Modified Tone Reservation for PAPR Reduction in OFDM Systems

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***Abstract*—One of the main drawbacks of orthogonal frequency division multiplex modulation is its high peak-to-average power ratio (PAPR) which can induce poor power efficiency at high power amplifier. Tone reservation (TR) is the most popular PAPR mitigation technique that uses a set of reserved tones to design peak cancelling signal for PAPR reduction. Finding an effective peak cancelling for PAPR reduction in the time domain by using only a small number of reserved tones, is not straightforward. Therefore, we are led to a trade-off between computational complexity and PAPR reduction. The TR method based on the gradient projection algorithm gives the best compromise. In this paper, we propose to modify the classical TR structure. The new proposed method achieves an improvement up to $1.2\,dB$ in terms of PAPR performance without increasing the complexity. The effectiveness of this solution is confirmed through theoretical analysis and simulation results.**

***Index Terms*—OFDM, PAPR, Tone Reservation, CCDF**

I. Introduction

Orthogonal frequency division multiplex (OFDM) modulation, although being used in standards such as IEEE 802.16, IEEE 802.11a/g., HIPERLAN/2, and digital video broadcasting terrestrial (DVB-T2) [1], suffers from high peak-to-average power ratio (PAPR). A signal with high PAPR requires a linear high power amplifier (HPA), which is inefficient in terms of power consumption. To overcome this downside, a larger number of PAPR mitigation techniques have been proposed in the scientific community [2]. Tone reservation (TR) [3] is the most popular adding signal technique for PAPR reduction without bit error rate (BER) distortion and out-of-Band pollution. In fact, the TR approach consists in using a set of reserved tones in order to design the peak cancelling signal. As TR works on reserved tones, no additional signal processing is required at the receiver to extract the data information. Due to these reasons, TR is quite popular for practical implementations and therefore, it was adopted for commercial standards such as (DVB-T2).

There are several studies in the literature about finding the suitable peak cancelling subject to TR constraint [3], [4], [5], [6], [7]. In [3] and [4] the authors propose to use a quadratic constrained quadratic program (QCQP) and a second order cone program (SOCP) respectively, to set the appropriate values on the reserved tones for PAPR mitigation. However, these approaches increase drastically the mean power of the signal and their complexity is very high, which make them not adequate for real time systems. To lower the computation complexity, various methods have been proposed to design the peak cancelling signal subject to TR principle, such as the active-set method [5], one tone-one peak (OPTOP) method [6], and transformation of clipping method as TR method [8]. Actually, the best technique that provides the best compromise between PAPR reduction and computational complexity is the TR method that uses the gradient project algorithm to compute the adding signal by maximizing the signal to clipping ratio, which is equivalent to minimizing the clipping noise [7]. In this paper, a new TR method using the gradient project algorithm is proposed. The main idea of this approach is to modify the classical TR structure by exploiting the decomposition of the useful OFDM symbol as a mixture of two times shorter multicarrier symbols, each containing two times less carriers. Thus, by decomposing conjointly the clipping noise with respect to the decomposition of the useful OFDM symbol, this new approach named modified tone reservation (MTR) consists in minimizing these partial clipping noise subject to the TR constraint with respect to the different components of the useful OFDM symbol after the decomposition.

The remainder of this paper is organized as follows: Section II, briefly reviews the TR principle. Section III describes the MTR principle. The simulation results are presented in Section IV, while in Section V a conclusion is drawn.

II. Overview of TR Techniques

A. Notations and Definitions

Throughout this paper, an OFDM symbol $x(t)$ of duration T_u is used and expressed as follows

$$x(t) = \sqrt{\frac{1}{M}} \sum_{m=0}^{M-1} X_m e^{j2\pi mFt} \;,\; 0 \leq t \leq T_u. \quad (1)$$

Where M is the total number of carriers, $F = \frac{1}{T_u}$ is the inter-carrier spacing and X_m the symbol carried out by the $m-th$ carrier during T_u. After oversampling the signal by a factor L, x_l, $\{l = 0, \ldots, LM-1\}$ are the discrete time domain samples at the instant lT_e where $T_e = \frac{T_u}{LM}$.
We denote $\mathbf{x} = [x_0, \ldots, x_{LM-1}]$ the vector of the samples of $x(t)$ after the oversampling operation. This vector can be efficiently computed by using an inverse fast Fourier transform (IFFT), and it can be expressed as follows

$$\mathbf{x} = \mathbb{F}_M \check{\mathbf{X}}. \quad (2)$$

Where $\mathbb{F}_M$ is the normalized IFFT matrix scaled by $\sqrt{L}$ and $\check{\mathbf{X}}$ is the vector obtained by zero-padding on $\mathbf{X}$. Let $\mathcal{R} = [m_1, \dots, m_{M_R}] \subset [0, \dots, M-1]$ be a set of M_R reserved tones. Then, $\mathbb{F}_{M,\mathcal{R}}$ denotes a sub-matrix of $\mathbb{F}_M$ indexed by the column vectors that belong to $\mathcal{R}$. The matrix $\mathbb{F}_{M,\mathcal{R}}$ has LM rows and M_R columns and is used in [7] for the computation of the suitable frequencies data carried by the reserved tones for PAPR reduction.

Hereafter, $\mathbb{F}_K$ represents, more generally, the normalized IFFT matrix of size LK scaled by $\sqrt{L}$ and $\mathbb{F}_{K,\mathcal{X}}$ a sub-matrix of $\mathbb{F}_K$ containing only the column that belong to $\mathcal{X}$.

B. Overview of TR based approaches for PAPR mitigation

The main idea of the TR approach is to use a set of M_R reserved tones (named $\mathcal{R}$) to design the peak cancelling signal. TR approach prevents then BER degradation and out-of-band emissions [7]. In fact, let $\mathbf{c}$ be the peak cancelling addressed to reduce the PAPR of the useful symbol $\mathbf{x}$. Therefore, under the TR constraint $\mathbf{x}+\mathbf{c}$ satisfies the following equation

$$\frac{1}{L}(\mathbf{F}^{\text{Col}}_{M,m})^H(\mathbf{x}+\mathbf{c}) = X_m + C_m = \begin{cases} X_m & \text{if } m \notin \mathcal{R} \\ C_m & \text{if } m \in \mathcal{R} \end{cases}, \quad (3)$$

where $\mathbf{F}^{\text{Col}}_{M,m}$ is the m-th column of the matrix $\mathbb{F}$. The scaling by $\frac{1}{L}$ is due to $(\mathbb{F}_M)^H\mathbb{F}_M = L\mathbb{I}$ with $\mathbb{I}$ is the identity matrix. From the (3), it can be remarked that $\mathbf{c} = \mathbb{F}_{M,\mathcal{R}}\mathbf{C}$. Then the TR method rely on finding the suitable frequency vector $\mathbf{C}$. For this purpose, several studies have been proposed in the literature, in order to find the effective peak cancelling signal $\mathbf{c}$ for PAPR reduction [3], [4], [5], [7], [8]. All these approaches are focused on the way of the effective $\mathbf{c}$ can be computed. For instance, in [7] the authors have formulated the computation of $\mathbf{c}$ as a Quadratically Constrained Quadratic Program (QCQP). This approach allows to achieve a good performance in terms of PAPR reduction. Nevertheless, this approach requires high numerical complexity. For real time systems, the best technique that provides the best compromise between PAPR reduction and computational complexity is the TR-Gradient Project (TR-GP) [7] which consists in minimizing the clipping noise by adding a peak cancelling signal subject to TR constraint [3]. This algorithm has been suggested for PAPR reduction in the DVB-T2 standard [1].

Let $\mathbf{x}+\mathbf{c}$ be the transmitted symbol after PAPR reduction. The clipping noise of this symbol is defined in [7] as follows:

$$J(.) = \|\mathbf{x}+\mathbf{c} - f(\mathbf{x}+\mathbf{c}, A)\|_2^2. \quad (4)$$

Where A is a clipping magnitude and $f(.,.)$ a hard clipping function. Fig. 1 gives an illustration of $J(.)$ that we want to minimize by adding a peak cancelling signal.

The TR-GP method, proposed in [7], relies on finding the peak cancelling signal $\mathbf{c}$ by minimizing $J(\mathbf{c})$ via the gradient project algorithm [9] subject to TR constraint, i.e,

$$\min_{\mathbf{C}} \quad (\|\mathbf{x}+\mathbb{F}_{M,\mathcal{R}}\mathbf{C} - f(\mathbf{x}+\mathbb{F}_{M,\mathcal{R}}\mathbf{C})\|_2^2) \quad (5)$$

Starting with the initial condition of $\mathbf{x}^{(0)} = \mathbf{x}$, it has been shown in [7] that the TR-GP method reduce iteratively the PAPR as follows,

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - \mu \sum_{|x_l^{(i)}|>A} \alpha_l^{(i)}\mathbf{P}_l, \quad (6)$$

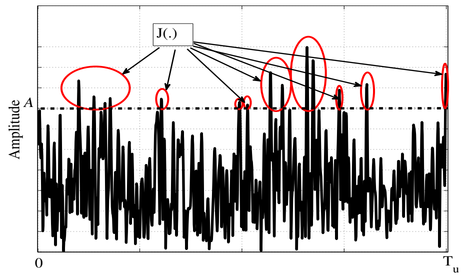

Fig. 1. Illustration of the cost function in TR-GP method.

where $\mathbf{P}_l$ is the l-th line of matrix $\mathbb{P} = \mathbb{F}_R(\mathbb{F}_R)^H$ and $\alpha_l^{(i)} = (x_l^{(i)} - Ae^{j\arg(x_l^{(i)})})$. Note that the matrix $\mathbb{P}$ can be pre-calculated and stored, the numerical complexity of the TR-GP is then $\mathcal{O}(ML)$.

III. Modified Tone Reservation Principle and Problem Formulation

The MTR approach consists in modifying the classical TR structure by using the decomposition of the useful OFDM symbol as a mixture of 4 multi-carriers (after first decomposition) signal or 16 multi-carriers signal (after the double decomposition), see Annexe III-A and Subsection III-B. Thus, by decomposing conjointly $J(.)$ as a sum of a partial clipping noise with respect to the decomposed parts of the useful OFDM symbol, the MTR approach propose to minimize conjointly these partial clipping noise by using the gradient project algorithm (MTR-GP) subject to prevent BER from degrading.

A. The MTR approach based on the first decomposition

Let $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^3$ and $\mathbf{x}^4$ be the multi-carriers symbols derived from the decomposition of $\mathbf{x}$, see Appendix A. Assuming that $\mathbf{x}$ satisfies the TR constraint, i.e, all reserved tones are set to zero, it can be easily verified that (use (21) and (24))

$$\begin{aligned} \frac{1}{L}(\mathbf{F}^{\text{Col}}_{M_1,m})^H\mathbf{x}^1 = X^1_m &= \begin{cases} X^1_m & \text{Si } m \notin \mathcal{K}^{(0)} \\ 0 & \text{if } m \in \mathcal{K}^{(0)} \end{cases} \\ \frac{1}{L}(\mathbf{F}^{\text{Col}}_{M_1,m})^H\mathbf{x}^3 = X^3_m &= \begin{cases} X^3_m & \text{Si } m \notin \mathcal{K}^{(0)} \\ 0 & \text{if } m \in \mathcal{K}^{(0)} \end{cases} \\ \frac{1}{L}(\mathbf{F}^{\text{Col}}_{M_1,m})^H(\mathbb{D}_{M_1})^H\mathbf{x}^2 = X^2_m &= \begin{cases} X^2_m & \text{Si } m \notin \mathcal{K}^{(1)} \\ 0 & \text{if } m \in \mathcal{K}^{(1)} \end{cases} \\ \frac{1}{L}(\mathbf{F}^{\text{Col}}_{M_1,m})^H(\mathbb{D}_{M_1})^H\mathbf{x}^4 = X^4_m &= \begin{cases} X^4_m & \text{Si } m \notin \mathcal{K}^{(1)} \\ 0 & \text{if } m \in \mathcal{K}^{(1)} \end{cases}, \end{aligned} \quad (7)$$

where $\mathcal{K}^{(0)} = \frac{\mathcal{R}^{(0)}}{2}$, $\mathcal{K}^{(1)} = \frac{\mathcal{R}^{(1)}-1}{2}$ and $\mathbf{F}^{\text{Col}}_{M_1,m}$ is the m-th row of the matrix $\mathbb{F}_{M_1}$ which is a normalized IFFT matrix of size M_1L. The construction of such a matrix is given in Section II-A.

According to the useful OFDM symbol decomposition, the clipping noise $J(.)$ (cost function in TR-GP method) is also decomposed as a sum of the partial clipping noise $J_1(.)$ and $J_2(.)$.

$$\begin{aligned} J_1(.) &= \|(\mathbf{x}^1+\mathbf{x}^2) - f(\mathbf{x}^1+\mathbf{x}^2, A)\|_2^2 \\ J_2(.) &= \|(\mathbf{x}^3+\mathbf{x}^4) - f(\mathbf{x}^3+\mathbf{x}^4, A)\|_2^2 \end{aligned} \quad (8)$$

Fig. 2 gives an illustration of the partial clipping noise derived from the decomposition of the clipping noise represented in the Fig. 1.

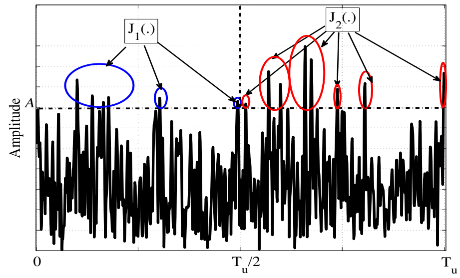

Fig. 2. Illustration of the clipping partial noise $J_1(.)$ and $J_2(.)$.

By using the reserved tones of the symbols derived from the decomposition of the useful symbol $\mathbf{x}$, see (7), the MTR approach consists in finding the signal $\mathbf{c}^1$ and $\mathbf{c}^2$ ($\mathbf{c}^3$ and $\mathbf{c}^4$ respect.) by minimizing $J_1(\mathbf{c}^1+\mathbf{c}^2)$ ($J_2(\mathbf{c}^3+\mathbf{c}^4)$ respect.). The objective is to construct the peak cancelling signal for PAPR reduction $\mathbf{c}$ using $\mathbf{c}^1$, $\mathbf{c}^2$, $\mathbf{c}^3$ and $\mathbf{c}^4$ derived from its decomposition, i.e

$$\mathbf{c} = [\mathbf{c}^1 \bullet \mathbf{c}^3] + [\mathbf{c}^2 \bullet \mathbf{c}^4]. \tag{9}$$

Where $\bullet$ stands for the vectors concatenation operator.

In other words, the MTR-GP method consists in solving the following optimization problems

$$\min_{\mathbf{C}^1,\mathbf{C}^2} \quad (\|\mathbf{x}^1+\mathbf{x}^2+\mathbb{F}_{M_1,\mathcal{K}^{(0)}}\mathbf{C}^1+\tilde{\mathbb{F}}_{M_1,\mathcal{K}^{(1)}}\mathbf{C}^2 - f(\mathbf{x}^1+\mathbf{x}^2+\underbrace{\mathbb{F}_{M_1,\mathcal{K}^{(0)}}\mathbf{C}^1}_{\mathbf{c}^1}+\underbrace{\tilde{\mathbb{F}}_{M_1,\mathcal{K}^{(1)}}\mathbf{C}^2}_{\mathbf{c}^4},A)\|_2^2), \tag{10}$$

and,

$$\min_{\mathbf{C}^3,\mathbf{C}^4} \quad (\|\mathbf{x}^1+\mathbf{x}^2+\mathbb{F}_{M_1,\mathcal{K}^{(0)}}\mathbf{C}^3+\tilde{\mathbb{F}}_{M_1,\mathcal{K}^{(1)}}\mathbf{C}^4 - f(\mathbf{x}^1+\mathbf{x}^2+\underbrace{\mathbb{F}_{M_1,\mathcal{K}^{(0)}}\mathbf{C}^3}_{\mathbf{c}^3}+\underbrace{\tilde{\mathbb{F}}_{M_1,\mathcal{K}^{(1)}}\mathbf{C}^4}_{\mathbf{c}^4},A)\|_2^2). \tag{11}$$

Let $\mathbb{A}$ be a matrix of dimension $(LM, 2M_R)$ defined by (12) and $\bar{\mathbf{C}} = \left[\mathbf{C}^1 \bullet \mathbf{C}^2 \bullet \mathbf{C}^3 \bullet \mathbf{C}^4\right]$.

$$\mathbb{A} = \begin{bmatrix} \mathbb{F}_{M_1,\mathcal{K}^{(0)}} & \mathbf{0}_{LM_1,K_0} & \tilde{\mathbb{F}}^1_{M_1,\mathcal{K}^{(1)}} & \mathbf{0}_{LM_1,K_1} \\ \mathbf{0}_{LM_1,K_0} & \mathbb{F}_{M_1,\mathcal{K}^{(0)}} & \mathbf{0}_{LM_1,K_1} & \tilde{\mathbb{F}}^2_{M_1,\mathcal{K}^{(1)}} \end{bmatrix} \tag{12}$$

where $\mathbf{0}_{t,z}$ represents a null matrix of size (t,z), K_0 and K_1 denote the number of even and odd tones in $\mathcal{R}$ respectively.

The optimization problems (10) and (11) can be conjointly formulated as follows, thanks to the matrix $\mathbb{A}$

$$\min_{\check{\mathbf{C}}} \left(\|\mathbf{x}+\mathbb{A}\check{\mathbf{C}} - f(\mathbf{x}+\mathbb{A}\check{\mathbf{C}})\|_2^2\right) \tag{13}$$

It may be noticed that $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^3$ and $\mathbf{x}^4$ are not used in the computation of the peak cancelling signal $\mathbf{c} = \mathbb{A}\bar{\mathbf{C}}$, see (13). The decomposition of the OFDM symbol presented in the Annex allows us only to modify the classical TR structure.

It is important to highlight that $\mathbf{C}^1 \neq \mathbf{C}^3$ and $\mathbf{C}^2 \neq -\mathbf{C}^4$ (the cost functions $J_1(.)$ and $J_2(.)$ are a priori different). Therefore, the peak cancelling signal $\mathbf{c}$ will interfere with the useful carriers of the symbol $\mathbf{x}$. To prevent BER degradation, a digital frequency domain filtering described in Fig. 3 are proposed.

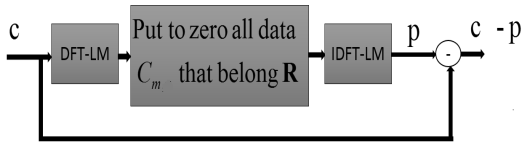

Fig. 3. Digital Frequency Domain Filtering scheme.

Let $\mathbf{c}^{ev} = [\mathbf{c}^1 \bullet \mathbf{c}^3]$ and $\mathbf{c}^{odd} = [\mathbf{c}^2 \bullet \mathbf{c}^4]$, from Fig. 3, it can be shown that $C_m^{ev} = \mathbf{F}^{\text{Col}}_{M,m}(\mathbf{c}^{ev})$ and $C_m^{od} = \mathbf{F}^{\text{Col}}_{M,m}(\mathbf{c}^{od})$ satisfy (14) and (15) respectively.

$$C_m^{ev} = \begin{cases} \underbrace{\dfrac{\mathbf{F}^H_{M_1,p}\mathbb{D}^H_{M_1}\mathbb{F}_{M_1,\mathcal{K}^{(0)}}}{L\sqrt{2}}(\mathbf{C}^1-\mathbf{C}^3)}_{\neq 0} & \text{if } m \in \mathcal{R}^{(1)} \\ C^1_{\frac{m}{2}} + C^3_{\frac{m}{2}} & \text{if } m \in \mathcal{R}^{(0)} (\frac{m}{2} \in \mathcal{K}^{(0)}) \\ 0 & \text{if } m \notin \mathcal{R} \end{cases} \tag{14}$$

$$C_m^{od} = \begin{cases} \underbrace{\dfrac{\mathbf{F}^H_{M_1,p}\mathbb{D}^H_{M_1}\mathbb{F}_{M_1,\mathcal{K}^{(1)}}}{L\sqrt{2}}(\mathbf{C}^2-\mathbf{C}^4)}_{\neq 0} & \text{if } m \in \mathcal{R}^{(0)} \\ C^2_{\frac{m-1}{2}} + C^4_{\frac{m-1}{2}} & \text{if } m \in \mathcal{R}^{(1)} (\frac{m-1}{2} \in \mathcal{K}^{(1)}) \\ 0 & \text{if } m \notin \mathcal{R} \end{cases} \tag{15}$$

From (14) and (15), it can be observed that $\mathbf{c}^1 \neq \mathbf{c}^3$ and $\mathbf{c}^2 \neq \mathbf{c}^4$ despite the filtering stage. This property represents the new degree of freedom in the optimization problem that will allows us to outperform the classical TR-GP method in terms of PAPR reduction. In fact, subject to $\mathbf{c}^1 = \mathbf{c}^3$ and $\mathbf{c}^2 = \mathbf{c}^4$, i.e, $\mathbf{C}^1 = \mathbf{C}^3$ and $\mathbf{C}^2 = \mathbf{C}^4$, it can be shown through a change of variable that (13) can be formulated as the classical TR-GP method defined by (5). Due to the digital frequency domain filtering stage which requires an FFT and IFFT operation, the MTR-GP method is more complex than the classical TR-GP method. Then, to reduce the computational complexity of the MTR-GP method, we propose to include the frequency domain filtering process directly in the optimization problem. In fact, from the proposed digital frequency domain filtering (see Fig. 3) the signal $\mathbf{p}$ can be computed as follows

$$\mathbf{p} \quad = \quad \frac{1}{L}\left(\mathbb{F}_{M,\mathcal{R}^c}(\mathbb{F}_{M,\mathcal{R}^c})^H\mathbb{A}\right)\check{\mathbf{C}}. \tag{16}$$

Where $\mathcal{R}^c$ is the complementary of $\mathcal{R}$ including the carriers due to the zero padding operation. Therefore, to prevent BER degradation, the optimization problem (13) and the filtering stage can be simultaneously formulated through the following optimization problem

$$\min_{\check{\mathbf{C}}} \left(\|\mathbf{x}+\tilde{\mathbb{A}}\check{\mathbf{C}} - f(\mathbf{x}+\tilde{\mathbb{A}}\check{\mathbf{C}})\|_2^2\right), \tag{17}$$

where $\tilde{\mathbb{A}} = \mathbb{A} - \frac{1}{L}\left(\mathbb{F}_{M,\mathcal{R}^c}\mathbb{F}^H_{M,\mathcal{R}^c}\mathbb{A}\right)$.

As in [7], (17) can be solved thanks to (6) by replacing $\mathbb{P}$ by $\tilde{\mathbb{A}}\tilde{\mathbb{A}}^H$. These two methods have the same computational complexity because $\tilde{\mathbb{A}}\tilde{\mathbb{A}}^H$ can be pre-calculated, and stored. This method will be called as 1-MTR-GP and in following subsection we will briefly extend the MTR principle by iterating the OFDM symbol decomposition.

B. MTR based on the double decomposition: 2-MTR-GP

Going more deeply into the decomposition of the OFDM signal, we proceed, similarly as Appendix A, a second decomposition of the symbol $x(t)$. For this purpose, the symbols derived from the first decomposition are decomposed similarly as in Annexe A. Through this double decomposition, $\mathbf{x}$ can be expressed as a mixture of 16 multi-carrier symbols with a duration of $\frac{T_u}{4}$ and $M_2 = \frac{M}{4}$ carriers (useful and reserved). Conjointly to this double decomposition, the cost function $J(.)$ is also decomposed as a sum of 4 partial clipping noise $J_{1,1}(.)$, $J_{1,2}(.)$, $J_{2,1}(.)$ and $J_{2,2}(.)$. Fig. 4 gives an illustration of the partial clipping noise derived from the decomposition of the clipping noise depicted in Fig. 1.

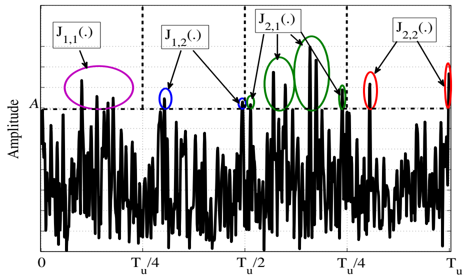

Fig. 4. Illustration of the double decomposition of $J_{[\mathbf{x}]}(.)$

As in Section III-A, the 2-MT-GP method consists in finding the substructures of the peak cancelling signal thanks to the reserved tones of the symbols derived from the double decomposition, by minimizing $J_{1,1}(.)$, $J_{1,2}(.)$, $J_{2,1}(.)$ and $J_{2,2}(.)$. Thus, similarly as in Section III-A, it can be shown that the 2-MTR-GP consists in finding the peak cancelling by solving the following optimisation problem

$$\min_{\tilde{\mathbf{C}}_n} \left(\|\mathbf{x} + \mathbb{B}\check{\mathbf{C}}_n - f(\mathbf{x} + \mathbb{B}\check{\mathbf{C}}_n)\|_2^2 \right), \tag{18}$$

where $\bar{\mathbf{C}}$ is a frequency vector constructed similarly as in (13), and $\mathbb{B}$ is a matrix of dimension $LM \times 4M_R$ defined as follows

$$\mathbb{B} = \begin{bmatrix} \mathbb{B}^1 & \mathbf{0}_{M_1L,K_0} & \mathbb{D}_{M_1L}\mathbb{B}^2 & \mathbf{0}_{M_1L,K_1} \\ \mathbf{0}_{M_1L,K_0} & \mathbb{B}^1 & \mathbf{0}_{M_1L,K_1} & -\mathbb{D}_{M_1L}\mathbb{B}^2 \end{bmatrix}. \tag{19}$$

The matrix $\mathbb{B}^1$ and $\mathbb{B}^2$ are defined in Appendix B.

To avoid BER degradation, a similar process as in (III-A) can be undertaken. In other words, this is equivalent to solve the following problem

$$\min_{\check{\mathbf{C}}} \left(\|\mathbf{x} + \tilde{\mathbb{B}}\check{\mathbf{C}} - f(\mathbf{x} + \tilde{\mathbb{B}}\check{\mathbf{C}})\|_2^2 \right), \tag{20}$$

where $\tilde{\mathbb{B}} = \mathbb{B} - \frac{1}{L}\left(\mathbb{F}_{M,\mathcal{R}^c}\mathbb{F}^H_{M,\mathcal{R}^c}\mathbb{B}\right)$. This problem can iteratively solved thanks to the gradient project algorithm as previously.

IV. Simulation Results

Simulations are performed with a 16-QAM modulated OFDM system that has $M = 256$ carriers wherein $M_R = 12$ carriers are reserved for PAPR reduction. The signal is over-sampled by a factor of four ($L = 4$). The set of reserved tones is $\mathcal{R} = [123, 124, \ldots, 133, 134]$.

Fig. 6 and 5, depict the CCDF before and after PAPR reduction using the TR-GP method , 1-MTR-GP and 2-MTR-GP methods after $N_{\max} = 10$ (number of performed iterations by the GP algorithm) featuring different normalized thresholds $\rho = 5$dB and 6dB. The simulation results depicted in Fig. 6 show that the proposed approaches outperform the classical TR-GP method in terms of PAPR reduction. In fact, with $\rho = 6\,dB$ ($\rho = 5\,dB$ respect.) and $N_{max} = 10$, the achieved PAPR at a clip rate 10^{-4} of the CCDF for the TR-GP, 1-MTR-GP, and 2-MTR-GP are $10.9\,dB$, $10.4\,dB$, and $9.9\,dB$ ($10.9\,dB$, $10.45\,dB$, and $10.12\,dB$ respect.) respectively. Thus, the MTR scheme allow us to improve the PAPR reduction of approximately 1dB. Besides, it can be also noticed that the deeper is the decomposition process of the signal $x(t)$, the better is the PAPR reduction performance achieved of the MTR-GP method.

Fig. (7) evaluates the BER versus E_b/N_0 curves in AWGN channels, where E_b denotes the average bit energy and N_0 is the one-sided power spectral density of the noise component. From these simulation results, it is worth noting that both proposed MTR-GP methods and the classical TR approach prevent the BER degradation as it is noticed in Subsection III-A and III-B.

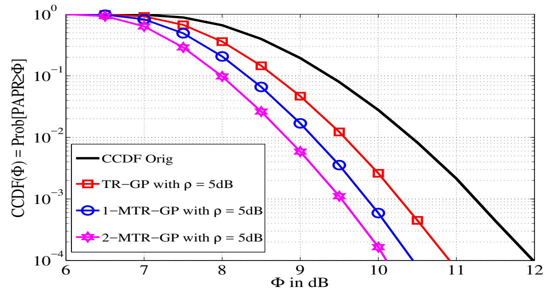

Fig. 5. Performance in term of PAPR reduction with $\rho = 5$dB.

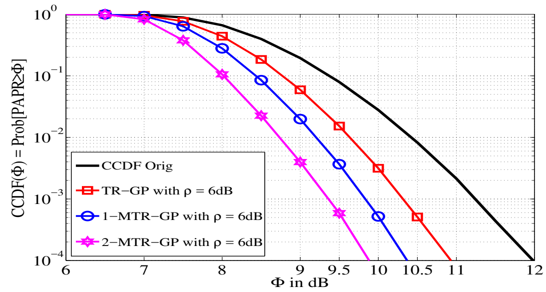

Fig. 6. Performance in term of PAPR reduction with $\rho = 6$dB.

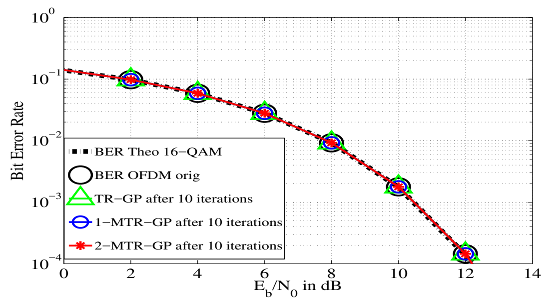

Fig. 7. BER performance of the signal before and after PAPR reduction.

V. Conclusion

In this paper, a new approach of designing the adding signal for PAPR mitigation by modifying the classical TR scheme is proposed. Using the gradient algorithm for the computation of the peak cancelling for PAPR mitigation, we have shown theoretically that it can be expected that our proposed solution outperforms the classical TR-GP method with the same computational complexity. Simulation results approved these theoretical analyses. Moreover, we have shown that going deeply in the decomposition process of the useful OFDM signal achieves better performance for the MTR-GP method. In our future works, the MTR will be analysed for higher decomposition levels.

Appendix A
OFDM symbol decomposition

In this section, we show that an OFDM symbol $x(t)$ can be seen as a mixture of 4 multi-carriers symbols.

Let $\mathbf{X}^1$, $\mathbf{X}^2$, $\mathbf{X}^3$ and $\mathbf{X}^4$ be the data vectors in frequency domain, of size $\frac{M}{2}$ defined as follows:

$$X_p^1 = X_p^3 = X_{2p} \text{ and } X_p^2 = X_p^4 = X_{2p+1}. \tag{21}$$

Note that $\mathbf{X}^1$ and $\mathbf{X}^3$ ($\mathbf{X}^2$ and $\mathbf{X}^4$ respect.) contain the data of $\mathbf{X}$ indexed by the even tones (odd tone respect.). Now, let $x^1(t)$, $x^2(t)$, $x^3(t)$ and $x^4(t)$ be the multi-carriers signals defined as

$$\begin{aligned}
x^1(t) &= \sqrt{\frac{1}{M_1}} \sum_{m=0}^{M_1-1} X_m^1 e^{j2\pi m F_1 t}, 0 \le t \le T_1, \\
x^3(t) &= \sqrt{\frac{1}{M_1}} \sum_{m=0}^{M_1-1} X_m^3 e^{j2\pi m F_1 t}, T_1 \le t \le 2T_1, \\
x^2(t) &= \sqrt{\frac{1}{M_1}} \sum_{m=0}^{M_1-1} X_m^2 e^{j2\pi (m F_1 + F) t}, 0 \le t \le T_1, \\
x^4(t) &= \sqrt{\frac{1}{M_1}} \sum_{m=0}^{M_1-1} X_m^4 e^{j2\pi (m F_1 + F) t}, T_1 \le t \le 2T_1.
\end{aligned} \tag{22}$$

with $T_1 = \frac{T_u}{2}$, $M_1 = \frac{M}{2}$ and $F_1 = 1/T_1$.

From 21 and 22 we have

$$x(t) = \begin{cases} x^1(t) + x^2(t), & \text{if } t \in \left]0, \frac{T_u}{2}\right] \\ x^3(t) + x^4(t), & \text{if } t \in \left[\frac{T_u}{2}, T_u\right[\end{cases}. \tag{23}$$

After oversampling using the same oversample rate, let $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^3$ and $\mathbf{x}^4$ be the vectors of the samples of the symbols $x^1(t)$, $x^2(t)$, $x^3(t)$ and $x^4(t)$ respectively. Therefore, after some derivations, we obtain:

$$\begin{aligned}
\mathbf{x}^1 &= \mathbb{F}_{M_1} \check{\mathbf{X}}^1, \quad \text{and} \quad \mathbf{x}^3 = \mathbb{F}_{M_1} \check{\mathbf{X}}^3, \\
\mathbf{x}^2 &= \tilde{\mathbb{F}}_{M_1} \check{\mathbf{X}}^2, \quad \text{and} \quad \mathbf{x}^4 = -\tilde{\mathbb{F}}_{M_1} \check{\mathbf{X}}^4.
\end{aligned} \tag{24}$$

where $\mathbb{F}_{M_1}$ is the normalized IFFT matrix of size $(M_1 L)$, $\tilde{\mathbb{F}}_{M_1} = \mathbb{D}_{M_1} \mathbb{F}_{M_1}$, and $\mathbb{D}_{M_1}$ is a diagonal matrix whose the diagonal is the vector $\mathbf{d} = [1, e^{j2\pi \frac{1}{M_1 L}}, \ldots, e^{j2\pi \frac{M_1 L - 1}{M_1 L}}]$. For $i = 1, .., 4$, the vector $\check{\mathbf{X}}^{(i)}$ is a vector obtained thanks to the zero-padding operation.

Therefore, using (23) $\mathbf{x}$ can be expressed versus $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^3$ and $\mathbf{x}^4$ as follows:

$$\mathbf{x} = [\mathbf{x}^1 \bullet \mathbf{x}^3] + [\mathbf{x}^2 \bullet \mathbf{x}^4]. \tag{25}$$

where $\bullet$ denotes the concatenation operation.

Appendix B
Construction of the MTR matrix

Let $\mathcal{K}^{(0),ev}$ and $\mathcal{K}^{(0),od}$ ($\mathcal{K}^{(1),ev}$ and $\mathcal{K}^{(1),od}$ respect.) be the subsets of $\mathcal{K}^{(0)}$ ($\mathcal{K}^{(1)}$ respect.) containing its even and odd tones (indices) respectively. From these subsets, let $\mathcal{P}^{(0)}$, $\mathcal{P}^{(1)}$, $\mathcal{Q}^{(0)}$, $\mathcal{Q}^{(1)}$ be the following subsets

$$\begin{aligned}
\mathcal{P}^{(0)} &= \frac{\mathcal{K}^{(0),ev}}{2}, \quad \text{and } \mathcal{P}^{(1)} = \frac{\mathcal{K}^{(0),od}}{2}, \\
\mathcal{Q}^{(0)} &= \frac{\mathcal{K}^{(1),ev} - 1}{2}, \quad \text{and } \mathcal{Q}^{(1)} = \frac{\mathcal{K}^{(1),od} - 1}{2}.
\end{aligned} \tag{26}$$

The matrix $\mathbb{B}^1$ and $\mathbb{B}^2$ are then defined as follows

$$\mathbb{B}^1 = \begin{bmatrix} \mathbb{F}_{M_2,\mathcal{P}^{(0)}} & \mathbf{0}_{M_2 L, P_0} & \tilde{\mathbb{F}}^1_{M_2,\mathcal{P}^{(1)}} & \mathbf{0}_{M_2 L, P_1} \\ \mathbf{0}_{M_2 L, P_0} & \mathbb{F}_{M_2,\mathcal{P}^{(0)}} & \mathbf{0}_{M_2 L, P_1} & \tilde{\mathbb{F}}^2_{M_2,\mathcal{P}^{(1)}} \end{bmatrix}. \tag{27}$$

where $M_2 = \frac{M}{4}$, P_0 is the cardinal of $\mathcal{P}^{(0)}$ (i.e number of the even tones in $\mathcal{K}^{(0)}$) and P_1 is the cardinal of $\mathcal{P}^{(1)}$ (i.e number of the odd tones in $\mathcal{K}^{(0)}$).

$$\mathbb{B}^2 = \begin{bmatrix} \mathbb{F}_{M_2,\mathcal{Q}^{(0)}} & \mathbf{0}_{M_2 L, Q_0} & \tilde{\mathbb{F}}^1_{M_2,\mathcal{Q}^{(1)}} & \mathbf{0}_{M_2 L, Q_1} \\ \mathbf{0}_{M_2 L, Q_0} & \mathbb{F}_{M_2,\mathcal{Q}^{(0)}} & \mathbf{0}_{M_2 L, Q_1} & \tilde{\mathbb{F}}^2_{M_2,\mathcal{Q}^{(1)}} \end{bmatrix}. \tag{28}$$

where Q_0 is the cardinal of $\mathcal{Q}^{(0)}$ (i.e number of the even tones in $\mathcal{K}^{(1)}$) and Q_1 is the cardinal of $\mathcal{Q}^{(1)}$ (i.e number of the odd tones in $\mathcal{Q}^{(1)}$).

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