FULL-RATE GENERAL RANK BEAMFORMING IN SINGLE-GROUP MULTICASTING NETWORKS USING NON-ORTHOGONAL STBC

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ABSTRACT

In this paper, we propose a novel single-group multicast beamforming technique using non-orthogonal space-time block coding (STBC). In the system, a multi-antenna base station broadcasts its information to a large group of single-antenna users. We introduce a modified max-min fair beamforming optimization problem, which maximizes the worst user's modified Euclidean distance instead of the conventional worst user's signal-to-noise ratio. Our beamforming formulation extends the traditional max-min fair problem, by taking the pairwise error probability of the employed STBC into consideration, which in turns improves the overall system performance. For the resulting non-convex quadratically constrained quadratic program, an iterative inner convex approximation method is devised, in which the non-convex part of the problem is linearized and a sequence of inner convex optimization problems is solved. Simulation results demonstrate considerable improvement for networks with a large number of users.

Index Terms— Space-time block codes, general-rank transmit beamforming, pairwise error probability minimization, iterative convex optimization.

I. INTRODUCTION

In recent years, multicasting networks attract tremendous attention due to their wide application spectrum in, e.g., data broadcasting, and video conferencing. In the context of a single-group multicasting network, a multi-antenna base station (BS) broadcasts its information to a single-group of users subscribed to a particular network service [1–5]. In order to steer the energy in the desired direction, optimal beamformers are generally computed based on two popular optimization designs [1, 6]. In the quality-of-service (QoS) based optimization design, the transmitted power is minimized under signal-to-noise ratio (SNR) constraints at each user. In contrast to this, in the max-min fair (MMF) beamforming design, the SNR of the worst user is maximized under a transmitted power constraint. Both optimization problems are equivalent to each other up to scaling, where the scaling constant can be easily determined [1, 6]. Moreover, both problems are also NP-hard and non-convex [1, 6–9].

As shown in [10], when the number of users does not exceed three, the semi-definite relaxation (SDR) technique [1] yields optimal rank-one solutions for both optimization problems. However, as the number of users further increases, it is generally observed that the rank of the beamforming matrix also increases [10]. To overcome the problem of the SDR performance degradation, many algorithms were proposed, see, e.g., [11–14]. The authors of [14] proposed a randomization technique, which was employed in networks with a large number of users to obtain feasible beamforming solutions, which are however suboptimal in general.

In [10, 15], orthogonal space-time block coding (OSTBC) based beamforming designs were introduced to increase the degrees of freedom, where the famous Alamouti code was utilized. This approach doubles the degrees of freedom due to the use of two beamformers with which the Alamouti code is transmitted. The beamformer optimization problem, if addressed using the SDR technique, yields optimal solutions when the rank of the solution matrix does not exceed two. This implies that the optimality of the beamformer design is guaranteed when the number of users does not exceed eight [10]. To further increase the degrees of freedom to serve more users, higher order OSTBC can be applied. The orthogonality property of the OSTBC enables the users to perform simple symbol-by-symbol detection [16]. However, the extension to higher order OSTBC is associated with a reduced code rate [16]. A general-rank beamforming extension involving higher order real-valued OSTBCs has recently been proposed in [17] for unicasting networks. Although this approach achieves a full code rate, it is not applicable in our considered single group multicasting scenario because it requires dedicated beamformers per user. A space-time trellis code (STTC) based single-group multicasting general-rank beamforming scheme was proposed in [18] to yield general-rank beamforming solution of the optimization problem. Despite the increased degrees of freedom and the improved performance compared with rank-one and rank-two approaches, STTC based beamforming is associated with a high decoding complexity.

In order to further increase the degrees of freedom in the beamformer designs and to accommodate more users in the network in this paper we extend the rank-one and the Alamouti based rank-two beamforming designs to general rank beamforming in single group multicast networks. This generalization is not straight forward as it involves the use of higher STBC for which full rate orthogonal STBC do not exist. Therefore we propose to use general (non-orthogonal) full-rate STBC sacrificing the orthogonality and ability to apply simple symbol-wise decoding at the receivers. In this case the general approach applied in common QoS based rank-one and rank-two beamforming design, to consider the symbol-wise post detection SNR is not longer meaningful and alternative design criteria need to be used to asses the users' QoS. In our work we propose to minimize the worst user's pairwise error probability (PEP) under a total power constraint at the transmitter. Towards this end, we formulate a modified max-min fair (MMMF) optimization problem. As this problem is non-convex and NP-hard, we solve it by employing an iterative inner approximation algorithm. At each iteration, the non-convex part of the problem is successively approximated by its first-order Taylor approximation, which yields a convex approximate problem that can be solved efficiently. Our design can be applied to any STBC. Simulation results show that our proposed design achieves a lower frame error rate (FER) than both of the rank-one and rank-two methods, in systems with a large number of users.

Throughout this paper, $(\cdot)^H$, $\mathrm{tr}(\cdot)$, $(\cdot)^*$, $\|\cdot\|_F$, $(\cdot)^T$, and $\bigotimes$ denote the Hermitian transpose, the matrix trace, the complex conjugate, the Frobenius norm, the matrix transpose, and the Kronecker product, respectively. Furthermore, $\mathbf{I}_n$ and $\overline{\mathrm{diag}}(e_1, \cdots, e_n)$, denote the $n \times n$ identity matrix and the anti-diagonal matrix with anti-diagonal elements equal to $e_1, \cdots, e_n$ starting from the upper right side to the lower left side of the matrix. The notation $\mathcal{CN}(\mathbf{a}, \mathbf{B})$ represents the complex multivariate Gaussian distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{B}$. Furthermore, we use $\mathrm{E}(\cdot)$ and $\mathrm{Pr}(\cdot)$

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to denote the statistical expectation and the probability function, respectively.

II. SYSTEM MODEL

We consider a wireless multicasting network, where a BS equipped with N_t antennas broadcasts common information symbols to M single-antenna users [1]. In this work, we assume that the CSI of each user is available both at the transmitter and the receiver side. Moreover, STBC is applied at the BS to obtain the $T \times K$ code matrix $\mathbf{C} \in \mathcal{K}_{\mathcal{P}}$, where T is the number of transmission time slots, K is the number of spatial streams, and $\mathcal{K}_{\mathcal{P}}$ is the set of STBC matrices of cardinality $\mathcal{P}$. The STBC matrix $\mathbf{C}$ is then broadcasted to all users using the $N_t \times K$ beamformer matrix $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_K]$. Hence the lth column of the code matrix is transmitted with beamforming vector $\mathbf{w}_l$, which can also be considered as a virtual MIMO antenna from which the code matrix $\mathbf{C}$ is transmitted. The use of K beamformers increases the degrees of freedom in the beamformer design and allows to accommodate a large number of users and a large number of constraints in our beamformer design. The $T \times 1$ received signal at the jth user $\mathbf{y}_j$ is given by

$$\mathbf{y}_j = \mathbf{C}\mathbf{W}^H\mathbf{h}_j + \mathbf{n}_j, \tag{1}$$

where $\mathbf{h}_j$ is the $N_t \times 1$ channel vector corresponding to the jth user, and $\mathbf{n}_j$ is the $T \times 1$ vector of additive white Gaussian receiver noise with $\mathbf{n}_j \sim \mathcal{CN}(\mathbf{0}, \sigma_j^2\mathbf{I}_T)$. Our objective is to design the beamformer matrix $\mathbf{W}$ at the BS under a total power constraint in order to minimize the pairwise error probability of the worst user, which we address in the next section.

III. SINGLE GROUP MULTICASTING BEAMFORMER DESIGN

We propose a non-orthogonal STBC based general rank beamforming approach, which enjoys full rate. However, in contrast to the existing Alamouti based beamforming, due to the non-orthogonality property symbol-by-symbol detection cannot be applied at the receiver side. Therefore, vector-wise detection MIMO detection must be applied at the receivers. In this case, the expressions for the symbol-wise post-detection SNR used in the existing rank-two beamforming designs to describe the QoS requirements in terms of decoding performance at the receiver do not apply to non-orthogonal STBC based beamforming. Therefore, in our beamformer design we propose to use the PEP expression which holds for vector-wise detection of any STBC matrix.

In this section, we first present the expression for the PEP at a given user. The worst user's PEP performance is then used as the objective in our beamformer design. Given the instantaneous CSI $\mathbf{h}_j$, the conditional PEP at the jth user under maximum-likelihood decoding is given by [16]

$$\begin{aligned}\text{PEP} &= \Pr\left(\mathbf{C}^m \rightarrow \mathbf{C}^n | \mathbf{h}_j\right) \\ &= \mathrm{Q}\left(\sqrt{\frac{\gamma}{2}\mathrm{tr}\left(\mathbf{h}_j^H\mathbf{W}\left(\mathbf{C}^m - \mathbf{C}^n\right)^H\left(\mathbf{C}^m - \mathbf{C}^n\right)\mathbf{W}^H\mathbf{h}_j\right)}\right),\end{aligned} \tag{2}$$

where $\mathbf{C}^m \in \mathcal{K}_{\mathcal{P}}$ denotes the mth STBC matrix transmitted by the BS, $\mathbf{C}^n \in \mathcal{K}_{\mathcal{P}}$ $(n \neq m)$ denotes the wrongly decoded STBC matrix at the jth user, $\gamma = 1/\sigma_j^2$ is the SNR, and $\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right)\mathrm{d}t$ is the Gaussian Q-function [16].

The objective of our approach is to design the beamformer matrix $\mathbf{W}$ to minimize the worst user's PEP performance under a total power constraint for the scenario of single-group multicasting networks using STBC. As the Q-function is monotonically decreasing, using (2) our optimization problem can be formulated as

$$\gamma_{\text{BF-STBC}} = \max_{\mathbf{W}} \min_{j=1,\cdots,M} \min_{\substack{\mathbf{C}^m,\mathbf{C}^n \in \mathcal{K}_{\mathcal{P}};\\ n\neq m}} \mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_{mn}\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right) \tag{3a}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right) \leq P, \tag{3b}$$

where P is the total transmitted power at the BS, $\tilde{\mathbf{H}}_j = \mathbf{h}_j\mathbf{h}_j^H$, $\mathbf{D}_{mn} = \mathbf{C}^m - \mathbf{C}^n$ is the code difference matrix, and $\tilde{\mathbf{D}}_{mn} = \mathbf{D}_{mn}^H\mathbf{D}_{mn}$ is the distance matrix [16]. Define the set of matrices $\tilde{\mathcal{D}} = \left\{\tilde{\mathbf{D}}_{mn}\right\}_{\substack{m,n=1\\ m\neq n}}^{\mathcal{P}}$, which contains all the distance matrices formed by distinct codewords.

Solving the MMMF problem (3) over all distance matrices $\tilde{\mathbf{D}}_{mn} \in \tilde{\mathcal{D}}$ requires very high computation complexity due to the fact that the cardinality of $\tilde{\mathcal{D}}$, which is $\mathcal{P}^2 - \mathcal{P}$, is large for higher-rank STBC (> 2). To reduce the computational complexity, we define the set of matrices $\tilde{\mathcal{D}}_{\text{sub}} = \left\{\tilde{\mathbf{D}}_1, \ldots, \tilde{\mathbf{D}}_{\mathcal{L}}\right\}$ as the largest subset of $\tilde{\mathcal{D}}$, for which there exists no pair $\tilde{\mathbf{D}}_l \in \tilde{\mathcal{D}}_{\text{sub}}$ and $\tilde{\mathbf{D}}_{mn} \in \tilde{\mathcal{D}}_{\text{sub}}$, such that $\tilde{\mathbf{D}}_l \succ \tilde{\mathbf{D}}_{mn}$, where $\tilde{\mathbf{D}}_l \succ \tilde{\mathbf{D}}_{mn}$ means that $\tilde{\mathbf{D}}_l - \tilde{\mathbf{D}}_{mn}$ is positive definite.

Then, it follows from **Lemma 1** given at the end of this section, and the definition of the subset $\tilde{\mathcal{D}}_{\text{sub}}$, that

$$\mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_{mn}\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right) \geq \mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right). \tag{4}$$

Therefore, we obtain that

$$\min_{\substack{\mathbf{C}^m,\mathbf{C}^n \in \mathcal{K}_{\mathcal{P}};\\ n\neq m}} \mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_{mn}\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right) = \min_{l=1,\cdots,\mathcal{L}} \mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right). \tag{5}$$

Making use of the identity (5), problem (3) can be equivalently written as

$$\gamma_{\text{BF-STBC}} = \max_{\mathbf{W}} \min_{j=1,\cdots,M} \min_{l=1,\cdots,\mathcal{L}} \mathrm{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right) \tag{6a}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right) \leq P. \tag{6b}$$

In general, for practical STBCs, $\mathcal{L}$ is much smaller than $\mathcal{P}^2 - \mathcal{P}$, and thus the computational complexity of the problem (6) is significantly lower than that of the problem (3).

The optimization problem (6) belongs to the class of non-convex quadratically constrained quadratic programs, which are difficult to solve and NP-hard in general. In the following section, we introduce an iterative algorithm, which consists in solving a sequence of inner convex approximation problems. Due to the use of inner approximations, the solutions of the approximate problems are always feasible for the original problem in (6), provided that the algorithm is initialized with a feasible point.

Lemma 1. *Given three positive definite Hermitian matrices $\mathbf{\Sigma}$, $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$. If $\mathbf{\Delta}_1 \succ \mathbf{\Delta}_2$, then $tr\left(\mathbf{\Sigma}\mathbf{\Delta}_1\right) > tr\left(\mathbf{\Sigma}\mathbf{\Delta}_2\right)$.*

Proof. Rewrite $\mathbf{\Sigma} = \mathbf{\Lambda}^H\mathbf{\Lambda}$. Given $\mathbf{\Delta}_1 \succeq \mathbf{\Delta}_2$, we have $\mathrm{tr}\left(\mathbf{\Sigma}\mathbf{\Delta}_1\right) - \mathrm{tr}\left(\mathbf{\Sigma}\mathbf{\Delta}_2\right) = \mathrm{tr}\left(\mathbf{\Lambda}^H\mathbf{\Lambda}\left(\mathbf{\Delta}_1 - \mathbf{\Delta}_2\right)\right) = \mathrm{tr}\left(\mathbf{\Lambda}\left(\mathbf{\Delta}_1 - \mathbf{\Delta}_2\right)\mathbf{\Lambda}^H\right) > 0$, which follows from the definition of positive definiteness. □

IV. CONVEX APPROXIMATION TECHNIQUE

In this section, we solve the optimization problem in (6) iteratively by using a first order Taylor approximation to transform the non-convex problem of (6) to a linear convex problem. By introducing an auxiliary variable t, the optimization problem in (6) can be equivalently written as

$$\min_{\mathbf{W},\ t} \quad t \tag{7a}$$
$$\text{s.t.} \quad t > 0 \tag{7b}$$
$$\text{tr}\left(\frac{\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j}{\sigma_j^2}\right) \geq \frac{1}{t},\ j = 1, \cdots, M,\ l = 1, \cdots, \mathcal{L} \tag{7c}$$
$$\text{tr}\left(\mathbf{W}\mathbf{W}^H\right) \leq P. \tag{7d}$$

Note that by solving the problem (7), a lower bound on $\gamma_{\text{BF-STBC}}$ in (6) is obtained and given by $1/t$. The trace constraint (7c) can be rewritten as

$$\lambda_{j,l}\left(\mathbf{W}, t\right) = \frac{\sigma_j^2}{t} - \text{tr}\left(\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j\right) \leq 0. \tag{8}$$

The first term σ_j^2/t in (8) is a convex function. From the definition of $\tilde{\mathbf{D}}_l$ and $\tilde{\mathbf{H}}_j$, it follows that the second term $\text{tr}\left(\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j\right)$ is also convex. Hence, $\lambda_{j,l}(\mathbf{W}, t)$ in (8) represents a difference of convex (DC) functions. Therefore, the optimization problem (7) belongs to the class of DC programs [19–23]. The problem of (8) can be approximately solved by following a similar procedure as the iterative approach proposed in [24]. Let us denote $\mathbf{W}^{(k)}$ as the beamformer matrix generated at the kth iteration. The beamformer matrix at the $(k+1)$th iteration is subsequently generated by

$$\mathbf{W}^{(k+1)} = \mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}, \tag{9}$$

where $\Delta\mathbf{W}^{(k)}$ is the update at iteration k. At each iteration k, we use the first order Taylor approximation of the $\lambda_{j,l}(\mathbf{W}, t)$ around the current point $\mathbf{W}^{(k)}$ to compute the update $\Delta\mathbf{W}^{(k)}$. Replacing the concave part in the constraint of (8) by its first order Taylor approximation and keeping the convex part unchanged, we can approximate the non-convex constraint (8) as

$$\begin{aligned}\bar{\lambda}_{j,l}^{(k)}\left(\Delta\mathbf{W}^{(k)}\right) = &\frac{\sigma_j^2}{t} - \text{tr}\left(\mathbf{W}^{(k)}\tilde{\mathbf{D}}_l{\mathbf{W}^{(k)}}^H\tilde{\mathbf{H}}_j\right)\\ &- 2\text{tr}\left(\text{Re}\left(\mathbf{W}^{(k)}\tilde{\mathbf{D}}_l\Delta{\mathbf{W}^{(k)}}^H\tilde{\mathbf{H}}_j\right)\right) \leq 0.\end{aligned} \tag{10}$$

Comparing (8) and (10), we observe that

$$\lambda_{j,l}^{(k)}\left(\mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}\right) \leq \bar{\lambda}_{j,l}^{(k)}\left(\Delta\mathbf{W}^{(k)}\right). \tag{11}$$

The term on the right hand side of (11) represents a linearization of the non-convex part of (8). Inserting (9) in (7) and considering $\Delta\mathbf{W}^{(k)}$ as the new optimization variable, problem (7) becomes

$$\min_{\Delta\mathbf{W}^{(k)},\ t^{(k)}} \quad t^{(k)} \tag{12a}$$
$$\text{s.t.} \quad t^{(k)} > 0 \tag{12b}$$
$$\bar{\lambda}_{j,l}^{(k)}\left(\Delta\mathbf{W}^{(k)}\right) \leq 0,\ j = 1, \cdots, M,\ l = 1, \cdots, \mathcal{L} \tag{12c}$$
$$\text{tr}\left(\left(\mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}\right)\left(\mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}\right)^H\right) \leq P. \tag{12d}$$

It can be noticed from (11) that the problem in (12) represents an inner approximation of the optimization problem of (7), and it can be solved by using standard convex optimization solvers without any further relaxations. The algorithm to solve the problem (12) is summarized in the following table. A feasible point is generated randomly, then the algorithm iterates until the step size ρ falls below a predetermined threshold value ϵ.

According to [25], the problem of (12) is globally convergent if the variables are within a compact set, which

Algorithm 1: Iterative algorithm

begin
  Set $k = 0$, choose $\mathbf{W}^{(0)}$ randomly.
  Set $t^{(0)} = 1/(\min_{j,l}(\mathbf{W}\tilde{\mathbf{D}}_l\mathbf{W}^H\tilde{\mathbf{H}}_j))$.
  Set $\rho > \epsilon$.
  while $\rho > \epsilon$ **do**
    $k = k + 1$.
    Compute $(\Delta\mathbf{W}^*,\ {t^{(k)}}^*)$ by solving (12).
    Update $\mathbf{W}^{(k+1)} = \mathbf{W}^{(k)} + \Delta\mathbf{W}^*$ by using $\mathbf{W}^{(k)}$.
    Calculate $\rho = |t^{(k)} - t^{(k+1)}|/t^{(k)}$.
  end
end
return $\mathbf{W}^{(k+1)}$, $t^{(k+1)}$.

is closed and bounded by definition. These conditions are met in our approach first because the power constraints $\text{tr}\left(\left(\mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}\right)\left(\mathbf{W}^{(k)} + \Delta\mathbf{W}^{(k)}\right)^H\right) \leq P$ impose an upper bound on the feasible weight vectors set, and second because the variable t is minimized towards a positive lower value.

V. SIMULATION RESULTS

In our simulations we consider a wireless single group multicasting network consisting of a single BS with $N_t = 4$ transmit antennas and a group of single-antenna users. We assume independent flat Rayleigh fading channels with circularly symmetric unit-variance channel coefficients. The BS applies a quasi-orthogonal STBC (QOSTBC) to transmit the $T \times K$ encoded matrix $\mathbf{C}$, where $T = 4$. We assume that the channel coefficients are quasi-static over one frame duration and may change arbitrarily from one frame to another. The encoded matrix $\mathbf{C}$ is given by [16]

$$\mathbf{C} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}, \tag{13}$$

with s_1, s_2, s_3, and s_4 being the quadratic phase-shift keying (QPSK) modulated input symbols. The set $\mathcal{K}_{\mathcal{P}}$ of the QOSTBC matrices has a cardinality $\mathcal{P} = 256$. Numerical results show that for this setup, $\mathcal{L} = 3$ in (6a), and thus the computational complexity of performing the minimization in (3a) is significantly reduced. The corresponding $\mathcal{L}$ distance matrices with the minimum distance are given by

$$\mathbf{D}_1 = \frac{1}{2}\sum_{i=1}^{T}|s_i|^2\mathbf{I}, \tag{14a}$$
$$\mathbf{D}_2 = \sum_{i=1}^{T}|s_i|^2\left(\mathbf{I} + \overline{\text{diag}}\left(1, -1, -1, 1\right)\right), \tag{14b}$$
$$\mathbf{D}_3 = \sum_{i=1}^{T}|s_i|^2\left(\mathbf{I} + \overline{\text{diag}}\left(-1, 1, 1, -1\right)\right), \tag{14c}$$

where the symbol transmission power is normalized to one, i.e., $|s_i|^2 = 1, (i = 1, 2, 3, 4)$. The simulation results are averaged over 10^4 Monte-Carlo runs and 100 independent channel realizations. In Algorithm 1, we assume a precision threshold value $\epsilon = 10^{-3}$ and the transmission power $P = 1$.

Fig. 1 illustrates the obtained lower and upper bounds on $\gamma_{\text{BF-STBC}}$ as defined in (6) versus the number of users for different approaches. In this figure, the theoretical upper bound is obtained

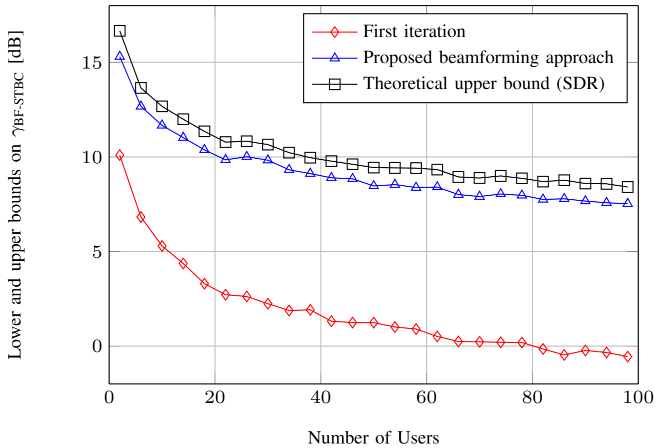

Fig. 1. Lower and upper bounds on $\gamma_{\text{BF-STBC}}$ at the worst user versus the number of users at SNR=10 dB and $N_t = 4$.

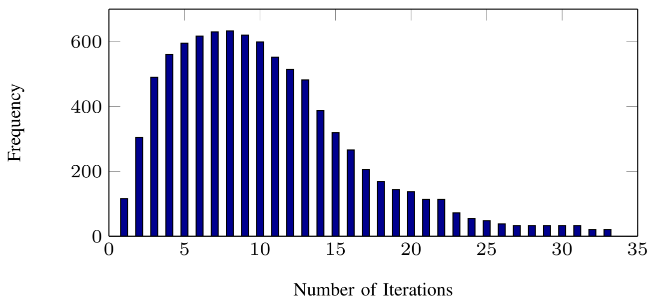

Fig. 2. A histogram of the number of iterations for a system of 64 users and $N_t = 4$.

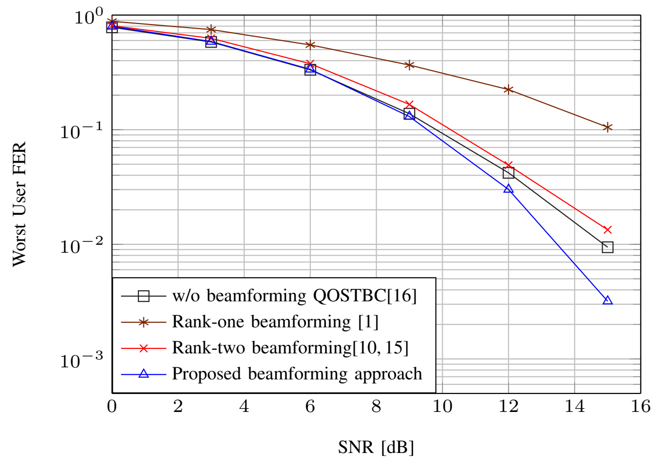

Fig. 3. System performance versus SNR for a system of 64 users and $N_t = 4$.

based on SDR by first equivalently writing the problem in (7) using the vectorization properties as

$$\max_{\mathbf{X}} \min_{\substack{j=1,\cdots,M;\\ l=1,\cdots,\mathcal{L}}} \operatorname{tr}\left(\frac{\mathbf{X}\left(\tilde{\mathbf{D}}_l \otimes \tilde{\mathbf{H}}_j\right)}{\sigma_j^2}\right) \tag{15a}$$

$$\text{s.t.} \quad \operatorname{tr}(\mathbf{X}) \le P,\ \mathbf{X} \succeq 0,\ \operatorname{rank}(\mathbf{X}) \le 1, \tag{15b}$$

where $\mathbf{X} = \operatorname{vec}(\mathbf{W}^T)(\operatorname{vec}(\mathbf{W}^T))^H \;\Leftrightarrow\; \mathbf{X} \succeq 0$ and $\operatorname{rank}(\mathbf{X}) \le 1$, and then dropping the rank 1 constraint in (15b).

In Fig. 1, the curve with the label "Proposed approach" represents the obtained value of $1/t^{(k+1)}$ in (7a) using Algorithm 1, and the "First iteration" curve represents $1/t^{(1)}$ obtained by running Algorithm 1 for only one iteration. This curve provides an indication of the convergence of Algorithm 1. From the figure, we observe that $1/t^{(k+1)}$ obtained by Algorithm 1 converges to the theoretical upper bound, which indicates that Algorithm 1 produces near optimal solutions of problem (7). It can also be observed that all the lower and upper bounds decrease by increasing the number of users.

Fig. 2 displays a histogram of the number of iterations required to achieve the precision threshold $\epsilon = 10^{-3}$ in Algorithm 1. In this simulation we consider a group of 64 users and an SNR= 10 dB. It can be observed that the proposed algorithm produces the solution after approx. $8 - 10$ iterations. We remark that a proper initialization of Algorithm 1 leads to a faster convergence.

In order to make a fair comparison with the existing algorithm, we use the FER as a performance metric, which is defined as the average probability a transmitted code matrix $\mathbf{C}$ is wrongly decoded at the receiver side. Fig. 3 displays the worst user's FER performance of the beamformer design using Algorithm 1, compared with the competing state-of-the-art beamforming techniques for this problem, i.e., the rank-one beamforming [1], and the rank-two beamforming [10, 15]. In Fig. 3, we also plot the FER performance of the conventional QOSTBC [16]. We assume a system with a group of 64 users. The worst user's FER of the rank-two beamforming is calculated for a frame composed of four symbols, each two symbols represent one Alamouti matrix as in [10, 15]. For the rank-one beamforming, the worst user's FER is shown using a frame which also contains four symbols. Each symbol is obtained from solving the conventional rank-one beamforming problem as in [1]. The transmitted power of each approach at each time slot is normalized to be 1. This ensures a fair comparison between our approach and other approaches. From Fig. 3, we see that the performance of the proposed approach significantly outperforms the other approaches, and a gain of approx. $1 - 2$ dB can be observed. Furthermore, our approach yields a higher diversity gain (in terms of the slope of the FER curve in the high SNR region) than the scheme without beamforming.

Fig. 4 demonstrates a similar comparison as in Fig. 3 for different numbers of users, assuming the SNR is 10 dB. It can be observed that rank-one beamforming exhibits the best performance for a group of 2 users. However, the performance of rank-one beamforming degrades severely as the number of users increases. As the number of users increases from 3 to 30, the rank-two beamforming outperforms all existing approaches. As the number of users further increases, our proposed approach begins to deliver the best performance compared with both of the rank-one and rank-two beamforming. The performance of our proposed approach converges to that of the scheme without beamforming for a system with more than 74 users at SNR= 10 dB. Moreover, the performance of our proposed approach does not degrade severely by increasing the number of users. We remark that due to the increased degrees of freedom in the proposed approach, more constraints, i.e., additional shaping constraints [17, 26] can be included in the optimization problem (6), which applies in many practical applications, e.g., in the cognitive radio context. However, including more constraints in the existing QOSTBC design without beamforming is not possible. This marks an important advantage of the proposed approach over the existing QOSTBC techniques.

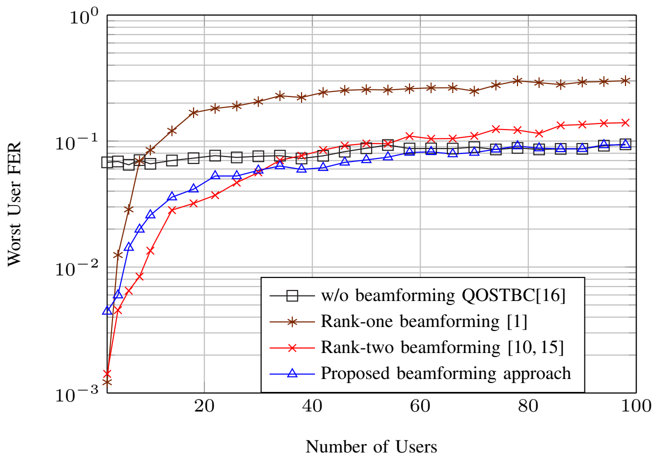

Fig. 4. System performance versus the number of users at SNR= 10 dB and $N_t = 4$.

VI. CONCLUSION

In this paper, we considered general rank beamforming in single-group multicasting networks, to increase the number of degrees of freedom in the beamformer design and accommodate a large number of users in the network. Our proposed beamformer design is a nontrivial generalization of the recently developed rank-one and rank-two beamforming approaches. In order to guarantee full-rate transmission, OSTBC instead of OSTBC is employed at the beamforming transmitter, however, at the cost of orthogonality. In this case the users' symbol-wise post detection SNR is no longer a meaningful objective in the QoS based beamformer design problem. Therefore, in our general rank beamformer design we proposed to maximize the worst users' pairwise error probability.

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