Low Complexity Hybrid Precoding in Finite Dimensional Channel for Massive MIMO Systems

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***Abstract*—Massive multiple-input multiple-output (MIMO) is an emerging technology for future wireless networks, scaling up conventional MIMO to an unprecedented number of antennas at base stations. Such a large antenna array has the potential to make the system achieve high channel capacity and spectral efficiency, but it also leads to high cost in terms of hardware complexity. In this paper, we consider a finite dimensional channel model in which finite distinct directions are applied with M angular bins. In massive multi-user MIMO systems, a hybrid precoding method is proposed to reduce the required number of radio frequency (RF) chains at the base station, employing a single antenna per mobile station. The proposed precoder is partitioned into a high-dimensional RF precoder and a low-dimensional baseband precoder. The RF precoder is designed to obtain power gain with phase-only control and the baseband precoder is designed to facilitate multi-stream processing. For realistic scenarios, we consider the situation where the RF phase control is quantized up to B bits of precision. Furthermore, an upper bound on spectral efficiency is derived with the proposed precoding scheme. The simulation results show that hybrid precoding achieves desirable performance in terms of spectral efficiency, which approaches the performance of zero-forcing precoding.**

***Index Terms*—Massive MIMO, precoding, finite dimension, RF chain, hardware complexity**

I. Introduction

The blossoming research on massive multiple-input multiple-output (MIMO) technology has captured the attention of researchers all over the world [1] [2] [3], as it is proposed to deal with the challenges of the exponentially growing communication traffic and spectrum bands with wider bandwidth [4] [5]. In order to achieve high array gain and high spatial multiplexing gain, massive MIMO employs an unprecedented number of base station antennas simultaneously to serve a smaller number of single-antenna user terminals in the same channel. In massive MIMO systems, linear precoding methods such as zero-forcing (ZF) and minimum mean square error (MMSE) are able to achieve near optimal performance [1]. However, traditional precoding schemes process the complex signals digitally at the baseband and then upconvert to the carrier frequency, thus every antenna element needs to be coupled with one radio frequency (RF) chain, which includes the digital-to-analog convertors, mixers and power amplifiers. When the number of antennas at the base station is very large, a large number of RF chains will result in excessively high hardware cost and power consumption.

To address the RF hardware constraints [6], variable phase shifters with high-dimensional phase-only RF processing are exploited to control the phases of the upconverted RF signal [7] [8]. The variable phase shifters are digitally controlled and changed in reasonably low time scale for variable channels. Therefore, a hybrid precoding scheme is proposed, which exploits a phase-only RF precoder in the analog domain and a baseband precoder in the digital domain. In [9], phase only RF precoding is proposed to implement maximization of the average spectral efficiency of single-antenna mobile stations for massive MIMO systems. Furthermore, in millimetre wave (mmWave) MIMO systems, [10] and [11] use hybrid methods to obtain near optimal SVD (singular value decomposition) performance, which decomposes the optimal precoder and combiner through the concept of orthogonal matching pursuit. Due to the high complexity of searching columns of the overcomplete matrix in [11], a low-complexity hybrid sparse precoding method is proposed in [12] using a greedy method with the element-wise normalization of the first singular vector of the residual. In [13], limited feedback hybrid precoding is proposed with small training and feedback overhead to achieve near optimal block diagonalization performance. In addition, not only the number of RF chains but also phase shifters can be reduced to cut energy consumption based on successive interference cancelation hybrid precoding with sub-connected architecture in [14]. In terms of mmWave MIMO systems, the channel model in [11], [13], [14] focuses on point-to-point channels using the clustered channel model. However, for the multiple users situation, where each mobile station is equipped with a single antenna, a finite dimensional channel model with M angular bins can be more applicable to obtain generally correlated or not asymptotically orthogonal channel [15]. The reason is that the finite dimensional channel is able to reflect the property of the poor scattering channel environment caused by high pathloss at high frequency [16] and the angles of arrival at the mobile stations can be neglected.

In this paper, we consider the downlink transmission of massive multi-user MIMO systems with a finite dimensional channel model, in which a limited number of dimensions M is defined to model the channel vectors. Unfortunately, the precoding methods in [11] and [16] can not be applied to such a finite dimensional channel. Thus, a low complexity hybrid precoding scheme is proposed for an M-dimensional channel model. By analyzing the structure of the channel model, we

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combine the beamsteering codebooks with extracting the phase of the conjugate transpose of the fast fading matrix [17] to design the RF precoder, which thereby harvests the large array gain achieved by an unprecedented number of base station antennas. Then a baseband precoder is designed based on the equivalent channel with ZF precoding. Furthermore, we derive a tight upper bound on the achievable rate and investigate the performance of hybrid precoding when the number of base station antennas is much larger than M. It is shown that the hybrid precoding is actually more feasible than traditional ZF precoding, because of the near optimal performance and low hardware complexity. In consideration of practical constraints of the phase shifter, the performance of the proposed precoding is also investigated in the simulation based on quantized RF phase control.

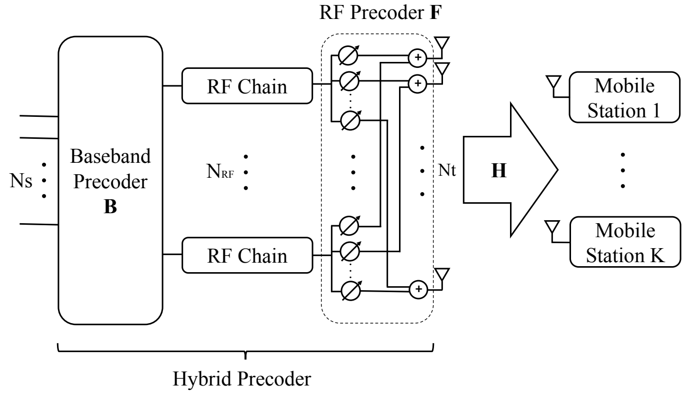

Fig. 1. System diagram of massive multi-user MIMO systems with hybrid precoding for a finite dimensional channel.

II. System Model

A. System Model

We consider the downlink of a massive multi-user MIMO system. As shown in Fig. 1, we denote the number of base station antennas as N_t and assume there are only N_{RF} RF chains at the base station to communicate with K mobile stations. For simplicity, the base station utilizes K RF chains to serve the mobile stations and each mobile station is equipped with a single antenna. Single data streams are transmitted from the base station to every mobile station, therefore we assume K transmitted data streams are handled by the base station.

On the downlink, we propose a new hybrid precoding scheme, for which $\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_K]$ is the RF precoder of dimension $N_t \times K$ and $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_K]$ is the baseband precoder of dimension $K \times K$. The baseband precoder modifies both amplitude and phase while only phase changes can be made by the RF precoder with a network of variable analog phase shifters. Thus, the entries $\mathbf{F}_{m,n}$ in the RF precoder are of constant modulus, which are normalized to satisfy $|\mathbf{F}_{m,n}|^2 = \frac{1}{N_t}$. Due to the total transmit power constraint, the entries of $\mathbf{B}$ are normalized to satisfy $\|\mathbf{FB}\|_F^2 = K$, where $\|\cdot\|_F$ is the Frobenius norm. The transmit signal is given by

$$\mathbf{x} = \mathbf{FBs}, \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \cdots, s_K]^T$. $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the vector of signal for K mobile stations, and $[\cdot]^T$ denotes vector transpose. $\mathbb{E}[\mathbf{ss}^H] = \frac{P_t}{K}\mathbf{I}_K$ where P_t is the total transmit power and $\mathbb{E}[\cdot]$ is used to denote expectation operator. We further assume that perfect channel state information is known at the base station. The signal received at the kth mobile station is given by

$$y_k = \mathbf{H}_k \sum_{n=1}^{K} \mathbf{FB}_n s_n + n_k, \tag{2}$$

where $\mathbf{H}_k$ represents the $1 \times N_t$ downlink channel gains between the base station and kth mobile station and n_k represents additive white Gaussian noise with zero mean and unit variance, i.e. $\mathcal{CN}(0, \sigma^2)$. The general channel matrix is set as $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K]^T$. In this channel model, combiners are not used at the mobile stations, which reduces the hardware cost and power consumption. The achievable spectral efficiency of this transmission can be expressed as

$$R_k = \log_2\left(1 + \frac{\frac{P_t}{K}|\mathbf{H}_k\mathbf{FB}_k|^2}{\frac{P_t}{K}\sum_{n=1,n\neq k}^{K}|\mathbf{H}_k\mathbf{FB}_n|^2 + \sigma^2}\right), \tag{3}$$

The sum spectral efficiency of the system is $R = \sum_{k=1}^{K} R_k$.

B. Channel Model

In reality, the channel environment suffers from poor scattering which will influence the performance of systems and the dimension of the physical channel is finite [18] [19]. Moreover, large antenna arrays operating at high frequency will imply channel vectors are correlated, hence many statistical fading models of traditional MIMO systems are not applicable [11]. Due to the mobile stations being equipped with a single antenna, the angles of arrival at the mobile stations are not considered, so the clustered channel model is also inaccurate. In this paper, we consider a geometric channel model with finite dimension. The M-dimensional channel divides the transmitted angular region into M directions, where $M < N_t$. The angles of departure θ_m are applied in each direction, where $\theta_m \in [-\pi/2, \pi/2]$ and $m = 1, \cdots, M$. We employ the uniform linear array (ULA) in our work. The vector $\mathbf{a}_{BS}(\theta_m)$ presents the transmit antenna array structure and the $N_t \times 1$ array response vector $\mathbf{a}_{BS}(\theta_m)$ can be defined as

$$\mathbf{a}_{BS}(\theta_m) = \frac{1}{\sqrt{M}}[1, e^{-j\frac{2\pi}{\lambda}dsin\theta_m}, \cdots, e^{-j(N_t-1)\frac{2\pi}{\lambda}dsin\theta_m}]^T, \tag{4}$$

where λ is the wavelength of the signal and d is the distance between antenna elements. Under this model, the M-dimensional channel can be expressed as

$$\mathbf{H} = \mathbf{GA}^H, \tag{5}$$

where $\mathbf{A} = [\mathbf{a}_{BS}(\theta_1)\ \mathbf{a}_{BS}(\theta_2)\cdots\mathbf{a}_{BS}(\theta_M)]$ is an $N_t \times M$ matrix, $\mathbf{G}$ presents the propagation coefficient matrix of dimension $K \times M$ from the base station to mobile stations and $(\cdot)^H$ denotes Hermitian transpose. The propagation matrix $\mathbf{G}$

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is used to model fast fading, geometric attenuation and shadow fading [15]. Then, we have propagation matrix

$$\mathbf{G} = \mathbf{D}\mathbf{W}, \tag{6}$$

where $\mathbf{D}$, of dimension $K \times K$, is the diagonal matrix of the geometric attenuation and shadow fading coefficients for the multi-cell scenario. The diagonal elements of $\mathbf{D}$, which are given by β_{kk}, are assumed to be constant, because they vary slowly with time. β_{kk} represents the (k,k)th entry of $\mathbf{D}$. The entries in the $K \times M$ matrix $\mathbf{W}$ represent the fast fading coefficients between the base station and mobile stations, which are assumed to have zero mean and unit variance [15]. Therefore, assuming $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_K]^T$, (2) can be written as

$$y_k = \beta_{kk}\mathbf{W}_k\mathbf{A}^H \sum_{n=1}^{K} \mathbf{F}\mathbf{B}_n s_n + n_k, \tag{7}$$

III. Hybrid Precoding Design

In this section, we propose a new design of hybrid precoding algorithm in the finite dimensional channel. The main objective is to achieve optimal spectral efficiency with low hardware complexity. The simplest precoding scheme is to invert the channel matrix using pseudoinverse, which is referred to as ZF precoding. However, in massive MIMO systems, although ZF precoding achieves near optimal capacity performance, a large number of RF chains are required to perform analog-to-digital conversion and frequency translation between baseband and RF, which suffers from high hardware cost and restricts the array size at the base station from scaling to a large dimension. Thus, the proposed precoding scheme leverages the structure of the finite dimensional channel to address the hardware constraints.

A. Hybrid Precoder Design

As shown in Fig. 1, the hybrid precoder is composed of an RF precoder $\mathbf{F}$ and a baseband precoder $\mathbf{B}$. $\mathbf{F}$ utilizes the phase shifters to couple K RF chains with N_t base station antennas based on phase-only control of the upconverted RF signal and $\mathbf{B}$ performs low-dimensional multiple stream processing to change the amplitude and phase of the transmitted complex signal based on ZF precoding.

First, we design the RF precoding matrix $\mathbf{F}$. Perfect channel state information is assumed known at the base station and we extract the phases of the conjugate transpose of the fast fading coefficient matrix $\mathbf{W}$ as $\widetilde{\mathbf{W}}_{m,n} = \frac{1}{\sqrt{M}} e^{j\varphi_{m,n}}$, where $\varphi_{m,n}$ is the phase of the $(m,n)th$ entry of the conjugate transpose of $\mathbf{W}$. Then beamsteering codebooks are required and have similar form to the array response vector. We assume $\mathcal{A}$ to be the beamsteering codebook and because of the phase-only processing, it is designed by the normalized array response vector. We select $\widetilde{\mathbf{A}}$ from $\mathcal{A}$ to maximize the diagonal entries of matrix $\mathbf{A}^H\widetilde{\mathbf{A}}$ and then use $\widetilde{\mathbf{W}}$ to maximize the desired signal power of mobile stations. Therefore, the design of the RF precoder $\mathbf{F}$ can be given by

$$\mathbf{F} = \widetilde{\mathbf{A}}\widetilde{\mathbf{W}}, \tag{8}$$

In realistic application scenarios, the practical constraints of phase shifters makes the phase of each entry of $\mathbf{F}$ suffer from the impact of coarse quantization, which influences the performance of hybrid precoding. Therefore, we also consider the situation where the phases of the KN_t entries are quantized up to B bits of precision, i.e. $B = 4$ or 5. In Section IV, the performance with quantized $\mathbf{F}$ is investigated to compare with the unquantized situation.

Then an equivalent channel $\overline{\mathbf{H}}$ is applied to design the baseband precoder $\mathbf{B}$. The dimension of $\overline{\mathbf{H}}$ is $K \times K$, which is much lower than the $N_t \times K$ dimension of the original channel. $\overline{\mathbf{H}}$ is given by

$$\overline{\mathbf{H}} = \mathbf{H}\mathbf{F}, \tag{9}$$

where $\overline{\mathbf{H}} = [\overline{\mathbf{H}}_1, \overline{\mathbf{H}}_2, \cdots, \overline{\mathbf{H}}_K]$. After ZF precoding, the direct baseband precoding matrix $\overline{\mathbf{B}}$ based on the equivalent channel is then defined as

$$\overline{\mathbf{B}} = \overline{\mathbf{H}}^H(\overline{\mathbf{H}}\overline{\mathbf{H}}^H)^{-1}, \tag{10}$$

The normalized baseband precoder $\mathbf{B}$ is

$$\mathbf{B} = \frac{\overline{\mathbf{B}}}{\sqrt{\|\mathbf{F}\overline{\mathbf{B}}\|_F^2}}, \tag{11}$$

where $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_K]$ to satisfy the precoding power constraints $\|\mathbf{F}\mathbf{B}_k\|_2^2 = 1$. From (8) and (11), $\mathbf{F}$ and $\mathbf{B}$ of the hybrid precoding are both obtained. As shown in Fig. 1, the transmitter supports simultaneous transmission of K data streams and the hardware complexity is obviously reduced because there are only K RF chains using the proposed precoding instead of N_t RF chains required by the traditional ZF precoding and $K \ll N_t$. For example, in Section IV $N_t = 128$ and $K = 4$, giving a significant reduction of hardware complexity.

B. Spectral Efficiency Analysis

In this section, we analyze the spectral efficiency with perfect channel state information. For the finite dimensional channel model, tight upper bound on spectral efficiency achieved by hybrid precoding is derived and expressed as the spectral efficiency for the kth mobile station which is upper bounded by

$$R_k \leq \log_2(1 + \frac{\text{SNR}}{K}\lambda_{app}(\mathbf{A}\mathbf{A}^H)(\mathbf{W}_k\mathbf{W}_k^H)), \tag{12}$$

where $\text{SNR} = \frac{P_t}{\sigma^2}$ and $\lambda_{app}(\mathbf{A}\mathbf{A}^H)$ is the appropriate eigenvalue of matrix $\mathbf{A}\mathbf{A}^H$.

Proof: The spectral efficiency for the kth mobile station is

$$\begin{aligned} R_k &= \log_2(1 + \frac{\text{SNR}}{K}|\overline{\mathbf{H}}_k\mathbf{B}_k|^2) \\ &= \log_2(1 + \frac{\text{SNR}}{K}|\overline{\mathbf{H}}_k\frac{\overline{\mathbf{B}}_k}{\sqrt{\|\mathbf{F}\overline{\mathbf{B}}_k\|_2^2}}|^2) \\ &= \log_2(1 + \frac{\text{SNR}}{K}\frac{1}{\|\mathbf{F}\overline{\mathbf{B}}_k\|_2^2}). \end{aligned} \tag{13}$$

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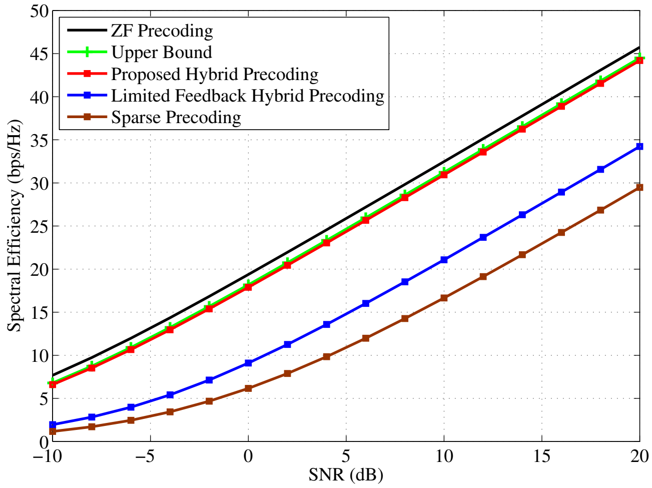

Fig. 2. Spectral efficiency achieved by different precoding schemes with infinite resolution in downlink massive MU-MIMO systems where $N_t = 128, K = N_{RF} = 4$ and the finite dimension M is 64.

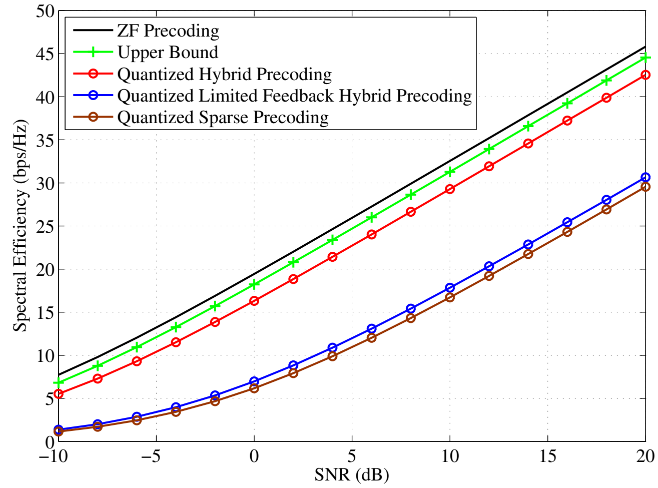

Fig. 3. Spectral efficiency achieved by different quantized precoding schemes with 4 bits of precision in downlink massive MU-MIMO systems where $N_t = 128, K = N_{RF} = 4$ and the finite dimension M is 64.

Based on (8) (9) and (10), we have

$$\mathbf{F}\overline{\mathbf{B}}_k = \widetilde{\mathbf{A}}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\widetilde{\mathbf{A}}^H\mathbf{A}\mathbf{W}_k^H(\mathbf{W}_k\mathbf{A}^H\widetilde{\mathbf{A}}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\widetilde{\mathbf{A}}^H\mathbf{A}\mathbf{W}_k^H)^{-1}$$
$$= \mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}\mathbf{W}_k^H(\mathbf{W}_k\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}\mathbf{W}_k^H)^{-1}, \quad (14)$$

where $\widetilde{\mathbf{A}} = \sqrt{\frac{M}{N_t}}\mathbf{A}$ based on the beamsteering codebooks. Next,

$$\frac{1}{\|\mathbf{F}\overline{\mathbf{B}}_k\|_2^2} = \frac{|\mathbf{W}_k\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}\mathbf{W}_k^H|^2}{(\mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}\mathbf{W}_k^H)^H(\mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}\mathbf{W}_k^H)}. \quad (15)$$

Briefly, by defining $\mathbf{X} \triangleq \mathbf{W}_k\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{W}}^H$ and $\mathbf{Y} = \widetilde{\mathbf{W}}\mathbf{A}^H\mathbf{A}$, (15) can be rewritten as

$$\frac{1}{\|\mathbf{F}\overline{\mathbf{B}}_k\|_2^2} = \frac{|\mathbf{X}\mathbf{Y}\mathbf{W}_k^H|^2}{\mathbf{X}\mathbf{Y}\widetilde{\mathbf{W}}^H\mathbf{X}^H}$$
$$\overset{(a)}{\leqslant} \frac{\mathbf{X}\mathbf{Y}(\mathbf{X}\mathbf{Y})^H\mathbf{W}_k\mathbf{W}_k^H}{\mathbf{X}\mathbf{Y}\widetilde{\mathbf{W}}^H\mathbf{X}^H}, \quad (16)$$

where (a) is valid by applying the Cauchy-Schwarz inequality [20]. We substitute $\mathbf{X}$ and $\mathbf{Y}$ into (16) and define $\mathbf{Z} = \mathbf{W}_k\mathbf{A}^H\mathbf{A}\widetilde{\mathbf{W}}^H\widetilde{\mathbf{W}}\mathbf{A}^H$. Then we have

$$\frac{1}{\|\mathbf{F}\overline{\mathbf{B}}_k\|_2^2} \leqslant \frac{\mathbf{Z}\mathbf{A}\mathbf{A}^H\mathbf{Z}^H}{\mathbf{Z}\mathbf{Z}^H}\mathbf{W}_k\mathbf{W}_k^H$$
$$\overset{(b)}{\leqslant} \lambda_{app}(\mathbf{A}\mathbf{A}^H)\mathbf{W}_k\mathbf{W}_k^H, \quad (17)$$

where the Rayleigh-Ritz theorem [21] is used in (b) because $\mathbf{A}\mathbf{A}^H$ is a Hermitian matrix. Then we obtain the upper bound on the spectral efficiency in (12) by the proposed hybrid precoding scheme. The upper bound separates the dependence on the array response vectors and the fast fading coefficients matrix and shows the optimality of the hybrid precoding.

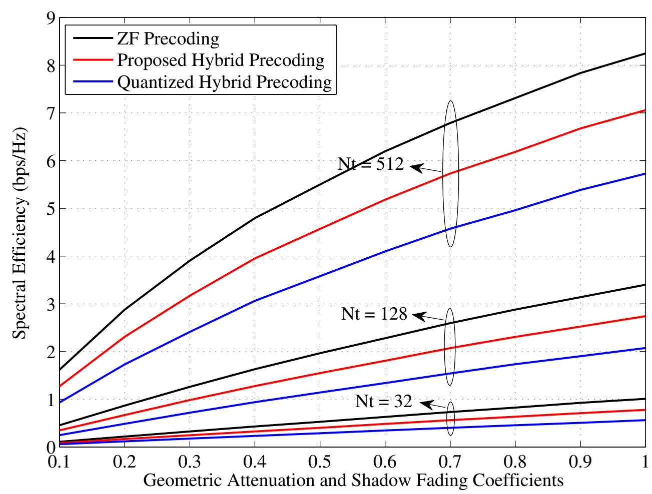

Fig. 4. Spectral efficiency of precoding schemes versus the geometric attenuation and shadow fading coefficients.

IV. Simulation Results

In this section, we evaluate the performance of the proposed hybrid precoding scheme. In our simulation, the relative element spacing of the ULA is $\frac{d}{\lambda} = 0.3$ and the angles of departure are uniformly distributed as $\theta_m = -\pi/2 + (m-1)\pi/M$, where $m = 1, 2, \cdots, M$.

First, we compare the spectral efficiency achieved by different precoding schemes with infinite resolution, together with the tight upper bound in Fig. 2. The base station is assumed to employ the ULA with $N_t = 128$ antennas, serving $K = 4$ mobile stations each with a single antenna. There are $N_{RF} = 4$ RF chains and the finite dimension M is 64. Although ZF precoding is optimal in the simulation, it is infeasible because of the requirement of costly RF chains in the large antenna array. It is observed that the proposed hybrid precoding achieves spectral efficiency close to that achieved by the optimal ZF precoding with less than 1 dB loss. The tight upper bound derived in Section III-B is also plotted. Obviously, the proposed hybrid precoding method performs better than the limited feedback hybrid precoding and the sparse precoding.

Then Fig. 3 illustrates the spectral efficiency achieved by different quantized precoding schemes, considering the same setup as Fig. 2. For the quantized phase, we simulate with $B = 4$ bits of precision and the phase control candidates are $(2\pi n)/(2^B)$ where $n \in \{0, \cdots, 2^B - 1\}$. We observe that the quantized hybrid precoding schemes all suffer degradation, while the proposed hybrid precoding still has better performance than the limited feedback hybrid precoding and the sparse precoding.

In Fig. 4, we consider the effort of the multi-cell transmission at SNR $= -10$dB, but with different numbers of base station antennas, $N_t = 32$, 128 and 512. The spectral efficiency for different precoding schemes versus the geometric attenuation and shadow fading coefficients is simulated with $K = 4$ mobile stations. The number of RF chains is equal to the number of mobile stations and the finite dimension M is 16. We assume that β_{kk} are equal to 1 when the mobile stations are in the cell of the transmitting base station, but β_{kk} are equal to a when the mobile stations are in other cells, where $0 < a < 1$. We can see that when the value of the coefficients is low, the effect of the antenna number at the base station tends to be small, while when the coefficients becomes higher, the spectral efficiency gap between different numbers of base station antennas is increasingly obvious.

V. Conclusion

In this paper, we proposed a low complexity hybrid precoding scheme for multiple mobile stations for the finite dimensional channel. The geometric channel model with finite dimension is applicable to obtain generally correlated or not asymptotically orthogonal channel. The proposed precoding with non-iterative design gives a significant reduction of RF chains, which reduces the hardware complexity in massive MIMO systems. In terms of the performance of spectral efficiency, the proposed method approaches the optimal ZF performance and performs better than other precoding schemes, which is showed in the simulation results. We also show that the proposed precoding can be quantized and the numerical results on the performance are still better than other precoding schemes.

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