WEIGHTED SHOOTING METHOD FOR HIGH-RESOLUTION DOA ESTIMATION BASED ON SPARSE SPECTRUM FITTING

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ABSTRACT

This paper presents a weighted version of shooting method for high-resolution Direction Of Arrival (DOA) estimation based on Sparse Spectrum Fitting (SpSF). SpSF is one of the sparse DOA estimation methods based on ℓ_1-regularization which uses the penalty term in optimization. The regularization process often takes long time and is very sensitive to the penalty term, but it is difficult to know its appropriate value in advance. We recall that the shooting method is a computationally efficient ℓ_1-regularization algorithm but still sensitive to the penalty term. We try to modify the shooting method into the weighted ℓ_1-regularization problem so that it does not become sensitive. Performance of the proposed method is evaluated in comparison with several conventional methods through some computer simulation.

Index Terms— direction of arrival estimation, array signal processing

1. INTRODUCTION

Direction-Of-Arrival (DOA) estimation plays an important role in radar, sonar, indoor and outdoor wireless communications [1]–[3]. High resolution DOA estimation methods like MUSIC [4], Root-MUSIC [5], ESPRIT [6] are based on eigenvalue decomposition of sample covariance matrix of array input signal, and MODE [7] is based on maximum likelihood estimation and achieves high DOA estimation accuracy. However, in the case of severe environments like low SNR, small number of snapshots or large number of sources, DOA estimation accuracy often becomes worse due to the lack of effective information or the limited degree of freedom [8]–[10].

DOA estimation can also be regarded as a sparse optimization problem to specify DOAs from all the possible directions. The DOA estimation problem is reformulated under the framework of sparse signal reconstruction and can be solved by Sparse Spectrum Fitting (SpSF) [11] with the expression of Lasso compensation [12]. The ℓ_1-regularization problem is often solved by ℓ_1-singular value decomposition (ℓ_1-SVD) [13],[14] which is basically based on the convex optimization [15], [16]. It often gives good DOA estimation accuracy as a result of ℓ_1-regularization, however its computational cost often becomes very large. Also the regularization process is very sensitive to the penalty term in optimization. Therefore it is important to find an appropriate value of the penalty term to achieve good DOA estimation results, but it is difficult to know such value in advance of optimization. We recall that the shooting method [17] is computationally efficient and works well for ℓ_1-regularization problem, but it still sensitive to the value of the penalty term.

In this paper, we try to modify the shooting method into the the weighted ℓ_1-regularization problem for DOA estimation by Uniform Linear Array (ULA). First we roughly determine the weight values by the help of Beamformer method, and then tune the DOAs by the weighted ℓ_1-regularization. Then the optimization is no longer sensitive to the value of the penalty term. Performance of the proposed method is evaluated in comparison with several conventional methods through some computer simulation.

2. PRELIMINARIES

In this section, we first prepare signal model for ULA and then briefly introduce SpSF and Shooting method.

2.1. Signal Model

Consider the case that L incident waves arrive at M-element ULA of the half-wavelength interelement spacing ($d = \lambda/2$) under an Additive White Gaussian Noise (AWGN) environment. The received signal $x_m(n)$ at the m-th element / n-th time snapshot is given by

$$x_m(n) = \sum_{\ell=1}^{L} s_\ell(n) e^{j(m-1)\phi_\ell} + v_m(n), \quad m = 1, 2, \ldots, M,$$

where $v_m(n)$ is the receiver noise at m-th element, $s_\ell(n)$ is the ℓ-th incident wave coming from the angle θ_ℓ, and $\phi_\ell = -\frac{2\pi d}{\lambda} \sin\theta_\ell$ denotes the corresponding phase difference.

With the array input vector $\boldsymbol{x}(n) = [x_1(n), \ldots, x_M(n)]^T$, the array covariance matrix $\boldsymbol{R}_x$ (approximated by its sampled version) is defined as

$$\boldsymbol{R}_x = \mathrm{E}\left[\boldsymbol{x}(n)\boldsymbol{x}^H(n)\right] \simeq \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}(n)\boldsymbol{x}^H(n),$$

where N and $\mathrm{E}[\cdot]$ denote the number of snapshots and the expectation operator, respectively. The matrix $\boldsymbol{R}_x$ can also be written as $\boldsymbol{R}_x = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma^2\boldsymbol{I}$, where σ^2 denotes the noise power, and

$$\begin{aligned}\boldsymbol{A} &= \left[\boldsymbol{a}(\phi_1), \boldsymbol{a}(\phi_2), \ldots, \boldsymbol{a}(\phi_K)\right],\\ \boldsymbol{a}(\phi_k) &= \left[1, e^{-j\frac{2\pi}{\lambda}d\sin\phi_k}, \ldots, e^{-j\frac{2\pi}{\lambda}(M-1)d\sin\phi_k}\right]^T,\\ \boldsymbol{s}(n) &= \left[s_1(n), \ldots, s_L(n)\right]^T,\end{aligned}$$

where K corresponds to the angular resolution which satisfy $K \gg L$ and $K > M^2$, and $\boldsymbol{R}_s = \mathrm{E}[\hat{\boldsymbol{s}}(n)\hat{\boldsymbol{s}}^H(n)]$ denotes the extended source covariance matrix with $\hat{\boldsymbol{s}}(n) = [\hat{s}_1(n), \ldots, \hat{s}_K(n)]^T$, where $\hat{s}_k = s_i$ if the k-th angle corresponds to i-th source, otherwise $\hat{s}_k = 0$ [11].

2.2. Sparse Subspace Fitting (SpSF)

Sparse Spectrum Fitting (SpSF) [11] is briefly introduced in this subsection. The DOA estimation problem can be formulated as a constrained ℓ_1-regularization, i.e.,

$$\tilde{\boldsymbol{R}}_s = \underset{\boldsymbol{R}_s}{\operatorname{argmin}}\left\{\|\boldsymbol{R}_x - \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H\|_2^2 + \beta\|\mathrm{vec}(\boldsymbol{R}_s)\|_1\right\}, \quad (1)$$

where $\beta(> 0)$ denotes the penalty term in optimization. Then the diagonal elements $\tilde{\boldsymbol{p}} = \mathrm{diag}(\tilde{\boldsymbol{R}}_s)$ of the estimated matrix $\tilde{\boldsymbol{R}}_s$ corresponds to the SpSF angular spectrum.

The matrix $\boldsymbol{R}_s$ becomes diagonal in the case of uncorrelated sources; then we can rewrite (1) into

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}}{\operatorname{argmin}}\left\{\|\boldsymbol{r}_x - \boldsymbol{B}\boldsymbol{p}\|_2^2 + \beta\|\boldsymbol{p}\|_1\right\}, \quad (2)$$

where

$$\begin{aligned}\boldsymbol{r}_x &= \mathrm{vec}(\boldsymbol{R}_x),\\ \boldsymbol{B} &= [\boldsymbol{b}_{11}, \ldots, \boldsymbol{b}_{KK}],\\ \boldsymbol{b}_{kj} &= \mathrm{vec}[\boldsymbol{a}(\phi_k)\boldsymbol{a}^H(\phi_j)].\end{aligned}$$

The obtained vector $\tilde{\boldsymbol{p}}$ becomes an angular spectrum which has L peaks at DOAs.

2.3. Shooting Method

The shooting method [17], one of the computationally efficient ℓ_1-norm optimization methods, is briefly introduced. In the case of uncorrelated sources, the target cost function G to be minimized here is given by

$$G = \|\boldsymbol{r}_x - \boldsymbol{B}\boldsymbol{p}\|_2^2 + \beta\|\boldsymbol{p}\|_1.$$

Calculate its partial differential by j-th element p_j of $\boldsymbol{p}$, we have

$$\frac{\partial G}{\partial p_j} = \sum_i 2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{ki}p_i - 2\boldsymbol{b}_{kj}^T\boldsymbol{r}_x + \beta\cdot\mathrm{sign}(p_j),$$

where the function $\mathrm{sign}(p_j)$ takes the sign of p_j. Recall that $\partial G/\partial p_j = 0$ holds when G is minimized, we have

$$\sum_i 2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{ki}p_i - 2\boldsymbol{b}_{kj}^T\boldsymbol{r}_x = -\beta\cdot\mathrm{sign}(p_j),$$

which can be rewritten as a function of p_j:

$$2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{kj}p_j + S_0 = -\beta\cdot\mathrm{sign}(p_j), \quad (3)$$

with the intercept S_0:

$$S_0 = \sum_{i\neq j} 2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{ki}p_i - 2\boldsymbol{b}_{kj}^T\boldsymbol{r}_x.$$

Equation (3) can be regarded as a problem to find an intersection point between linear and weighted sign functions. By solving (3), we have

$$p_j = \begin{cases}\dfrac{\beta - S_0}{2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{kj}}, & S_0 > \beta,\\ 0, & |S_0| \leq \beta,\\ \dfrac{-\beta - S_0}{2\boldsymbol{b}_{kj}^T\boldsymbol{b}_{kj}}, & S_0 < -\beta,\end{cases} \quad (4)$$

which works as an alternating update function for the vector $\boldsymbol{p}$ in the iterative ℓ_1-regularization problem.

The initial value $\boldsymbol{p}_0$ for the iterative optimization is given by an mean square solution for $\boldsymbol{B}\boldsymbol{p}_0 = \boldsymbol{r}_x$, i.e.,

$$\boldsymbol{p}_0 = \boldsymbol{B}^+\boldsymbol{r}_x = \left(\boldsymbol{B}^T\boldsymbol{B}^T\right)^{-1}\boldsymbol{B}^T\boldsymbol{r}_x.$$

Then the elements $\{p_j\}_{j=1}^K$ of the vector $\boldsymbol{p}$ is alternately updated by (4) until convergence.

3. PROPOSED APPROACH

Here we try to modify the shooting method described in subsection 2.3 so that it does not become sensitive to the penalty term β.

First we confirm the relation between the SpSF spectrum and the penalty term β through some example spectrums. Figure 1 shows the example spectrums by SpSF with the shooting method in the case of 8-elements half-wavelength ULA, 2 DOAs from 0° and $-40°$, 0dB SNR and 300 snapshots, where $\beta = 1, 60$ and 200. Figure 1 says that a well-sparse spectrum is obtained in the case the value of β is adequate (e.g., $\beta = 60$). However, the spectrum has many pseudo peaks for a small value of β (e.g., $\beta = 1$), and always becomes zero

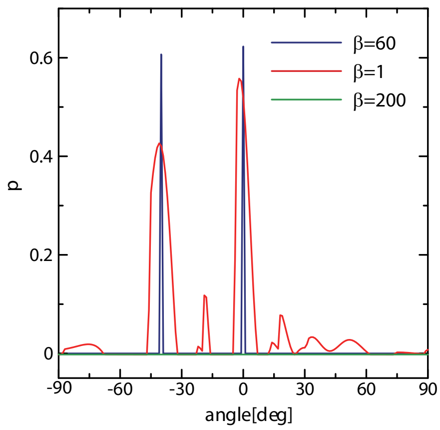


Fig. 1. Behavior of the angular spectrums by SpSF with Shooting method for various values of β.

without any peak for a large value of β (e.g., $\beta = 200$). We see from Fig. 1 that the SpSF is very sensitive to the penalty term β, and it is significant to choose an appropriate value of β in advance.

SpSF gives the spectrum vector $\boldsymbol{p}$ based on (3) where the penalty term β is a constant irrespective to angle. Here we newly introduce a penalty term which depends on angle. Using the angular weight vector $\boldsymbol{w} = [w_1, w_2, \ldots, w_K]$, equation (2) can be rewritten as

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}}{\operatorname{argmin}} \left\{ \|\boldsymbol{r}_x - \boldsymbol{B}\boldsymbol{p}\|_2^2 + \alpha \boldsymbol{w}^T \boldsymbol{p}_a \right\}, \qquad (5)$$

where α is the scaling factor, and $\boldsymbol{p}_a = [|p_1|, |p_2|, ..., |p_K|]^T$ is the vector with the absolute values of p_j. Here the term $\boldsymbol{w}^T \boldsymbol{p}_a$ can be regarded as the weighted ℓ_1-norm of $\boldsymbol{p}$. The elements p_j of the vector $\boldsymbol{p}$ are updated by

$$p_j = \begin{cases} \dfrac{\alpha w_j - S_0}{2\boldsymbol{b}_{kj}^T \boldsymbol{b}_{kj}}, & S_0 > \alpha w_j, \\ 0, & |S_0| \leq \alpha w_j, \\ \dfrac{-\alpha w_j - S_0}{2\boldsymbol{b}_{kj}^T \boldsymbol{b}_{kj}}, & S_0 < -\alpha w_j. \end{cases} \qquad (6)$$

It is desired from Fig. 1 that p_j becomes zero for non-DOA angles. Therefore a large value of the weight w_j should be assigned for those angles.

Based on the above discussion, we arrange the following two-step approach: (a) the first step is a rough estimation stage of non-DOA angles by any simple DOA estimation method just to determine weight values w_j, and (b) the next step is a fine tuning stage by the weighted ℓ_1-norm optimization based on (5) and (6). We employ the classical beamformer method as a simple DOA estimation method to be used in the first step. The angular spectrum $\boldsymbol{P}_{bf} = [P_{bf}(\phi_1), P_{bf}(\phi_2), \ldots, P_{bf}(\phi_K)]$ of the beamformer method is given by

$$P_{bf}(\phi_k) = \frac{\boldsymbol{a}^H(\phi_k)\boldsymbol{R}_x \boldsymbol{a}(\phi_k)}{\boldsymbol{a}^H(\phi_k)\boldsymbol{a}(\phi_k)}. \qquad (7)$$

Note that the angular spectrum $\boldsymbol{P}_{bf}$ in (7) becomes small for non-DOA angles. In order to make large values for those angles, the weight w_j is defined as a normalized version of the inverse beamformer spectrum, i.e.,

$$\boldsymbol{w} = P_{\max} \boldsymbol{P}'_{bf}, \qquad (8)$$

where

$$P_{\max} = \max_k (P_{bf}(\phi_k)),$$

$$\boldsymbol{P}'_{bf} = \left[\frac{1}{P_{bf}(\phi_1)}, \frac{1}{P_{bf}(\phi_2)}, \ldots, \frac{1}{P_{bf}(\phi_K)} \right].$$

Note that the weight $\boldsymbol{w}$ is normalized by the maximum value of the beamformer spectrum so that the minimum value becomes one. Figures 2(a) and 2(b) respectively show the example angular spectrum $\boldsymbol{P}_{bf}$ in (7), and the obtained weight vector $\boldsymbol{w}$ in (8) under the same specifications with Fig. 1. We see from Fig. 2 that an appropriate weight vector is certainly generated which has small values near DOAs and large values in non-DOA angles.

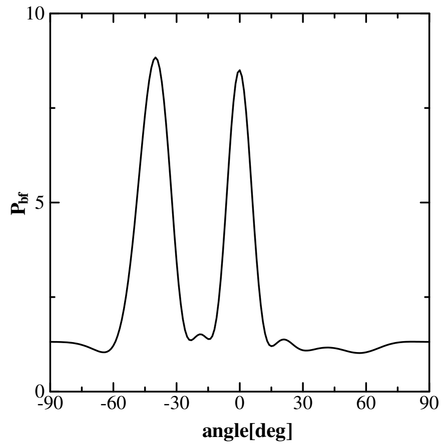
Thanks to the weight $\boldsymbol{w}$, the proposed method is no longer sensitive to the scaling term α in (5) while the original shooting method is very sensitive to the penalty term β. Indeed the value of α is simply determined so that the weight $\boldsymbol{w}$ has an enough large dynamic range (the difference between mainlobe and sidelobe levels of the beamformer spectrum). Note that the dynamic range of the weight $\boldsymbol{w}$ often becomes small in the case of low SNR due of the poor characteristics of the beamformer method. For such case, a large value of α is assigned to expand the dynamic range of the weight $\boldsymbol{w}$.

4. SIMULATION

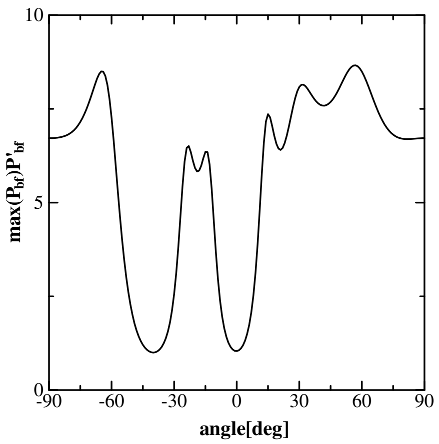
In this section, the performance of the proposed method is evaluated through some simulation in comparison with some conventional methods.

Table 1 lists the specifications of the simulation in this section. We try to estimate DOAs for the cases of two different scenarios. The DOA estimation performance is evaluated by RMSE (Root Mean Square Error) between the estimated and true DOAs. The angular spectrum is calculated at every 0.1° in the range $[-90^\circ, 90^\circ]$; which means the resolution parameter $K = 1801$. The value of the parameter α in the proposed method is simply determined so as to expand the dynamic range of the weight w_j to be more than 30dB.

Figure 3 shows the example angular spectrums by MUSIC and the proposed methods under the scenario #1 for the cases of (a) 5dB and (b) -15dB. We see from Fig. 3 that the spectrum by the proposed method well separate close-angle

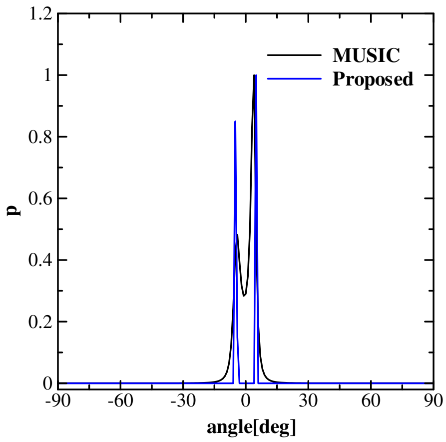


(a) Spectrum $\boldsymbol{P}_{bf}$ by Beamformer method

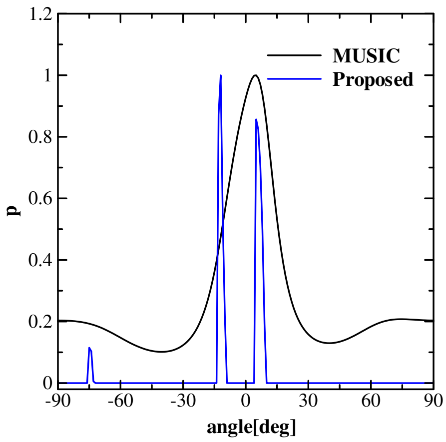


(b) Weight $\boldsymbol{w}$ obtained by (8)

Fig. 2. Example beamformer spectrum and the weight obtained by (8).



(a) in the case of SNR 5dB



(b) in the case of SNR -15dB

Fig. 3. Comparison of angular spectrums

Table 1. Specifications of simulation

scenario	#1	#2
array configuration	half-wavelength ULA	
# of array elements, M	4	8
# of sources, L	2	3
correlation of sources	uncorrelated	
DOAs, θ_ℓ	$-10°, 5°$	$-10°, 0°, 10°$
# of snapshots, N	300	
# of trials, N	100	

waves, especially in low SNR situation in Fig. 3(b). The weighted approach works well even in a severe environment.

To objectively evaluate DOA estimation performance, we compared the averaged RMSE for multiple sources as a function of SNR in low SNR environments from -15dB to 5dB as in Fig. 4. The optimum value of the penalty term β is studied in advance and used in each DOA estimation for the standard shooting method. We see from Fig. 4 that the proposed method gives smaller RMSE than the conventional MUSIC and the standard shooting method, especially in the case of low SNR where the RMSE of the conventional methods does not become close to CRLB [18].

5. CONCLUDING REMARKS

This paper investigated the weighted shooting method for high-resolution DOA estimation based on sparse spectrum fitting. The present method achieves smaller RMSE than the conventional approaches in severe environments. Application of the present method to underdetermined DOA estimation problem still remains as one of future studies.

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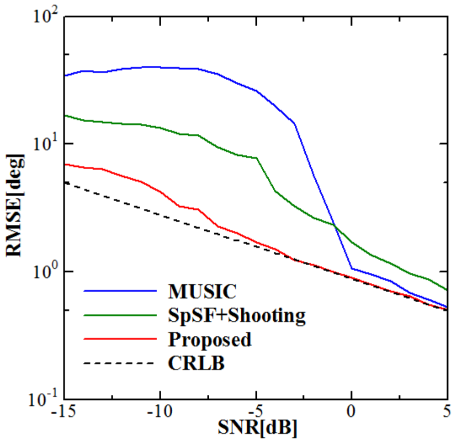
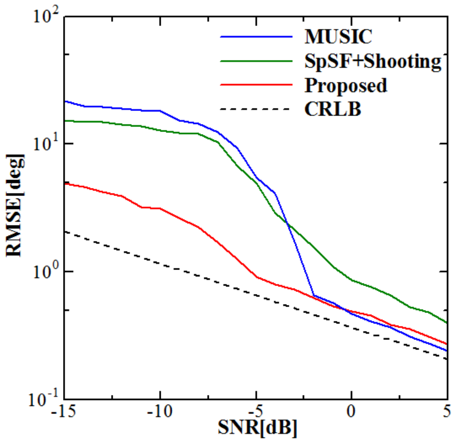


(a) in the case of scenario #1



(b) in the case of scenario #2

Fig. 4. Behavior of RMSEs as a function of SNR

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