Genetic Approach and HOTELLING Criterion for Selecting the Optimal Attribute Vector: Application to a Supervised Classification of Texture Images

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Abstract—Selecting the parameters for the classification is a delicate procedure. We present in this work a new method for selecting the parameters based on the genetic approach which optimizes the choice of parameters by minimizing a cost function. This function is defined by a new criterion that we have proposed. The proposed approach is validated on some texture images. The experimental results show the good performance of the proposed method.

I. INTRODUCTION

A texture is spatial repetition of a basic pattern in different directions [1]. The classification [2] of texture images requires a robust selection of the specific parameters [3][4]. Selecting the parameters for the classification is a delicate procedure [5][6][7][8][9][10][11]. There exist various methods for selecting the parameters [1]. We present in this work a new method for selecting the parameters [12][13].

II. TEXTURE PARAMETERS

Various methods of texture characterization of images have been proposed. HARALICK has proposed 14 descriptive texture parameters all extracted from the cooccurrence matrix. Most frequently used are [14]: Hom, HomL, Ent, Uni, Dir, Cont and Cor. GALLOWAY has defined 5 parameters for characterizing textures from the matrix of run length, which are [15]: SRE, LRE, GLN, RLN and RP.

III. GENETIC ALGORITHMS

Genetic algorithms are techniques for optimizing functions. GA are based on the evolution of a population of solutions which under the action of some precise rules optimize a given behavior, which initially has been formulated by a given specified function fitness function [16]. A GA manipulates a population of constant size. This population is formed by chromosomes. Each chromosome represents the coding of a potential solution to the problem to be solved, it is formed by a set of genes belonging to an alphabet [17]. At each iteration is created a new population by applying the genetic operators: selection, crossover and mutation [18]. The algorithm chooses in selection the most pertinent candidates. Crossover consists in building 2 new chromosomes from 2 old ones referred to as the parents. Mutation realizes the inversion of one or several genes in a chromosome [18].

Random generation of the initial population
Fitness evaluation of each chromosome
Repeat
Selection
Crossover

Mutation
Fitness evaluation of each chromosome
Until Satisfying the stop criterion

Fig. 1. Basic genetic algorithm.

IV. PARAMETERS SELECTION PROBLEM FORMULATION

A. Descriptive elements

Let consider a set of \( M \) texture images \( \{\mathbf{I}_i\}_{i=1}^M \) characterized by \( N \) texture parameters written in a line vector \( \mathbf{V} = (a_j) \). Let \( \mathbf{R}_i = (a_{ij})_{i \in \mathbb{N}} \) be a line vector of \( \mathbb{R}^N \) where \( a_{ij} \) is the value of \( a_j \) for \( \mathbf{I}_i \). Let \( \mathbf{E}_v = \{a_j\}_{i \in \mathbb{N}} \). Let \( \mathbf{mat}_va : \)

\[
\mathbf{mat}_va(\mathbf{a}_{ij})_{i \in \mathbb{N}}
\]

\( V \) is the attribute vector, \( \mathbf{R}_i \) is the realization of \( V \) for \( \mathbf{I}_i \), \( \mathbb{R}^N \) is the parameter space \([2][1][10]\), \( \mathbf{E}_v \) is the set associated with \( V \) and \( \mathbf{mat}_va \) is the observation matrix associated with \( V \). The \( i \)th line of \( \mathbf{mat}_va \) is \( \mathbf{R}_i \). Each \( \mathbf{R}_i \) belongs to the class \( \mathbb{C}_{\alpha}, s=1, \ldots, C \). Let \( \mathbf{col}_{a_j} \) a vector of \( M \) dimension associated with \( a_j \). Let \( \mathbf{m}_{a_j} \) be the mean value and let \( \sigma_{a_j} \) be the standard deviation of \( a_j \):

\[
\mathbf{col}_{a_j} = (a_{ij})_{i \in \mathbb{N}}
\]

\[
\mathbf{m}_{a_j} = \frac{1}{M} \sum_{i=1}^{M} a_{ij}
\]

\[
\sigma_{a_j} = \frac{1}{M} \left( \sum_{i=1}^{M} (a_{ij} - \mathbf{m}_{a_j})^2 \right)^{1/2}
\]

The linear correlation coefficient between \( a_j \) and \( a_l \) is :

\[
r(a_j, a_l) = \frac{\text{cov}(\mathbf{col}_{a_j}, \mathbf{col}_{a_l})}{\sigma_{a_j} \sigma_{a_l}}
\]

Let \( \mathbf{m}_{(a_j, \mathbb{C}_{\alpha})} \) be the mean value and let \( \sigma_{(a_j, \mathbb{C}_{\alpha})} \) be the standard deviation of \( a_j \) of the \( \mathbb{C}_{\alpha} \) class realizations:

\[
\mathbf{m}_{(a_j, \mathbb{C}_{\alpha})} = \frac{1}{\text{card}(\mathbb{C}_{\alpha})} \sum_{R_{ij} \in \mathbb{C}_{\alpha}} a_{ij}
\]

\[
\sigma_{(a_j, \mathbb{C}_{\alpha})} = \left( \frac{1}{\text{card}(\mathbb{C}_{\alpha})} \sum_{R_{ij} \in \mathbb{C}_{\alpha}} \left( a_{ij} - \mathbf{m}_{(a_j, \mathbb{C}_{\alpha})} \right)^2 \right)^{1/2}
\]
Let $V_{(m)}$ be the vector formed by the mean value of each $a_j$ and let $V_{(m,CL)}$ be the vector formed by the mean value of each $a_j$ of the $CL_s$ class realizations:

$$V_{(m)} = \left[ \frac{m}{a_j} \right]_{1 \leq j \leq N}$$

$$V_{(m,CL_s)} = \left[ \frac{m}{a_j(\text{CL}_s)} \right]_{1 \leq j \leq N}$$

The total variance matrix $T_V$ associated with $V$ is [1] [12]:

$$T_V = \frac{1}{M} \sum_{i=1}^{M} \left( R^j_i - V_{(m)} \right) \left( R^j_i - V_{(m)} \right)^T$$

The intra-class matrix $W_V$ associated with $V$ is [1] [12]:

$$W_V = \frac{1}{M} \sum_{j=1}^{C} \sum_{R^j_i \in \text{CL}_s} \left( R^j_i - V_{(m,\text{CL}_s)} \right) \left( R^j_i - V_{(m,\text{CL}_s)} \right)^T$$

The inter-class matrix associated with $V$ is $B_V = T_V - W_V$.

**B. Formulation of the optimization problem**

We start with $V_{\text{init}} = (a_1, a_2, \ldots, a_q, \ldots a_k)$, and with $M$ realizations $R^j_i$ of this vector. Each $R^j_i$ associated with $I_s$, belongs to $CL_s$, $s = 1, \ldots, C$. The $CL_s$ classes are:

$$\left( R^j_i \right)_{1 \leq h - 1 \leq \frac{M}{b_h}} \in \text{CL}_s$$

where $b_0 = 0, b_C = M$

$b_1, b_2, \ldots, b_{C-1}$: integers delimiting the $C$ classes.

The objective is to extract, among the $N$ parameters $(a_j)_{1 \leq j \leq N}$ of $V_{\text{init}}$, the $q$ (where $q < N$) most pertinent parameters in the sense of the classification performances. The $q$ parameters retained constitute $V_{\text{opt}}$.

**V. THE HOTELLING CRITERION**

HOTELLING criterion was proposed by HOTELLING in 1951 [13][7] [12]. This criterion estimates the discriminate power of the set of parameters based on a measurement of the classes separability and compacity:

$$J_{\text{Hot}}(q, V_k) = \text{trace}(B_{V_k} W_{V_k}^{-1})$$

with $V_k$ of $q$ dimensions extracted from $V_{\text{init}}$. $J_{\text{Hot}}$ is to be maximized [12], it was used in [6].

**VI. THE PROPOSED CRITERION**

We have improved the HOTELLING criterion by introducing the correlation as an additional factor for selecting the parameters. We have thus proposed the following criterion $J$:

$$J(q, V_k) = \frac{\text{trace}(B_{V_k} W_{V_k}^{-1})}{\sum_{r(a_j, a_l) \in E^2} \frac{r(a_j, a_l)}{V_k}}$$

$J$ is to be maximized.

**VII. GENETIC ALGORITHM STRATEGY FOR THE ATTRIBUTE VECTOR OPTIMIZATION CASE**

**A. The proposed coding**

$V_k$ may have one to $N$ components, then for each $V_k$ is associated a chromosome $\text{chr}_k$ of $N$ binary genes $g_{kj}$:

$$\text{chr}_k=(g_{kj})_{1 \leq j \leq N}$$

where

$$g_{kj}=\begin{cases} 1 & \text{if } a_j \in E_{V_k} \\ 0 & \text{otherwise} \end{cases}$$

$V_k$ may not be a possible solution unless if:

$$\text{Dim}(V_k) = \sum_{j=1}^{N} g_{kj} = q$$

**B. The proposed fitness functions**

We define the following fitness functions corresponding to two criteria $J_{\text{Hot}}$ and $J$:

$$F_{\text{Hot}}(\text{chr}_k) = \frac{1}{\sum_{r(a_j, a_l) \in E^2} \frac{r(a_j, a_l)}{V_k}}$$

$$F(\text{chr}_k) = \frac{1}{\sum_{r(a_j, a_l) \in E^2} \frac{r(a_j, a_l)}{V_k}}$$

$V_k$ is the optimal solution according to the $J$ (respectively $J_{\text{Hot}}$) criterion if $F(\text{chr}_k)$ (respectively $F_{\text{Hot}}(\text{chr}_k)$) is minimal.

**VIII. EXPERIMENTAL RESULTS AND EVALUATIONS**

For both experiments, the value of $q$ retained is 2 and $V_{\text{init}}$ is:

$$V_{\text{init}}=\{\text{Hom, Ent, Uni, Dir, Cont, SRE, LRE, GLN, RLN, RP}\}$$

**A. First experiment**

Each class contains 40 texture images.

![Fig. 2. Three test images representing the three classes of textures.](image)

![Fig. 3. Fitness evolution for the $J_{\text{Hot}}$ criterion.](image)
Fig. 4. Fitness evolution for the $J$ criterion

Fig. 5. Representation of the observations in $(Uni\ Cont)$.

Fig. 6. Representation of the observations in $(Uni\ SRE)$.

Fig. 7. Classes obtained in the $(Uni\ Cont)$ space.

Fig. 8. Classes obtained in the $(Uni\ SRE)$ space.

Fig. 9. Three test images representing the three classes of textures.

Fig. 10. Fitness evolution for the $J_{Hot}$ criterion.

Fig. 11. Fitness evolution for the $J$ criterion.

Tab. 1. Parameters space and correlation coefficient.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>$J_{Hot}$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{opt}$</td>
<td>$(Uni\ Cont)$</td>
<td>$(Uni\ SRE)$</td>
</tr>
<tr>
<td>$r$</td>
<td>0.924</td>
<td>0.332</td>
</tr>
</tbody>
</table>

Tab. 2. Error rate of classification in each space.

The discrimination of the 3 classes for $J$ is realized without information redundancy.

B. Second experiment

Each class contains 40 texture images.

$I_i\ 1\leq i\leq 40, i\in CL_1\ \ \ \ \ 41\leq i\leq 80, i\in CL_2\ \ \ \ \ 81\leq i\leq 120, i\in CL_3$

Fig. 11. Fitness evolution for the $J$ criterion.

Criterion   | $J_{Hot}$ | $J$ |
------------|-----------|-----|
$V_{opt}$   | $(Uni\ GLN)$ | $(Uni\ Cont)$ |
$r$         | 0.894     | 0.757 |
With respect to the separability, $J$ and $J_{Hot}$ behave the same way. With respect to the compacity $J$ behaves in a better way than $J_{Hot}$. $J$ has obtained a space for which the parameters are less correlated than those obtained by $J_{Hot}$. 

IX. CONCLUSION

We have presented a new selection criterion for the parameters inspired from the HOTELLING criterion and the correlation approach. We have used the genetic technique for optimizing the two criteria. We have performed two experiments on some texture images. The experimental results confirm the good performance of our approach with respect to the HOTELLING criterion.

REFERENCES