Spatial and Time diversities for Canonical Correlation Significance Test in Spectrum Sensing

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***Abstract*—In this paper, we present a new detector for cognitive radio system based on the Canonical Correlation Significance Test (CCST). Unlike existing CCST approaches, which can only be applied on Multi-Antenna System (MAS), our algorithm can be extended for both Single Antenna System (SAS) and MAS. For SAS, the proposed algorithm exploits the time diversity of cyclostationary signals in order to detect the Primary User (PU) signal. Our simulation results shows that our algorithm outperforms well-known cyclostationary algorithm [9]. For MAS, our algorithm uses both spatial and time diversities to apply the CCST. Numerical results are given to illustrate the performance of our algorithm and verify its efficiency for special noise cases (spatially correlated and spatially colored). The simulation results show the superiority of the performance of the proposed detector compared to the recently CCST proposed algorithm [1].**

Keywords*—*Canonical Correlation Significance Test, Single Antenna System, Multi-Antenna System, Spatial and Time diversities, Spectrum Sensing, Cognitive Radio.

I. INTRODUCTION

The Cognitive Radio (CR) has been recently proposed in order to solve the scarcity in frequency bandwidths [2]. CR uses spectrum sensing in order to share spectrum between two classes of users, Primary User (PU) and Secondary User (PU). PU has the spectrum license. When PU is idle, a SU can access the channel. When the PU becomes again active, SU should immediately vacate the channel, to avoid any interference. The monitoring of the PU activities is allocated to the spectrum sensing part of a CR.

In the literature, many Spectrum Sensing techniques can be identified [3], [4], [5]. The widely used Energy Detection (ED) method consists in comparing the energy of the received signal to a predefined threshold that is suffering from the noise uncertainty. This uncertainty leads to the SNR wall phenomenon [6], which prevents the ED to make an accurate decision on the channel even with infinite time observation.

Other methods are well known in this context such as Waveform Detection (WFD) that requires a perfect knowledge about the PU signal, which makes WFD not applicable in CR which should deal with a great variety of signals [3], [5]. The Autocorrelation Detection (ACD) exploits the correlation of the PU signal samples in order to detect it, assuming that the noise samples are white [7]. Eigenvalues based detection (EBD) is based on testing the greatest eigenvalue of the correlation matrix of the signals received on several antennas. EBD can also be applied for a system of one antenna when the PU signal is oversampled [8].

The cyclostationary detector (CSD) shows its robustness against the noise uncertainty and the low SNR [1], [9], [11]. Thanks to the fact that most communication signals are cyclostationary due to the modulation process, the carrier frequency, the pilot signal, *etc.*, CSD becomes a good candidate to detect the PU in CR, and therefore, to differentiate between signals and noise since the noise does not exhibit any cyclostationarity.

In order to enhance the Spectrum Sensing performance, multi-antennas system (MAS) has been proposed and exploited for various Spectrum Sensing strategies, such as Cooperative Spectrum Sensing for hard and and soft combining schemes [5], [10]. Recently, MAS has been used used to perform the cyclostationary detection in Spectrum Sensing. MAS [12] is used to detect multi cyclic frequencies PU's signals. In their approach [12], each antenna tests the cyclostationarity of the received signal at one cyclic frequency, then the cooperative antennas send their decisions to the Fusion Center (FC) to make the final decision. In [1], [11], the Canonical Correlation Significance Test (CCST) is used to examine the canonical correlation among the observed at M antennas and the shifted copies of these signals at a given cyclic frequency.

In this paper, we aim at extending CCST for both Single-Antenna System (SAS) and MAS. We refer to our algorithm for SAS by CCST-S and for that of MAS by CCST-M. Our two algorithms exploit both time and spatial diversities. Time diversity help us to develop CCST-S, which tests the canonical correlation of the time shifted versions of the received signal at a given cyclic frequency. The numerical results shows that CCST-S outperforms the Generalized Likelihood Ratio Test (GLRT) cyclostationary detector of [9]. Hereinafter, we extend our algorithm to the MAS. Our algorithm is tested under various scenarios, for spatially uncorrelated, spatially correlated and spatially colored noise. In those different scenarios, our algorithm outperforms significantly the existing CCST algorithm.

II. SYSTEM MODEL

The problem formulation on the presence/absence of the PU can be presented in a classic Bayesian detection problem as follows:

$$H_\eta : x_i(n) = \eta h_i s(n) + w_i(n) \tag{1}$$

Where $\eta \in \{0;1\}$. H_0 stands for the case where PU is absent, whereas under H_1 PU is transmitting. $x_i(n)$ is a $1 \times N$ vector representing the observation at the ith SU receiving antenna, N stands for the total number of received samples, $s(n)$ is the PU signal, $w_i(n)$ is the noise at the ith SU receiving antenna and assumed to be stationary zero mean White Gaussian Noise (AWGN), with a variance $\sigma^2_{w_i}$ and the channel gain, h_i, between the PU base station and the ith SU receiving antenna is assumed to be constant during the Spectrum Sensing Process.

Let $\mathbf{x}(n)$ be the vector collecting the observations on M antennas:

$$\mathbf{x}(n) = [x_1(n),\ x_2(n),\ ...\ x_M(n)]^T \tag{2}$$

In [1], [11], CCST requires MAS in order to be applied, where the CCST is done over $\mathbf{x}(n)$ and $\mathbf{x}(n-\tau)e^{j2\pi\alpha n}$. The lag τ is chosen offline in order to maximize $\sum_{n=1}^{N} s(n)s^*(n-\tau)e^{-j2\pi\alpha n}$ at a non-zero cyclic frequency α, where $s^*(n)$ stands for the conjugate of $s(n)$. CCST determines the number of signals having non-zero cyclic statistics at α. In this manuscript, CCST is applied on the set of multiple shifted versions of the received signal over SAS. When the PU signal is absent (*i.e.* H_0), the noise does not exhibit any cyclic statistics; Whereas under H_1, CCST should confirm the presence of PU thanks to the cyclic statistics of the PU signal. Hereinafter, this system is extended for MAS, where both spatial and time diversities are exploited, unlike [1], [11], where only the spatial diversity was exploited.

III. SPECTRUM SENSING DETECTOR BASED ON CCST

CCST is based on the canonical correlation theory (CCT), which aims at finding common factors between two sets of data, $\mathbf{y}(n)$ and $\mathbf{z}(n)$. The number of common factors between $\mathbf{y}(n)$ and $\mathbf{z}(n)$ is equal to the rank of the following matrix [11], [13], [14].

$$R = R_{\mathbf{yy}}^{-1} R_{\mathbf{yz}} R_{\mathbf{zz}}^{-1} R_{\mathbf{zy}} : \tag{3}$$

Where $R_{\mathbf{yz}} = \mathrm{Cov}[\mathbf{y}(n), \mathbf{z}(n)]$ and can be estimated by $\hat{R}_{\mathbf{yz}}$

$$\hat{R}_{\mathbf{yz}} = \frac{1}{N}\mathbf{y}(n)\mathbf{z}^H(n) \tag{4}$$

Where $\mathbf{z}^H(n)$ is the Transpose Conjugate of $\mathbf{z}(n)$.

CCST uses similar techniques to identify the common factors between $\mathbf{x}(n)$ and $\mathbf{x}(n)\exp(-j2\pi\alpha n)$, where α is a known cyclic frequency. The number of common factors is the number of signals having a cyclic frequency α [1], [11]. In our context, under H_0 there is no signal having a cyclic frequency α; whereas under H_1, we should have only one signal, which is the PU signal. According to this discussion, the application of the CCST is depending on the presence of a multi-antenna system to ensure the vector $\mathbf{x}(n)$.

IV. PROPOSED CCST ALGORITHM FOR A SINGLE ANTENNA SYSTEM

The received signal in SAS under H_0 and H_1 is presented as follows:

$$\begin{cases} H_0 : x_1(n) = w_1(n) \\ H_1 : x_1(n) = h_1 s(n) + w_1(n) \end{cases} \tag{5}$$

Let us define the vector, Γ, containing the lag values:

$$\Gamma = [\tau_1, \tau_2, ..., \tau_P] \tag{6}$$

Where P stands for the length of Γ, which is chosen offline in such a way $\sum_{n=1}^{N} s(n-\tau_m)s^*(n-\tau_k)e^{-j2\pi\alpha n} \neq 0$, $\forall\ \tau_m, \tau_k \in \Gamma$.

A vector of shifted signals, $\mathbf{r}_1(n,m)$, is defined as follows:

$$\mathbf{r}_1(n,m) = [x_1(n-\tau_1),\ x_1(n-\tau_2),\ ...\ x_1(n-\tau_m)]^T \tag{7}$$

Where $m \leq P$. The CCST will be estimated on $\mathbf{r}_1(n,p_1)$ and $\mathbf{q}_1(n,p_2,\alpha) = \mathbf{r}_1(n,p_2)e^{j2\pi\alpha n}$, $\forall\ p_1, p_2 \in [1;P]$, to obtain the matrix $\hat{R}_{SAS}$:

$$\hat{R}_{SAS} = \hat{R}_{rr}^{-1}\hat{R}_{rq}\hat{R}_{qq}^{-1}\hat{R}_{qr} \tag{8}$$

where $\hat{R}_{rq}$ is estimated as presented in (4).

Under H_0, the cyclic autocorrelation matrix, $\hat{R}^0_{rq}$, of the shifted versions of the noise is obtained as follows:

$$\hat{R}^0_{rq}(\alpha) = \begin{bmatrix} \hat{R}_{w_{11}}(\alpha) & \hat{R}_{w_{12}}(\alpha) & \dots & \hat{R}_{w_{1p_2}}(\alpha) \\ \hat{R}_{w_{21}}(\alpha) & \hat{R}_{w_{22}}(\alpha) & \dots & \hat{R}_{w_{2p_2}}(\alpha) \\ \vdots & \vdots & \dots & \vdots \\ \hat{R}_{w_{p_1 1}}(\alpha)\dots & \hat{R}_{w_{p_1 2}}(\alpha) & \dots & \hat{R}_{w_{p_1 p_2}}(\alpha) \end{bmatrix} \tag{9}$$

Where $\hat{R}_{w_{ij}}(\alpha)$ is defined by:

$$\hat{R}_{w_{ij}}(\alpha) = \frac{1}{N}\sum_{n=1}^{N} w_1(n-\tau_i)w_1^*(n-\tau_j)e^{-j2\pi\alpha n} \tag{10}$$

Since $w_1(n)$ is purely stationary and does not exhibit any cyclic correlation for all $\alpha \neq 0$, then $\hat{R}^0_{rq} \simeq \mathbf{0}$.

Under H_1, the cyclic autocorrelation matrix, $\hat{R}^1_{rq}$, is presented as follows:

$$\hat{R}^1_{rq}(\alpha) = \hat{R}_{ss}(\alpha) + \hat{R}_{sw}(\alpha) + \hat{R}_{ws}(\alpha) + \hat{R}^0_{rq}(\alpha) \tag{11}$$

Where $\hat{R}_{ws}(\alpha)$ and $\hat{R}_{sw}(\alpha)$ are the cyclic autocorrelation matrices between the noise and the PU signal, and they should be equal to zero, and $\hat{R}_{ss}(\alpha)$ is defined as follows:

$$\hat{R}_{ss}(\alpha) = |h|^2 \begin{bmatrix} \hat{R}_{s_{11}}(\alpha) & \hat{R}_{s_{12}}(\alpha) & \dots & \hat{R}_{s_{1p_2}}(\alpha) \\ \hat{R}_{s_{21}}(\alpha) & \hat{R}_{s_{22}}(\alpha) & \dots & \hat{R}_{s_{2p_2}}(\alpha) \\ \vdots & \vdots & \dots & \vdots \\ \hat{R}_{s_{p_1 1}}(\alpha) & \hat{R}_{s_{p_1 2}}(\alpha) & \dots & \hat{R}_{s_{p_1 p_2}}(\alpha) \end{bmatrix} \tag{12}$$

Where $\hat{R}_{s_{ij}}(\alpha)$ is the estimated cyclic autocorrelation of $s(n)$ at two lags τ_i and τ_j and it can be found similarly to (10). $R_{ss}(\alpha)$ is a non-zero matrix thanks to the cyclic autocorrelation of $s(n)$ at the cyclic frequency α.
In order to diagnose the channel status, we can examine the existence of cyclostationary signal (PU signal) or not (only noise). The CCST can help us to estimate the number of the signals having a cyclic frequency α using (8). Since one PU signal can be existing in the channel, the challenge becomes to differentiate between two cases: noise-only or signal plus noise.
The test statistic, T_{SAS}, that leads to determine the vacancy of the channel is defined as follows [13]:

$$T_{SAS} = -N \log \prod_{i=1}^{l} (1 - \lambda_i) \tag{13}$$

Where $\{\lambda_i\},\ i = 1, 2, ..., l$ are the greatest l eigenvalues of $\hat{R}_{SAS}$, and $1 \geq \lambda_1 \geq \lambda_2 \geq ... \geq \lambda_l$. $l \leq M$ is the number of signals to be detected. According to our hypothesis, one PU signal can exist, therefore $l = 1$ in our application. T_{SAS} will be compared to a certain threshold, ξ, in order to examine an existing vacancy of the bandwidth.

$$T_{SAS} \underset{H_0}{\overset{H_1}{\gtrless}} \xi \tag{14}$$

The following algorithm summarizes the steps followed to calculate T_{SAS} and to make a decision on the channel status.

Algorithm 1 Spectrum Sensing using CCST

1. Estimate the covariance matrix R_{SAS} using (8)
2. Calculate the eigenvalues of $\hat{R}_{SAS}$
3. Evaluate the test statistic according to (13).
4. Compare the test statistic to a threshold to make a decision on the channel opportunity

V. Spectrum Sensing Using CCST under Multi-Antenna System

In this section, we develop the detector CCST-S in order to be applied in the Multi-Antenna System (MAS). Let us denote by $\mathbf{X}(n, m)$ and $\mathbf{Y}(n, p, \alpha)$ the two following vectors respectively:

$$\mathbf{X}(n, m) = [\mathbf{r}_1(n, m), \mathbf{r}_2(n, m), ..., \mathbf{r}_M(n, m)]^T \tag{15}$$

$$\mathbf{Y}(n, p, \alpha) = [\mathbf{q}_1(n, p, \alpha), \mathbf{q}_2(n, p, \alpha), ..., \mathbf{q}_M(n, p, \alpha)]^T \tag{16}$$

Where $\mathbf{r}_i(n, m),\ 1 \leq i \leq M$, is the vector containing the shifted versions of the signal received at the ith antenna, and is defined according to (7), and $\mathbf{q}_i(n, p, \alpha),\ 1 \leq i \leq M$, is equal to $\mathbf{r}_i(n, p)e^{j2\pi\alpha n}$.
In order to find the number of cyclostationary signals that have a cyclic frequency α in the two data sets $\mathbf{X}(n, m)$ and $\mathbf{Y}(n, p, \alpha)$, the CCST is applied:

$$\hat{R}_{MAS} = \hat{R}_{XX}^{-1} \hat{R}_{XY} \hat{R}_{YY}^{-1} \hat{R}_{YX} \tag{17}$$

The test statistic evaluated to examine the channel is presented as follows:

$$T_{MAS} = -N \log(1 - \rho_1) \tag{18}$$

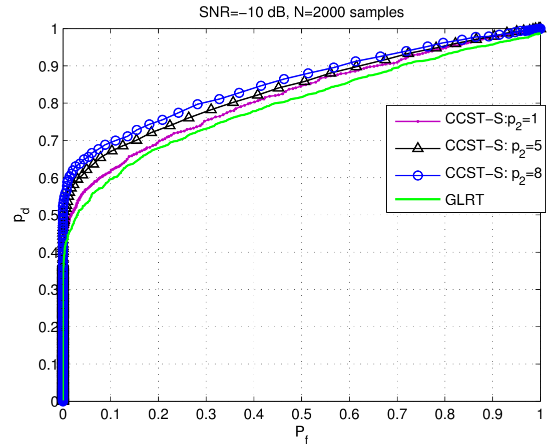

Fig. 1. ROC curves of GLRT and CCST-S for various values of the length of the lag vector

Where ρ_1 is the greatest eigenvalue of $\hat{R}_{MAS}$.
The advantage of this proposed detector with respect to that of [1] is that our algorithm exploits both spatial and temporal diversities, while the detector of [1] is only based on the spatial diversity as the CCST is done over $\mathbf{X}(n, 0)$ and $\mathbf{Y}(n, \tau_1, \alpha)$.

VI. Numerical Results

In this section, we examine the performance of our proposed detectors. The performance of CCST-S is compared to the GLRT cyclostationary detector of [9], and the performance of CCST-M is compared with the CCST detector of [1] that we refer to it by CCST-D. Throughout the simulations, the PU signal is assumed to be down-converted 16-QAM modulated signal. The symbol duration is $1\mu s$ and the sampling frequency, F_s, is 8 MHz. A square-root raised cosine shape is used with a roll-off factor of 0.5. the channel between the PU base station and the ith SU receiver is modeled as flat-fading Rayleigh. The lag vector used in this simulation is $\Gamma_u = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7] = [0, T_s, 2T_s, 3T_s, 4T_s, 5T_s, 6T_s, 7T_s]$ is assigned, where $T_s = \frac{1}{F_s}$.

A. *CCST over SAS*

In figure (1), the number of samples is 2000 and the lag vector length of $\mathbf{r}_1(n, p_1)$ is to fixed to $p_1 = 8$, whereas various values are assigned for the lag vector length, p_2, of $\mathbf{q}_1(n, p_2, \alpha)$. This figure shows the ROC curve which is the variation of the probability of detection (p_d) with respect to the probability of false alarm (p_{fa}). Our proposed algorithm outperforms the GLRT algorithm of [9] for various values of the $\mathbf{q}_1(n)$ lag vector length, p_2, and a fixed $p_1 = 8$. Furthermore, the performance of our algorithm is enhanced by increasing the number of lags p_2.

To show the time diversity effect on the performance of CCST-S, we examine this algorithm performance for various values of Γ's length of the two vectors $\mathbf{r}_1(n, p_1)$ and $\mathbf{q}_1(n, p_2, \alpha)$ which are assumed to have the same length (*i.e.* $p_1 = p_2 = p$). Our simulations are done under various

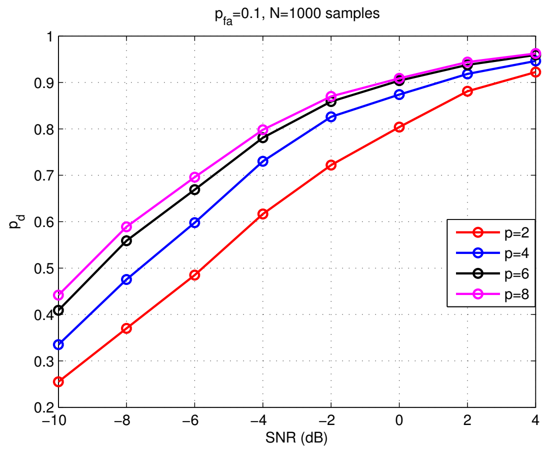

Fig. 2. Time Diversity effect on the performance of CCST-S for $p_{fa} = 0.1$

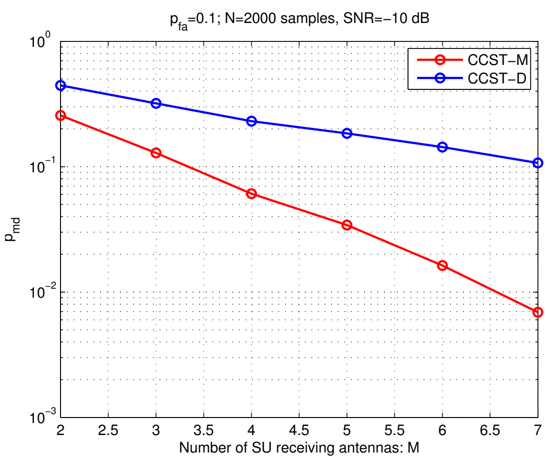

Fig. 3. The probability of missed detection, $p_{md} = 1 - p_d$, for various number of receiving antennas (M) under $p_{fa} = 0.1$

SNR and a constant $p_{fa} = 0.1$. Figure (2) shows the interdependence between CCST-S and the time diversity, where p_d increases progressively when the length of the lag vector increases.

B. CCST over MAS

In this section we evaluate CCST-M for different types of noise: the spatially uncorrelated noise, the spatially correlated noise and the spatially colored but uncorrelated noise. Through the following simulations, $\mathbf{X}(n,p)$ and $\mathbf{Y}(n,p,\alpha)$ are assumed to have the same lag vector which is the same as Γ_u.

a) Spatially Uncorrelated Noise: Figure (3) shows the probability of missed detection (p_{md}) for different number of receiving antennas, M. The number of received samples at each antenna is considered as $N = 1000$ samples, the SNR is fixed to -10 dB and ($p_{fa} = 0.1$). For different M, our algorithm achieves a lower p_{md} than the one of CCST-D. When M increases the gap between CCST-M and CCST-D becomes larger. For M=5 antennas, $p_{md} \simeq 0.2$ for CCST-D and $p_{md} \simeq 0.06$ for CCST-M. When M=7, p_{md} becomes 0.1 approximatly for CCST-D while CCST-M reaches $p_{md} = 0.004$.

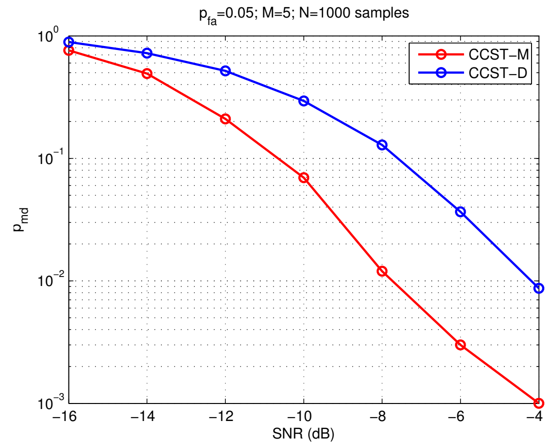

Fig. 4. The probability of missed detection, $p_{md} = 1 - p_d$, for various SNR under $p_{fa} = 0.05$

b) Spatially correlated Noise: Figure (4) shows the simulation results of CCST-M under spatially correlated noise. The correlation among the noise components at the SU receiving antennas is defined as follows:

$$E[w_i(n)w_j^*(n)] = \begin{cases} \sigma_w^2 & i = j \\ \sigma_w^2 \gamma^{|i-j|} & i \neq j \end{cases} \quad ; 1 \leq i, j \leq M; \tag{19}$$

Where γ is the correlation factor and $0 \leq \gamma \leq 1$.

In this simulation the number of SU receiving antennas is $M = 5$, the number of received samples at each antenna is 1000. Figure (4) shows that our algorithm slightly outperforms CCST-D by more than 2 dB. For example, our algorithm reaches $p_{md} = 0.5$ at SNR $= -14dB$, whereas CCST-D reaches this probability at SNR $= -12dB$.

c) Spatially Uncorrelated but colored Noise: In this simulation, we assume that the noise components on the M receiving antennas are spatially uncorrelated but colored. The average SNR is fixed to -12 dB, M=6 antennas and $N = 2000$ samples. As shown in figure (5), CCST-M has a lower Complementary ROC curve than CCST-D. CCST-M achieve $p_{md} = 0.1$ for a $p_{fa} = 0.03$, whereas CCST-D achieve the same p_{md} for $p_{fa} = 0.5$.

VII. Conclusion

In this paper, we presented a new algorithm based on the Canonical Correlation Significance Test (CCST). The main objective of this work was to apply CCST for Single-Antenna System. For that, the time diversity is manipulated. For Multi-Antenna System, both spatial and time diversities are exploited to detect the PU signal. A performance analysis was carried out by simulation to show the effectiveness of our proposed algorithms which outperform other existing ones for various noise models.

References

[1] P. Urriza, E. Rebeiz and D. Cabric, *Multiple Antenna Cyclostationary Spectrum Sensing Based on the Cyclic Correlation Significance Test*, IEEE Journal on Selected Areas in Communications, Vol. 31, pp. 2185-2195, November 2013.

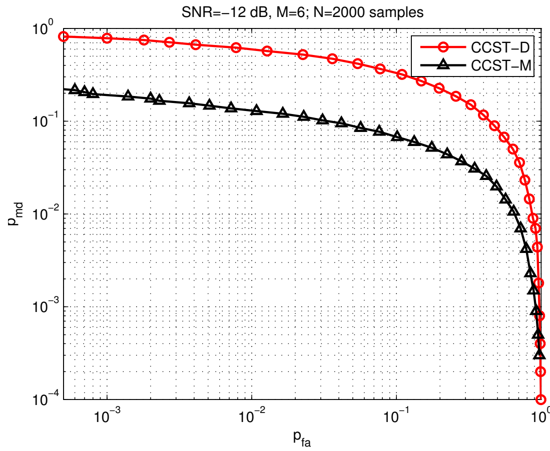

Fig. 5. The probability of missed detection, $p_{md} = 1 - p_d$, for various SNR under $p_{fa} = 0.05$

[2] J. Mitola, *Cognitive radio: Making software radios more personal*,IEEE Pers. Commun., vol. 6, No.4, pp. 13-18, Aug. 1999.

[3] A. Nasser, A. Mansour, K. C. Yao, H. Charara, and M. Chaitou, *Efficient spectrum sensing approaches based on waveform detection*, Third International Conference on e-Technologies and Networks for Development (ICeND), April 2014.

[4] A. Nasser, A. Mansour, K. C. Yao, H. Charara, and M. Chaitou, *Spectrum Sensing for Full-Duplex Cognitive Radio Systems*, 11th International Conference on Cognitive Radio Oriented Wireless Networks, May 2016 (*accepted*).

[5] T. Yucek and H. Arslan, *A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications*, IEEE Communications Surveys & Tutorials, Vol. 11, No. 1, pp. 116 - 130, First Quarter 2009.

[6] R. Tandra and A. Sahai, *SNR Walls for Signal Detection,* IEEE Journal of Selected Topics in Signal Processing, Vol. 2, pp.4-17, February 2008.

[7] M. Naraghi-Poor and T. Ikuma, *Autocorrelation-Based Spectrum Sensing for Cognitive Radio*, IEEE transactions on Vehicular Technology. Vol. 59, No. 2, pp. 718 - 733, February 2010.

[8] Y. Zeng and Y.-C. Liang, *Eigenvalue-Based Spectrum Sensing Algorithms for Cognitive Radio,* IEEE Transactions on Communications, Vol. 57, No. 6, pp. 1784 - 1793, June 2009.

[9] A. Dandawate and G. Giannakis, *Statistical tests for presence of cyclostationarity,* IEEE Transactions on Signal Processing, Vol. 42 , pp. 23552369, September 1994.

[10] I. F. Akyildiz, B. F. Lo and R. Balakrishnan, *Cooperative spectrum sensing in cognitive radio networks: A survey*, Physical Communication, Vol 4, pp. 40 - 62, March 2011.

[11] S. Schell and W. Gardner, *Detection of the number of cyclostationary signals in unknown interference and noise,* 24th Asilomar Conference on Signals, Systems and Computers, Nov 1990.

[12] M. Derakhshani, T. Le-Ngoc, and M. Nasiri-Kenari, *Efficient Cooperative Cyclostationary Spectrum Sensing in Cognitive Radios at Low SNR Regimes*, IEEE Transactions on Wireless Communications, Vol. 10, No. 11, pp. 3754 - 3764, November 2011.

[13] W.Chen, J .P. Rei1 ly and K. M. Wong, *Detection of the number of signals in noise with banded covariance,* IEEE Proceedings - matrices, Radar, Sonar and Navigation, Vol. 143, pp. 289-294, October 1996.

[14] D. N. Lawley, *Tests of significance in canonical analysis,* Biometrika, vol. 46, pp. 5966, Jun. 1959.