Error Characterization of Duty Cycle Estimation for Sampled Non-Band-Limited Pulse Signals With Finite Observation Period

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***Abstract*—In many applications the pulse duration of a periodic pulse signal is the parameter of interest. Thereby, the non-band-limited pulse signal is sampled during a finite observation period yielding to aliasing and windowing effects, respectively. In this work, the pulse duration estimation based on the mean value of the samples is considered, and an exact expression of the mean squared estimation error (averaged over all possible time shifts) is derived. The resulting mean squared error expression depends on the observation period, the pulse period and the pulse duration. Analyzing the effect of these parameters shows that the mean squared error can be reduced (i) if the observation period is a multiple of the pulse period, (ii) if the pulse period is not a multiple of the sampling period, and (iii) if the total number of samples is a prime number. All results were validated with simulation results.**

***Index Terms*—sampling process, band width, signal reconstruction, sampling error, wireless sensor networks (WSN), synchronization, localization, ultrasonic.**

I. Introduction

Following the established sampling theory, band-limited signals can be perfectly reconstructed from a set of samples that are collected with a sampling frequency larger than the double occupied bandwidth [1], [2]. However, this is only true if the signal is sampled during its infinitely long duration. In practical applications neither an infinite observation time can be achieved, nor the underlying signals are band-limited (e.g., discrete valued and continuous-time signals are not band-limited).

Consider time-duration measurements in periodic signals. For example, in ultrasonic reflection measurements for ranging [3], [4], pulses are emitted periodically, and a device counts the number of clock cycles between the emission of the pulse and the reception of the reflection. This setting corresponds to the sampling of a rectangular pulse signal, that is high between emission and reception, and low until the next emission. The pulse duration, i.e., the time that the pulse is on high, is used to determine the range. Another example is round-trip time (RTT) based clock synchronization for which clocks are modeled with discrete events. The corresponding RTT measurements are samples of a pulse function. In that scenario, the pulse duration is related to the propagation delay and the clock offset between two communication nodes [5], [6].

The two examples mentioned use a pulse shaped signal to determine key system parameters. Thereby, both conditions of the sampling theorem are violated, the bandwidth of the signal is infinite and the observation period is limited. Although the signals are considered as noise free, sampling and aliasing distorts the signal reconstruction. In this work the question is raised how accurately signal parameters can be estimated from non band-limited signals with a finite number of samples. Previous contributions have already derived upper bounds on the squared error [7]. Here, for the function space of sampled pulse signals, an exact formulation of the averaged squared error on the estimation of the pulse duration is derived. Moreover, the error analysis gives rise to the following design suggestions in technical systems:

- Predicting the estimation accuracy for a given number of samples in a given observation period;
- Selecting the number of pulse signal periods (i.e., the total observation period) to achieve a desired estimation accuracy.
- Selecting the number of samples to achieve the minimum estimation error for a given sampling period and observation period.

II. Preliminaries

Sampling is the obvious starting point in all discrete signal processing applications that have a relation to real world processes. In the following, signals $x(t)$ are considered that are integrable in the sense of Lebesgue, i.e.,

$$L^1(\mathbb{R}) = \left\{ x(t) \middle| \int_{-\infty}^{\infty} \mid x(t) \mid \mathrm{d}t < \infty \right\}. \tag{1}$$

Sampling $x(t)$ with a period T yields the sampled signal

$$x_s(n) \in \left\{ x(nT) \middle| n \in \mathbb{Z} \wedge T = \frac{1}{2B} \wedge B < \infty \right\}. \tag{2}$$

To reconstruct $x(t)$, the samples $x_s(n)$ are interpolated, i.e.,

$$x_i(t) = \sum_{n=-\infty}^{\infty} x_s(n) \frac{\sin(2\pi B(t - nT))}{\pi(t - nT)}, \tag{3}$$

is the interpolated signal. As commonly known [1], [2]

$$x_i(t) \equiv x(t), \tag{4}$$

if and only if $x(t)$ is band-limited by $\mathcal{B} = [-B, B[$.

A. Signals of interest

In this work, the rectangular pulse function

$$x(t) = \sum_{l=-\infty}^{\infty} \text{rect}\left(\frac{t - l\,P - \phi}{D}\right) \tag{5}$$

with $t, P, \phi \in \mathbb{R}$, $0 < D < P$ and $l \in \mathbb{Z}$ are considered. The signal period P is considered to be a fixed parameter, and the time shift ϕ and the pulse duration D are considered to be uniformly distributed variables on the mentioned support. The considered signal is a low-pass signal, and since it is periodic in P, it is wide-sense cyclo stationary and ergodic.

Note that $x(t)$ in (5) is discrete valued and hence not band-limited. The condition (4) can not be achieved due to aliasing induced by the sampling process.

B. Bounds on the Aliasing Error of Aribtrary Functions

The aliasing error is the difference between the original signal $x(t)$ and the interpolated signal $x_i(t)$, i.e., $|x(t) - x_i(t)|$. In [7] a upper bound is derived for sampling of one dimensional low-pass and band-pass signals. The analysis was extended to multidimensional signals in [8] yielding similar results. For one-dimensional signals, the main finding is that the aliasing error is bounded by the out-of-band signal contribution (cf. [9], [10]) with

$$|x(t) - x_i(t)| \leq 2 \int\limits_{\mathbb{R}\setminus\mathcal{B}} |X(f)| \mathrm{d}f\,, \tag{6}$$

where $X(f)$ is the spectral representation of $x(t)$, and where the band $\mathcal{B}$ depends on the sampling period in (2).

While the formulation in (6) provides an upper bound on the aliasing error of arbitrary signals, this work focuses on pulse-shaped signals as in (5). Hence, although P and ϕ are unknown, the general shape (e.g., its discrete amplitude) is known and can be interpreted as prior knowledge.

The aim of this work is to derive an exact formulation of the reconstruction error similar to (6). Thereby, first the frequency domain representation of the periodic infinite time-continuous time signal (5) is derived, and then the frequency representation of the sampled version of this signal. Finally, a rectangular windowing function is included to account for a finite observation period.

C. Time and Frequency Domain Pulse Signal Representation

The signal of (5) is an even function with a time shift ϕ and can be represented by the Fourier coefficients for $k \geq 1$ and for $k = 0$, respectively,

$$a_k = \frac{2}{k\pi} \sin\left(\frac{\pi k D}{P}\right)\,, \quad \text{and} \;\; a_0 = \frac{D}{P}\,. \tag{7}$$

Hence, (5) can be rewritten as

$$\begin{aligned} x(t) &= a_0 + \sum_{k=1}^{\infty} a_k \cos\left(\frac{2\pi k}{P}(t - \phi)\right) = \\ &= \frac{D}{P} + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{\pi k D}{P}\right) \cos\left(\frac{2\pi k}{P}(t - \phi)\right) \end{aligned} \tag{8}$$

and the corresponding frequency domain representation is given by

$$\begin{aligned} X(f) = &\frac{D}{P}\delta(2\pi f) + \\ &\sum_{k=-\infty}^{\infty} \frac{2}{k\pi} \sin\left(\frac{\pi k D}{P}\right) \delta\left(2\pi f - k\frac{2\pi}{P}\right) e^{j\frac{2\pi k \phi}{P}}\,. \end{aligned} \tag{9}$$

D. Frequency Domain Representation of Sampled Pulse Signals

In this section the sampling of the non-time-limited signal $x(t)$ in (8) is considered. In contrast to [11], the considered periodic pulse functions are unbounded in frequency domain. Nevertheless, it is required that the pulse period P has to be at least twice the sampling period $P \geq 2T$. Due to sampling, all out-of-band components of $X(f)$ are folded into $\mathcal{B}$ according to the Poisson sum formula [10], [12], [13]

$$X_s(f) = \frac{1}{T} \sum_{m=-\infty}^{\infty} X\left(f - m\frac{1}{T}\right)\,, \tag{10}$$

where $X_s(f)$ is the spectral representation of $x_s(n)$. Note that $X_s(f)$ is periodic in f with a periodicity of $1/T$. Hence, in the following $X_s(f)$ will only be considered for $f \in \mathcal{B}$ with $\mathcal{B} = [-B, B[$ and $B = 1/2T$.

$X(f)$ from (9) can be plugged into (10) yielding

$$\begin{aligned} X_s(f) = &\frac{1}{T} \sum_{m=-\infty}^{\infty} \frac{D}{P}\,\delta\left(2\pi f - m\frac{2\pi}{T}\right) + \\ &\frac{1}{T} \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \frac{2}{k\pi} \sin\left(\frac{\pi k D}{P}\right) \delta\left(2\pi f - m\frac{2\pi}{T} - k\frac{2\pi}{P}\right) e^{j\frac{2\pi k\phi}{P}}\,, \end{aligned} \tag{11}$$

for $f \in \mathcal{B}$. The first delta-dirac function is non-equal to zero in $\mathcal{B}$ only if $f = 0$ and $m = 0$. The second delta-dirac function is non-equal to zero if

$$f = m\frac{1}{T} + k\frac{1}{P}\,, \quad \text{for} \;\; f \in \mathcal{B}\,. \tag{12}$$

For each k, there exists exactly one m that fulfills (12). Defining the round modulo function [14] with

$$\oslash \colon \mathbb{R} \times (\mathbb{R} \setminus \{0\}) \to \mathbb{R}\,, (r, p) \mapsto r \oslash p := r - \left\lfloor \frac{r}{p} + 0.5 \right\rfloor p\,, \tag{13}$$

where $\lfloor \cdot \rfloor$ denotes the rounding to the next lower integer, allows to rewrite (12) as a function of k only, with

$$f_k = \left(m\frac{1}{T} + k\frac{1}{P}\right) \oslash \left(\frac{1}{T}\right) = \left(k\frac{1}{P}\right) \oslash \left(\frac{1}{T}\right)\,. \tag{14}$$

Finally, (11) can be rewritten to

$$\begin{aligned} X_s(f) = &\frac{D}{PT}\delta(2\pi f) \\ &+ \sum_{k=-\infty\setminus 0}^{\infty} \frac{2}{k\pi T} \sin\left(\frac{\pi k D}{P}\right) \delta\left(2\pi(f - f_k)\right) e^{j2\pi f_k \phi}\,. \end{aligned} \tag{15}$$

The sampled time-domain signal for arbitrary ϕ is finally obtained analogue to (8) with

$$x_s(n) = \frac{D}{PT} + \sum_{k=1}^{\infty} \frac{2}{k\pi T} \sin\left(\frac{\pi k D}{P}\right) \cos\left(2\pi f_k (nT - \phi)\right) . \tag{16}$$

The sampled signal representation of $x_s(n)$ in (16) includes all aliasing components due to sampling. It will be the starting point for considering a limited observation period in the following section.

E. Finite Observation Period

To include a finite observation period NT in the frequency domain representation of $X_s(f)$ in (15), we consider a non-causal rectangular window yielding

$$X_s(f) = \sum_{n=-\lfloor \frac{N}{2} \rfloor}^{\lfloor \frac{N-1}{2} \rfloor} x_s(n)\, e^{-j2\pi f n T} , \tag{17}$$

with $x_s(n)$ from (16) for arbitrary ϕ. Because of its linearity, we can transform each a_k weighted term of the Fourier series of (16) separately

$$X_s(f) = \sum_{k=0}^{\infty} a_k X_s^{f_k}(f) . \tag{18}$$

The finite discrete Fourier transform of each cosine function of (16) can be written by

$$\begin{aligned} X_s^{f_k}(f) &= \sum_{n=-\lfloor \frac{N}{2} \rfloor}^{\lfloor \frac{N-1}{2} \rfloor} \cos(2\pi f_k (nT - \phi))\, e^{-j2\pi f n T} \\ &= e^{-j(2\pi f_k \phi)} \frac{\sin(\pi N T (f - f_k))}{2 \sin(\pi T (f - f_k))} , \end{aligned} \tag{19}$$

where the simplification from the first to the second line applies the generalized formula of the geometric series. Finally, the frequency domain representation for $f \in \mathcal{B}$ of the sampled pulse signal with finite observation period NT is

$$\begin{aligned} X_s(f) = &\frac{D}{PT} \frac{\sin(\pi N T (f))}{\sin(\pi T (f))} \\ &+ \sum_{k=-\infty \backslash 0}^{\infty} e^{-j(2\pi f_k \phi)} \sin\left(\frac{\pi k D}{P}\right) \frac{\sin(\pi N T (f - f_k))}{k\pi T \sin(\pi T (f - f_k))} . \end{aligned} \tag{20}$$

III. Estimation Error of the Pulse Duration

The pulse duration D determines the mean value of the periodic time-continous signal $x(t)$ in (5), and appears in $X(f = 0) = D/P$ in (9). A natural choice of estimating D is using the mean value[1] of the sampled signal. Hence, the estimation error can be characterized by the influence of aliasing and windowing on $X_s(0)$. l'Hospital's rule yields

$$\begin{aligned} X_s(0) = &\frac{D}{PT} N \\ &+ \sum_{k=-\infty \backslash 0}^{\infty} e^{-j(2\pi f_k \phi)} \sin\left(\frac{\pi k D}{P}\right) \frac{\sin(\pi N T f_k)}{T k \pi \sin(\pi T f_k)} . \end{aligned} \tag{21}$$

[1]Based on the binary nature of the observed process this estimator is equivalent to an implementation of a nonlinear edge detector .

The estimator of the pulse duration can be written by

$$\widehat{D}(D, \phi; N, P) = \frac{PT}{N} X_s(0). \tag{22}$$

A. Mean Square Estimation Error

Considering (22) and (21), the estimation error e_D as a function of the random variables D, ϕ and the fixed parameters N, P, is given by

$$\begin{aligned} e_D(D, \phi; N, P) &= D - \widehat{D}(D, \phi; N, P) \qquad (23) \\ &= \frac{P}{N} \sum_{k=-\infty \backslash 0}^{\infty} e^{-j(2\pi f_k \phi)} \sin\left(\frac{\pi k D}{P}\right) \frac{\sin(\pi N T f_k)}{k\pi \sin(\pi T f_k)} \\ &= \sum_{k=-\infty \backslash 0}^{\infty} c_k e^{-j(2\pi f_k \phi)} , \qquad (24) \end{aligned}$$

with

$$c_k = \frac{P}{N} \sin\left(\frac{\pi k D}{P}\right) \frac{\sin(\pi N T f_k)}{k\pi \sin(\pi T f_k)} . \tag{25}$$

Note that (24) has the structure of a discrete Fourier transform with the coefficients c_k. Note that c_k only depends on Tf_k. Using the round modulo definition (13),

$$T f_k = k\frac{T}{P} - \left\lfloor \frac{kT}{P} + 0.5 \right\rfloor = k\frac{T}{P} - v(k)$$

with $v(k) = \left\lfloor \frac{kT}{P} + 0.5 \right\rfloor \in \mathbb{Z}$. The coefficients c_k in (25) can be simplified to

$$c_k = \frac{P}{N} \sin\left(\frac{\pi k D}{P}\right) \frac{(-1)^{(N-1)v(k)} \sin(\pi N k \frac{T}{P})}{k\pi \sin(\pi k \frac{T}{P})} . \tag{26}$$

To characterize the error independent of ϕ, the mean square error with respect to ϕ is evaluated. Due to the ergodic and wide-sense cyclo-stationary signal in (5), the expectation can be evaluated with $\Phi = mP$ and $\Phi \to \infty$ to

$$\begin{aligned} \overline{e_D^2(N)} &= E\{|e_D(D, \phi; N, P)|^2\} \\ &= \lim_{\Phi \to \infty} \frac{1}{\Phi} \int_{-\frac{\Phi}{2}}^{\frac{\Phi}{2}} |e_D(D, \phi; N, P)|^2 \mathrm{d}\phi \\ &= \lim_{\Phi \to \infty} \frac{1}{\Phi} \int_{-\Phi/2}^{\Phi/2} \left| \sum_{k=-\infty \backslash 0}^{\infty} c_k e^{-j(2\pi f_k \phi)} \right|^2 d\phi \\ &= \sum_{k=-\infty \backslash 0}^{\infty} |c_k|^2 \qquad (27) \\ &= \sum_{k=-\infty \backslash 0}^{\infty} \frac{P^2}{N^2} \sin^2\left(\frac{\pi k D}{P}\right) \frac{\sin^2(\pi N k \frac{T}{P})}{(k\pi)^2 \sin^2(\pi k \frac{T}{P})} , \qquad (28) \end{aligned}$$

where the simplification from the third to the fourth line is possible since the integration limits tend to infinity. The mean squared error on the estimation of the pulse duration D, i.e., $\overline{e_D^2(N)}$ is exactly characterized by (28). It is averaged over all possible time shifts ϕ, and it depends on the pulse period P, the number of samples N, and the sampling period T. In the following, special cases on the relation of the parameters P, N and T are discussed.

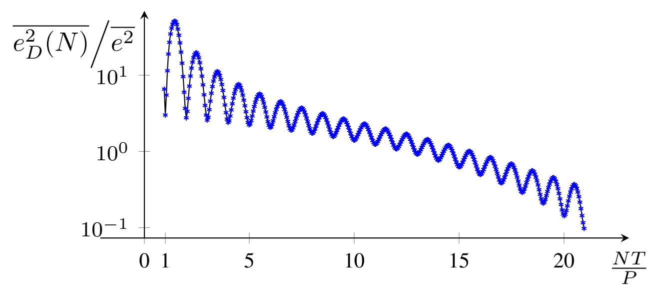

Fig. 1. Mean square error of duty cycle estimation depending on the observation length NT. The value $T = 25/501 = 0.0499\ldots$ was selected to guarantee that $mP \neq NT$ and $nT \neq P$. The error is decaying slowly to 0.1 of the quantization error at an observation interval of about 20 signal periods. The $(*)$ indicate the simulation results.

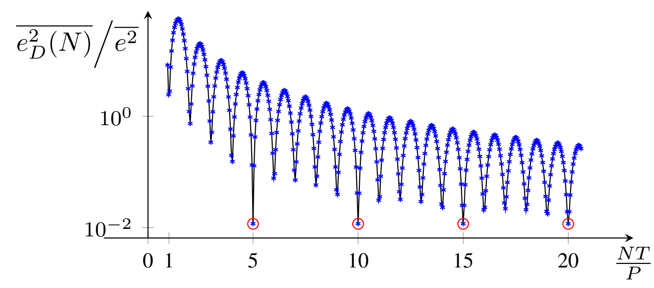

Fig. 2. Plot of the mean square error of duty cycle estimation over NT with $T = 5/107 = 0.04672\ldots$. The values for $mP = NT$ and $nT \neq P$ are indicated by (o). The error is decaying to 0.01 of the quantization error at an observation interval of about 5 signal periods due to the prime number N. The $(*)$ indicate the simulation results.

IV. Discussion on the Parameter Relation

By observing (28), one finds in the squared sine in the enumerator the relation NT/P, and in the squared sine in the denominator T/P. If $NT = mP$ with $m \in \mathbb{N}^+$, i.e., the observation period is a multiple of the pulse period, some coefficients in (28) will be zero. If the pulse period is a multiple of the sampling period, i.e., $P = nT$ with $n \in \mathbb{N}^+$, the enumerator and the denominator of some coefficients will tend to zero. In the following, four different cases on the relation of NT to mP and of nT to P are discussed.

The discussion on the parameter selection plays a key role in applications, e.g. the number of repeated pulses for ultrasonic distance measures and the sample frequency used to measure the time to incoming reflections. Both parameters are easy to determine and we can achieve higher estimation performance by optimized parameter relations.

In the following analysis, the pulse period is normalized to one second, i.e. $P = 1\,\mathrm{s}$, and the mean squared estimation error is normalized to the quantization error induced by T, i.e. $\overline{e_D^2(N)}/\overline{e^2}$ with $\overline{e^2} = T^2/12$ [15]. The normalized error is compared with simulation results using 4000 realizations.

A. Parameter: $mP \neq NT$ and $nT \neq P$

For the given parameter setting, Fig. 1 depicts the normalized error following the derived analytical expression (28), indicated by the solid line, and simulation results, indicated by the $(*)$ markers. Both results show the same behavior. Local minima can be observed whenever the NT is close to a multiple of the pulse period P. In contrast, local maxima can be observed in between. An evident conclusion is to use $NT = mP$ as observation length to have the lowest estimation error. This case will be considered in the following section.

B. Parameter: $mP = NT$ and $nT \neq P$

Considering $NT = mP$, the coefficients c_k in (28) can be rewritten to

$$|c_k|^2 = \frac{T^2}{m^2}\sin^2\left(\frac{\pi k m D}{NT}\right)\frac{\sin^2(\pi k m)}{k^2\pi^2\sin^2(\pi k \frac{m}{N})} \tag{29}$$

where the numerator is equal to zero. It is obvious, that also the denominator is zero for some $k \in \mathbb{Z}$, i.e., if

$$k = \frac{vN}{m}\,, \tag{30}$$

with $v \in \mathbb{N}^+$. For those $k \in \mathbb{Z}$, the coefficients c_k will not be equal to zero. Note that if N is a prime number, v must be a multiple of m and only a minimum number of coefficients c_k exists for which numerator and denominator tend to zero. Hence, for N is a prime number, the minimum mean squared estimation error $\overline{e_D^2(N)}$ is expected.

Applying de l'Hoptials rule to the non-zero c_k coefficients, i.e. for $kT \to mP$, yields

$$\lim_{kT\to mP}|c_k|^2 = \frac{P^2}{N^2}\sin^2\left(\frac{\pi k D}{P}\right)\frac{N^2}{k^2\pi^2} \tag{31}$$

which results for $k = i\,mN$ in

$$\left.\overline{e_D^2(N)}\right|_{N=\frac{mP}{T}} = \sum_{i=1}^{\infty}\frac{2T^2}{(i\,m\pi)^2}\sin^2\left(\frac{\pi i\,mD}{T}\right). \tag{32}$$

In Fig. 2, the analytical estimation error (solid line) and the simulated estimation error $(*)$ for $mP \neq NT$, and the special cases for $mP = NT$ (o) are depicted. Interestingly, the minimum value of the mean square error, i.e., for $mP = NT$, is constant also for multiples of fN, $f \in \mathbb{N}^+$. It can be concluded that (30) holds also for f multiples of N and leads to the same set of c_k. Therefore, increasing the observation period does not improve the estimation accuracy, because there is no dependence on N in (32).

C. Parameter: $mP = NT$ and $nT = P$

In some practical scenarios (e.g., the ultrasonic range measurement in [3]), the pulse period is equal to a multiple of the sampling period, and the observation period is a multiple of the pulse period. Reformulating $mP = NT$ to $N = mP/T$, i.e. N is a multiple of the pulse period due to $T = P/n$, and using (30) yields $k = vP/T \in \mathbb{N}^+$. As (31) holds also for $nT \to P$ it can be simplified to c_k not depending on N and used in (27) to

$$\overline{e_D^2(N)} = \sum_{v=1}^{\infty}\frac{2T^2}{\pi^2 v^2}\sin^2\left(\frac{\pi v D}{T}\right). \tag{33}$$

In Fig. 3, the (o) marker indicate the analytical results for $mP = NT$ and $nT = P$. The solid line and the $(*)$ marks in between indicate the analytical results and the simulation results, respectively, for $mP \neq NT$ and $nT = P$. It can be observed, that for $nT = P$, setting $mP = NT$ yields

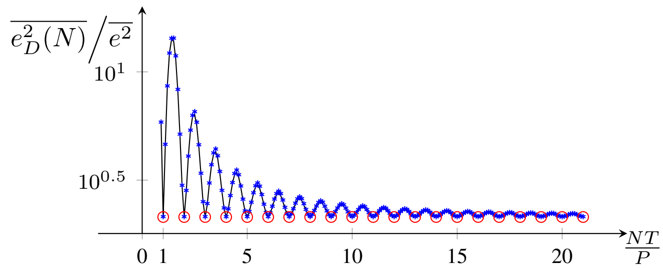

Fig. 3. Plot of the mean square error of duty cycle estimation over NT with $T = 0.01$. The values for $mP = NT$ (o), $mP \neq NT$ (-) and $nT = P$ are plotted. The error is not decaying beyond 2 times the standard quantization error. The $(*)$ indicate the simulation results.

the minimum estimation error. However, this error is constant regardless how many pulse periods are observed, and it is 200 times larger than the error obtained by setting $nT \neq P$.

D. Parameter: $mP \neq NT$ and $nT = P$

In Fig. 3 it could be observed that this parameter setting yields the highest mean squared error of all compared settings. It remains to find an analytical expression of the error. For the special setting of $mP \neq NT$ and $nT = P$, the sum in (28) consists of two types of factors: those that can be directly computed, i.e., $k \neq vP/T$; and those that require the application of de l'Hospital's rule. The latter is for $k = vP/T$. Finally, the analytical formulation of the mean squared error is

$$\overline{e_D^2(N)} = \sum_{v=1}^{\infty} \frac{2T^2}{\pi^2 v^2} \sin^2\left(\frac{\pi v D}{T}\right) + \sum_{k=-\infty \setminus 0 \wedge k \neq v\frac{P}{T}}^{\infty} \frac{P^2}{N^2} \sin^2\left(\frac{\pi k D}{P}\right) \frac{\sin^2(\pi N k \frac{T}{P})}{(k\pi)^2 \sin^2(\pi k \frac{T}{P})}. \quad (34)$$

The comparison of the analytical and the numerical results in Fig. 3 indicate the correctness of (34).

V. Conclusion

For sampled periodic pulse signals with finite observation period, an exact formulation of the mean squared error for estimating the pulse duration was derived. The resulting error formulation depends on the observation time, the pulse period and the pulse duration. As certain relations of these parameters prevent a direct evaluation of the mean squared error, exact expressions for these specific settings were presented using de l'Hospital's rule. The error formulations were validated with simulation results. A discussion on the parameter relations revealed that a minimum mean squared error is achieved if a full number of pulse periods is covered by the observation time, and if the number of used samples is equal to a prime number. For such system settings, an increase of the observation time can not improve the estimation accuracy. Moreover, it could be seen that technical systems, where the pulse period is a multiple sampling period have significant inferior performance. Both results are counterintuitive and may have an important impact to design systems.

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