{(j)}, V_m^{(j)} = (F_m^{(j)} + \overline{\alpha}_m^{(j)} I_M)^{-1},$$

$$\overline{m}_{c,k}^{(j)} = \overline{m}_{c,c,k}^{(j)},$$

$$F_m^{(j)} = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{m,j,i}^{(j)}}{1+\overline{m}_{m,j,i}^{(j)}}, \text{ with } \overline{\Theta}_{m,c,k}^{(j)} = \overline{d}_{c,k}^{(j-1)} \Theta_{m,c,k}.$$

$$e_{m,c,k,m,l}^{',(j)} = \frac{1}{M} tr\overline{\Theta}_{m,c,k}^{(j)} V_m^{(j)} (F_{m,m,l}^{',(j)} + \overline{\Theta}_{m,l}^{(j)}) V_m^{(j)},$$

$$\overline{\alpha}_m^{(j)} = \frac{\sum_i \overline{d}_{m,i}^{(j-1)}}{M\rho_m}, F_{m,m,l}^{',(j)} = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{m,j,i}^{(j)} e_{m,j,i,m,l}^{',(j)}}{(1+\overline{m}_{m,j,i}^{(j)})^2}.$$

$$\overline{\Psi}_c^{(j)} = \frac{1}{M} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} \frac{m_{c,k}^{',(j)}}{(1+e_{c,k}^{(j)})^2}, e_{c,k}^{',(j)} = \frac{1}{M} tr\overline{\Theta}_{c,k}^{(j)} V_c^{(j)} (F_c^{',(j)} + I_M) V_c^{(j)},$$

$$\overline{\Upsilon}_{c,k}^{(j)} = \frac{1}{M} \sum_{l=1,l\neq k}^{K} \frac{\overline{w}_{c,l}^{(j)}}{(1+\overline{m}_{c,l}^{(j)})^2} e_{c,c,k,c,l}^{',(j)}, F_c^{',(j)} = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{c,j,i}^{(j)} e_{j,i}^{',(j)}}{(1+\overline{m}_{c,j,i}^{(j)})^2},$$

$$\overline{a}_{c,k}^{(j)} = \frac{1}{\sqrt{\overline{P}_{c,k}^{(j-1)}}} \frac{\overline{\gamma}_{c,k}^{(j-1)}}{1+\overline{\gamma}_{c,k}^{(j-1)}}, \sqrt{\overline{P}_{c,k}^{(j-1)}} = \frac{1}{\overline{a}_{c,k}^{(j-1)}} \sqrt{\frac{P}{\overline{\Psi}_c^{(j-1)}}} \frac{\overline{m}_{c,k}^{(j)}}{1+\overline{m}_{c,k}^{(j)}},$$

$$\overline{w}_{c,k}^{(j)} = u_{c,k}(1+\overline{\gamma}_{c,k}^{(j-1)}), \overline{d}_{c,k}^{(j)} = \overline{w}_{c,k}^{(j)} \overline{a}_{c,k}^{2,(j)}$$

Step 3: $\overline{\gamma}_{c,k}^{(j)} = \frac{\overline{w}_{c,k}^{(j)} (\overline{m}_{c,k}^{(j)})^2}{\overline{\Upsilon}_{c,k}^{(j)} + \overline{\hat{\Upsilon}}_{c,k}^{(j)} + \overline{d}_{c,k}^{(j)} \frac{\overline{\Psi}_c^{(j)}}{\rho_c} (1+\overline{m}_{c,k}^{(j)})^2}; \rho_c = \frac{P_c}{\sigma^2}.$

Step 4: If converge stop and calculate $\overline{\widetilde{g}}_{c,k}^{(j)}$ as in Theorem 1, otherwise go to step 2.

*Note that all $e_{m,c,k} = \overline{m}_{m,c,k}$, $e'_{c,k}$ and $e'_{m,c,k,m,l}$ are obtained using the fixed-point iteration method as in [11].

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using the local channel information but uses the intercell interference given by Algorithm 1 using (22). c) The up-to-date intercell interference strategy where for every iteration of the precoders design each BS c calculates the quantities $\Upsilon_{intra,c,k}$ using the local channel information, calculates the intercell interferences $\Upsilon_{inter,m,c,k}$ in (19) and sends them to the corresponding BS and finally this BS c collects the interference leakages corresponding to each user and sums them. Clearly, the strategies (a) and (b) are suboptimal but less demanding than (c) w.r.t to the backhaul capacity. Although the strategies (a) and (b) are suboptimal, however they perform better than the approach in [1] by taking the MF initialization for a limited number of iterations. We recall that for each of the three strategies above, each BS calculates the $d_{c,k}$ for all served UEs and then send them to all the neighbouring BSs via the backhaul link. Furthermore, the fact that (c) requires that each BS m calculates the quantities in (19) for all users not served by m and then send them to the concerned BS consumes more backhaul capacity than (a) and (b). The overall mechanism is described briefly in Algorithm 2. The maximum number of iterations $iter_{max}$ is chosen very small, e.g., $iter_{max} = 2$ or $iter_{max} = 3$.

Algorithm 2 The Decentralized Algorithm

Step 1: Set $iter = 0$. All BSs estimate local channel matrices (from BS to served UEs and to the UEs of the neighbouring cells). The BSs distribute the channel covariance matrices to neighbouring cells via the backhaul link only at slow fading rate. They apply Algorithm 1 and then calculate (21). They calculate (22) for strategy (b) and the intercell inteference $\Upsilon_{inter,m,c,k}$with (21) for strategy (c) and exchange them with the concerned BS.

Step 2:
All the BSs calculate $\Upsilon_{intra,c,k}$, $a_{c,k}$, $w_{c,k}$ and $d_{c,k} = |a_{c,k}|^2 w_{c,k}$ using (17), (13) and (15) and send $d_{c,k}$ to the neighbouring BSs, at fast fading rate. Moreover, each BS calculates the interference leakages and collects the interference corresponding to its served users in strategy (c).

Step 3: All the BSs calculate their precoders using (16).

Step 4: $iter = iter + 1$, if $iter = iter_{max}$ stop, otherwise go to step 2.

VII. NUMERICAL RESULTS

In this section, results of simulations based on realistic settings with a finite number of transmit antennas corroborate the correctness of the proposed approximation. We compare the three strategies of our decentralized algorithm to the decentralized approach in [1], to the performance given by large system approximation in [11] which proposes an asymptotic approximation of the SINR of the WSR-WSMSE precoder at every iteration, and to the performance given directly by the precoder (21). The channel correlation matrix $[\Theta_{m,c,k}]_{ij} \forall i, \forall j$ can be modeled as in [10]. In our case, we take them as identity matrices. For the simulations we have used 200 channel realizations while the large system approximation in [11] needs only one channel realization. Furthermore, we have used $iter_{max} = 3$ iterations for the simulation of strategies (a), (b) and (c), 1 iteration for (21) and 3, 30 and 100 iterations for [1]. In Fig. 1 and Fig. 2, it can be observed that the curves 'performance corresponding to (21)', 'strategy (a)', 'strategy (b)' and 'strategy (c) are combined and that our precoders corresponding to (21), (a), (b) and (c) behave very efficiently in general, which means that they achieve higher rates than [1] with much less iterations (less information exchanged in the backhaul and smaller latencies). However, (21) has a better performance in Fig.1 corresponding to a non fully loaded system ($load = \frac{KC}{M} < 1$) than in Fig.2 corresponding to a fully loaded system ($load = 1$). Further explanations about the behavior of the large system approximations for fully loaded systems as in Fig. 1 can be found in [11].

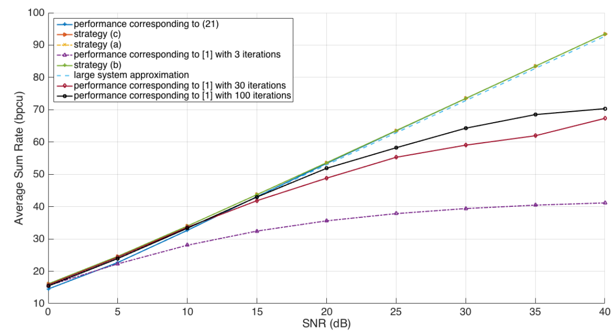

Fig. 1. Sum rate comparisons for C=3,K=2,M=15.

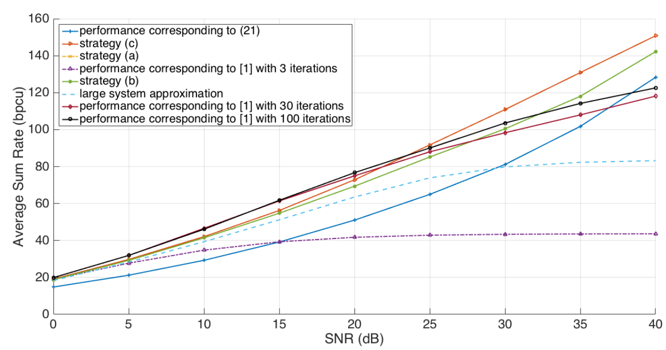

Fig. 2. Sum rate comparisons for C=3,K=5,M=15.

VIII. ANALYTIC SOLUTION

Let $\Theta_{m,c,k} = I_m$ for all m, c and k. Under that assumption the deterministic equivalents of $\overline{\gamma}_{c,k}$, $\overline{d}_{c,k}$, $\overline{a}_{c,k}$ and $\overline{w}_{c,k}$ for all c and k given by Algorithm 1 can be found in closed forms as the following:

$$\beta = \frac{M}{KC}; \beta_2 = \frac{M}{K}; \chi = \frac{1}{\beta} + \frac{1}{\beta_2} - 1; \tag{23}$$

$$\overline{\gamma} = \frac{-\chi + \sqrt{\chi^2 + \frac{4}{\beta_2 \rho}}}{\frac{2}{\beta_2 \rho}}; \overline{a} = \sqrt{\frac{\frac{\frac{\gamma^2}{\beta_2(1+\gamma)^2}}{1 - \frac{\gamma^2}{\beta(1+\gamma)^2}}}{P}}; \tag{24}$$

$$\overline{w} = 1 + \overline{\gamma}; \overline{d} = \overline{a}^2 \overline{w}. \tag{25}$$

IX. CONCLUSION

Inter-cell interference is a key parameter in the design of distributed beamforming algorithm as it couples the sub-problems at base stations. In this work, we approximated the inter-cell interference via large system analysis and tools from Random Matrix Theory (RMT) and introduced decentralized precoders that allow to achieve high data rates with limited information exchange between the different BSs of the network and hence with low latencies.

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APPENDIX

For the rest of this paper, note that Lemmas 1, 2, 3, 4 and Theorem 2 correspond respectively to [10, Lemma 1], [10, Lemma 2], [10, Lemma 4], [10, Lemma 6] and [10, Theorem 1]. Using Lemma 2,

$$\Gamma_m^{-1,(j)} = \Gamma_{m,[c,k],[m,l]}^{-1,(j)} + (\Gamma_m^{-1,(j)} - \Gamma_{m,[c,k],[m,l]}^{-1,(j)}); \quad (26)$$

$$(\Gamma_m^{-1,(j)} - \Gamma_{m,[c,k],[m,l]}^{-1,(j)}) = -\Gamma_m^{-1,(j)}(\Gamma_m^{(j)} - \Gamma_{m,[c,k],[m,l]}^{(j)})\Gamma_{m,[c,k],[m,l]}^{-1,(j)}; \quad (27)$$

$$(\Gamma_m^{(j)} - \Gamma_{m,[c,k],[m,l]}^{(j)}) = \frac{1}{M} h_{m,c,k}\overline{d}_{c,k}^{(j)} h_{m,c,k}^H + \frac{1}{M} h_{m,m,l}\overline{d}_{m,l}^{(j)} h_{m,m,l}^H$$
$$= \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} z_{m,c,k} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} + \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l} z_{m,m,l}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)}; \quad (28)$$

Denoting,

$$m_{m,l}^{(j)} = z_{m,m,l}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l}; \quad (29)$$

$$m_{m,c,k}^{(j)} = z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} z_{m,c,k}; \quad (30)$$

Using Lemmas 1,3 and 4 and (30),

$$z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} z_{m,c,k} = \frac{m_{m,c,k}}{1+m_{m,c,k}}; \quad (31)$$

Using (26), (27), (28), (29), (30) and (31),

$$z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l}$$
$$= z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l} -$$
$$z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} z_{m,c,k} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l} -$$
$$z_{m,c,k}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l} z_{m,m,l}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l}$$
$$= \frac{1}{1+m_{m,l}^{(j)}} \frac{1}{1+m_{m,c,k}^{(j)}} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l};$$
$$= L_{m,c,k,m,l}^{(j)} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l};$$

with

$$L_{m,c,k,m,l}^{(j)} = \frac{1}{1+m_{m,l}^{(j)}} \frac{1}{1+m_{m,c,k}^{(j)}}. \quad (32)$$

Thus,

$$\Upsilon_{inter,c,k} = \sum_{(m,l);m\neq c} h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k};$$

$$\frac{h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k} \times \overline{d}_{c,k}^{(j)}}{\xi_c^{2,(j)}}$$
$$= z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l} z_{m,m,l}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_m^{-1,(j)} \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} z_{m,c,k}$$

$$= |L_{m,c,k,m,l}^{(j)} z_{m,c,k}^H \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} z_{m,m,l}|^2$$
$$\xrightarrow{(a)} |L_{m,c,k,m,l}^{(j)}|^2 \times \frac{1}{M} |z_{m,m,l}^H \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,c,k}^{\frac{1}{2},(j)}|^2$$
$$\xrightarrow{(b)} |L_{m,c,k,m,l}^{(j)} \times \frac{1}{M}|^2 tr \overline{\Theta}_{m,c,k}^{(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)}$$
$$= |L_{m,c,k,m,l}^{(j)} \times \frac{1}{M}|^2 tr(\overline{\Theta}_{m,l}^{-\frac{1}{2},(j)} \overline{\Theta}_{m,c,k}^{(j)} \overline{\Theta}_{m,l}^{-\frac{1}{2},(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \times$$
$$\Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{-1,(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)})$$
$$= |L_{m,c,k,m,l}^{(j)} \times \frac{1}{M}|^2 tr\{(\overline{\Theta}_{m,l}^{-\frac{1}{2},(j)} \overline{\Theta}_{m,c,k}^{(j)} \overline{\Theta}_{m,l}^{-\frac{1}{2},(j)})$$
$$\times (\overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)})^{-1} \times (\overline{\Theta}_{m,l}^{\frac{1}{2},(j)} \Gamma_{m,[c,k],[m,l]}^{(j)} \overline{\Theta}_{m,l}^{\frac{1}{2},(j)})^{-1}\}$$
$$\xrightarrow{(c)} |L_{m,c,k,m,l}^{(j)}|^2 \times \frac{1}{M} tr\{$$
$$\overline{\Theta}_{m,c,k}^{(j)} (F_m + \overline{\alpha}^{(j)} I)^{-1} \times (F_{m,m,l}^{'} + \overline{\Theta}_{m,l}^{(j)}) \times (F_m + \overline{\alpha}^{(j)} I)^{-1}\}$$
$$= L_{m,c,k,m,l}^{(j),2} e_{m,c,k,m,l}^{'};$$

with $e_{m,c,k,m,l}^{'}$, F_m and $F'_{m,m,l}$ given as in Algorithm 2 which leads to (18). Note that (a), (b) and (c) above correspond to "using Lemma 3 and the fact that the matrices in a trace of a product can be switched", "using Lemma 3 and the property of trace" and "using Theorem 2 and [Appendix 2, 10]" respectively. Now, the proof is completed.