Performance Analysis of OFDM and CDMA Using Phase Noise and Frequency Error

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II. THEORETICAL BACKGROUND

Abstract - Two multiple access schemes used for high bit rate mobile wireless communication systems are compared. This is achieved by modelling OFDM and CDMA downlink waveform signals in order to allow phase noise and frequency error performance analysis. Computation results show that, they seem to suffer similarly from frequency offset errors however differs slightly as far as phase noise is concerned.

Keywords: CDMA, OFDM, phase noise, frequency offset, BER.

I. INTRODUCTION

In mobile wireless communications, the information signals are subjected to distortions caused by reflections and diffractions generated by the signals interacting with obstacles and terrain conditions. Because of the mobility of the receiver, the downlink channel (i.e. from network to users), is time varying [1]. This mobility is very low or even quasi absent in the case of wireless LAN (HiperlanII and IEEE 802.11a) and on the contrary very high for GSM and UMTS.

In the case of low mobility, multicarrier modulation schemes such as OFDM (orthogonal frequency division multiplex) may be suitable. On the other hand, for high mobility, CDMA (coded division multiple access) type systems are found to behave better.

It can be demonstrates that CDMA systems provide immunity to frequency selective fading and interferers by virtue of correlation property of the codes. On the other hand, OFDM is a promising technique for achieving high data rate and combating multipath fading in wireless communications. By transmitting several symbols in parallel, the symbol duration is increased proportionately, which reduces the effects of ISI caused by the dispersive Rayleigh-fading environment.

The comparison between these two approaches can be pushed further in order to adopt a system for given services constrain. This text aims at looking at the robustness as function of frequency error and phase noise usually experienced in actual systems. These parameters are to constrain future wireless communication systems. Behaviour of these access techniques waveforms in multipath propagation channel will guide future systems design.

Where N symbols are being transmitted during a time interval (symbol duration) $T_s$. A sampling time interval $T_e$ can be deduced as: $T_e = \frac{T_s}{N}$.

The expansion to several users may then be easily achieved by a matrix transformation of the vector $X(nT_s)$ to a vector $Y(nT_s)$ by means of a matrix $Z(nT_s)$. The latter consists of expansion sequences used for the symbols. The $k^{th}$ column of this matrix corresponds to the sequence associated with the symbol $x_k(nT_s)$.

\[ Y(nT_s) = Z(nT_s) X(nT_s) \] (2)

In this manner, the array approach may be used to study most of multiple access techniques. For instance:

- For TDMA the matrix $Z$ corresponds to identity, $Z(nT_s) = I$.
- For OFDM it corresponds to $Z(nT_s) = F$. Where $F$ denotes a Fourier Matrix with $F(n,m) = \exp(j2\pi nm/N)$.
- For DS-CDMA, the matrix $Z$ equals $D(nT_s)H$. Where $D(nT_s)$ represents a diagonal matrix whose elements belong to a maximal length pseudo random binary sequence. And $H$ is the Hadamard Matrix.
- For MC-CDMA $Z(nT_s) = FD(nT_s)H$.
- For OVSF (orthogonal variable spreading factor) used in UMTS the matrix $Z(nT_s) = D(nT_s)H_{ovsf}$. Where $H_{ovsf}$ is an array formed by columns of Hadamard sequences length less than $N$ and filled up with zeros.

By considering the radio channel impulse response as time invariant during the symbol duration $T_s$ and its interpolation at $T_e$, the response can be written as:

\[ C(nT_s) = \sum_{k=0}^{N-1} a_k(nT_s) \delta(nT_s - t_k) \] (3)

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With \( \tau_k = kT_s \).

Note that the insertion of guard intervals or spacing at transmission and their suppression at the reception make the influence of the propagation channel reduced to the vector \( X(nT_s) \) only. The channel may then be represented by:

\[
C(nT_s) = \sum_{k=0}^{L} \alpha_k(nT_s)J^k
\]

(4)

In this expression, the term \( J \) is a matrix, which has dimensions \( (N \times N) \) and defined in the following manner:

\[
J = \begin{pmatrix}
0 & 1 \\
I_{(N-\lambda N-\bar{N})} & 0
\end{pmatrix}
\]

(5)

Finally the resulting received signal can be expressed as:

\[
R(nT_s) = C(nT_s)ZX(nT_s) + B(nT_s)
\]

(6)

In this equation, \( B(nT_s) \) represents a complex vector of white noise independent samples. Each complex sample has a variance of \( \sigma^2 = N_0/T_s \) and \( N_0 \) is the noise spectral power density. For convenience, the time interval \( \{nT_s, (n+1)T_s\} \) is omitted from now on. The resulting equation will then be:

\[
R = CZX + B
\]

(7)

Because of the guard interval, all the useful information for estimating \( X \) is contained in the vector \( R \) that is therefore exhaustive estimate \( \hat{X} \) of the transmitted signal.

### III. RECEIVER MODELS

An optimal receiver according to least square measure (MMSE) is able to identify a matrix \( W \) of despreading sequences, applied to vector \( R \), yielding an error between \( X \) and \( \hat{X} \) such as:

\[
W/minE[|W^HR - X|^2]
\]

(8)

By assuming that all data symbols are independent and white having a unity signal power, otherwise:

\[
E[XX^H] = I
\]

(9)

The solution of Eq. (8) may then be written:

\[
W_{\text{MMSE}} = (CZZ^H(C^H\sigma^2 I)^{-1}) CZ
\]

(10)

It is possible to change the hypothesis of putting all data symbols power identical and equal to unity. The identity matrix of Eq. (9) may be then replaced by a diagonal array whose elements are the average signal power of each symbol. In some references such as in [2] the matrix \( W \) is split into a product of equalisation matrix \( G \) and a despreading matrix \( Z \). This gives:

\[
W = GHZ
\]

(11)

With

\[
G = (CZ^H(C^H\sigma^2 I)^{-1})
\]

(12)

In the case of OFDM [3], the receiver usually is FFT based followed by a division of each output by Fourier transform of the channel impulse response at the appropriate frequency. The model of this kind of receivers can be given in a matrix form by:

\[
W_{\text{OFDM}} = C^HZ
\]

(13)

This can be easily obtained by putting \( \sigma^2 = 0 \) in Eq. (10). This process corresponds to an ORC receiver (Orthogonal Restoring Combing) [2] and [4].

In the case of CDMA waveform schemes, the receiver is generally RAKE or MRC (maximum ratio combing) type, which results in:

\[
W_{\text{RAKE}} = CZ
\]

(14)

### VI. SENSITIVITY TO FREQUENCY ERROR

When there is relative motion between the transmitter and receiver, a Doppler shift of the RF carrier results and introduces a frequency error. Also, there can be a residual frequency error caused by frequency instabilities in the oscillators at the transmitter and receiver. The frequency difference or error between the transmitter and receiver carrier frequency synthesers is a serious problem encountered in radio communication systems. It is therefore legitimate to test the robustness of a given modulation scheme when this frequency offset occurs resulting in a lack of synchronisation. In order to check this parameter in the following DS-CDMA and OFDM waveforms are compared as far as problem is concerned [5, 10]. The frequency error can be expressed by first introducing a diagonal matrix defined as:

\[
\Phi_{nm} = e^{j2\pi nT_s \Delta f \pi N}
\]

(15)

Where the term \( \Delta f \) denotes the frequency offset or error. The received signal is then:

\[
R = \Phi(CZX + B)
\]

(16)

By letting the channel be an AWGN, the OFDM receiver and the RAKE type receiver are similar and waveform performances can be compared.

The estimate of the transmitted signal can be written as:

\[
\hat{X} = Z^H\Phi(ZX + B)
\]

(17)

It can be decomposed as:

\[
\hat{X} = \text{diag}(Z^H\Phi Z)X
\]
+ [Z^H \Phi Z - \text{diag}(Z^H \Phi Z)]X + Z^H \Phi B \quad (18)

The first term of this expression represents the useful component, the second one deals with the inter carrier interference; the last part is the additive Gaussian noise. Let

\[ M = \text{diag}(Z^H \Phi Z) \quad (19) \]

Eq. (18) can therefore become:

\[ \hat{X} = MX + [Z^H \Phi Z - M]X + XZ^H \Phi B \quad (20) \]

By introducing the symbol energy \( E_s \) and assuming white independent symbols of equal energy, Eq. (20) leads to expression (21).

\[ E[XX^H] = \frac{E_s}{T_s} I_d \quad (21) \]

If the useful signal power \( P_u \) is introduced then:

\[ P_u = \frac{1}{N} E[MX] \quad (22) \]

This can also be written as:

\[ P_u = \frac{1}{N} E[\text{trace}(MXX^H M^H)] \quad (23) \]

The inter channel interference (ICI) signal power is given by:

\[ P_{ici} = \frac{1}{N} E[|Z^H \Phi Z - M|X(nT_s)|^2] \quad (24) \]

In order to evaluate this power; note that the \( Z^H \Phi Z \) is a unity matrix, then:

\[ E|X|^2 = E[Z^H \Phi ZX|^2] = N \frac{E_s}{T_s} \quad (25) \]

The orthogonality between the vectors \( XM \) and \( [Z^H \Phi Z - M]X \) leads to:

\[ P_{ici} = \frac{E_s}{T_s} - P_u \quad (26) \]

The Gaussian noise power is:

\[ P_n = \frac{1}{N} E|Z^H \Phi B|^2 = \frac{N_0}{T_0} \quad (27) \]

In addition, because of the orthogonality between \( Z \) and \( \Phi \), it is possible to evaluate the signal to noise ratio as:

\[ \frac{S}{N} = \frac{\sum_{i=0}^{N-1} \sum_{n=0}^{N-1} |Z_{ni}|^2 e^{j\varphi_n}}{N - \sum_{i=0}^{N-1} \sum_{n=0}^{N-1} |Z_{ni}|^2 e^{j\varphi_n} + \frac{N N_0}{E_s}} \quad (28) \]

with \( \varphi_n = 2\pi n f T_s / N \).

In the case of OFDM and DS-CDMA:

\[ Z_0 f_T N S = \frac{1}{\sqrt{N}} \quad (29) \]

The expression (28) can be further developed and leads to:

\[ \frac{S}{N} = \sin^2 (\pi \Delta T_s ) \quad (30) \]

\[ N^2 (1 + \frac{N_0}{E_s} ) \sin^2 (\pi \Delta T_s / N) - \sin^2 (\pi \Delta T_s ) \]

Eq. (30) gives the degradation of the S/N ratio versus frequency error. Figure 1 illustrates this case for \( N=64 \) and \( E_s/N_0 = 10 \text{dB} \).

![Fig. 1: Signal to noise ratio versus frequency offset](image)

V. SENSITIVITY TO PHASE NOISE

The effects of noisy phase references on the performance of the demodulator are developed in this section. In order to study the phase noise effect, the model of Wiener-Levy is used [11] with zero mean and a variance of \( 2\pi \beta \), where \( \beta \) is the 3dB bandwidth. In a similar manner as in the case
of frequency error, the phase noise is taken into account by considering a diagonal matrix as follows:

\[ \Phi_{nn} = e^{J \theta_n} \]  

The terms \( \theta_n \) depict Gaussian and independent random variables of variance \( \sigma^2 = 2\pi \beta \). For convenience an AWGN channel is also considered in presence of phase noise. The estimated signal can be written as:

\[ \hat{X} = Z^H \Phi (ZX + B) \]  

Further mathematical manipulation gives:

\[ P_{ep} = \frac{E_s}{N T_s} \frac{\sigma_\theta^2}{6} (2N^2 + 3N - 5) \]  

The power associated to ICI is:

\[ P_{ic} = \frac{E_s}{N T_s} E [ \text{trace} \left( Z^H \psi Z - \Gamma \right) \left( Z^H \psi Z - \Gamma \right)^H ] \]

The expression can be further developed and leads to:

\[ P_{ep} = \frac{E_s}{N T_s} \left( \sum_{i=0}^{N-1} \sum_{n=0}^{N-1} Z^*_{mn} Z_{mn} \phi_n \right)^2 \]

Also

\[ P_{ic} = \frac{E_s}{N T_s} \sigma_\theta^2 \frac{N^2 + N - 2}{2} - P_{cpe} \]

The overall signal to noise ratio is then:

\[ S / N = \frac{E_s / T_s}{P_{cpe} + P_{ic} + N_0 / T_s} \]

Or simply:

\[ S / N = \frac{E_s}{N_0} \left( \frac{\sigma_\theta^2 \left( N^2 + N - 2 \right)}{2N} \left( \frac{E_s}{N_0} \right) + 1 \right)^{-1} \]

The expression in Eq. (46) gives the S/N ratio versus phase noise variance. It is possible to estimate by means of \( \text{erfc}(x) \) function, the impact of the degradation on the bit error rate. This is illustrated in figure 2. The behaviour of 4 modulation schemes are compared using \( N=8 \) and \( E_s/N_0=10 \)dB for an AWGN channel, the signal is BPSK modulated without coding.
BERs obtained for DC-CDMA are lower than OFDM in the vicinity of phase noise of $10^{-2}$. It is not that significant but it seems that DC-CDMA is better than OFDM whereas MC-CDMA is comparable as far as phase noise is concerned.

VI. CONCLUSION

This text deals with the analysis of multiple access schemes waveforms robustness in presence of Tx/Rx frequency error and phase noise. The mathematical approach is detailed. OFDM and DS-CDMA are found to respond similarly as far as the frequency offset is concerned. However, the phase noise is found to affect them differently for low phase noise values.

REFERENCES