

# Two or more mirrors for the omnidirectional stereovision?

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## I. ABSTRACT

Catadioptric omnidirectional stereovision system consists of a single camera and multi-mirrors (at least two mirrors). Many configurations have been designed in the past. In all the proposed systems, the centers of mirrors are often vertically aligned or on the same horizontal plane. In this paper, we propose to investigate the vertical and the horizontal shift of the mirrors's center positions. More precisely, we study the accuracy and view field for different configurations. We will also discuss the singularities problem in this kind of stereovision system.

## II. INTRODUCTION

Omnidirectional catadioptric computer vision is achieved by using convex mirrors and a conventional camera, offering large view. The conventional omnidirectional stereovision systems employ either a pair of rotating cameras simultaneously [2] or two omnidirectional catadioptric cameras [11]. The first solution is better for obtaining very good resolution, but it requires rotation of the cameras and this prevents treating scenes with moving objects. The second approach avoids this problem; but, it needs two cameras and two mirrors thus increasing the weight and size of the sensor. It also has all the conventional stereovision disadvantages: synchronization problems between the cameras and their calibration, optical response differences between cameras, and so on. Another way to recover stereovision is to exploit only one camera that observes several mirrors. This makes it possible to design sensors which have many advantages compared to the systems which use several cameras. These advantages are: single calibration, no synchronization problem, similar optical response, large view field, rigid link between mirrors, and finally the cost. Several works have dealt with a single camera and planar mirrors [3][7] [4] [?]. We restrict our overview to the stereo system based on a single camera (single lens) and convex mirrors. [5] was probably the first to use a stereo vision system based on convex mirrors (specular curved surfaces) and a single camera. In his report, Nayar proposed a system based on a single conventional camera (one lens) and two specular spheres "SPHEREO". His paper dealt with calibration by using four LEDs, explained how to recover depth by a classic triangulation, studied the characteristics of its accuracy and

briefly proposed a procedure to solve the matching problem by using epipolar constraints.

In [5], the authors study four stereo systems with a single camera looking mirrors. They discussed the case of all single view point systems (planar, ellipsoidal, hyperboloidal and paraboloidal). Southwell et al. [8], proposed a stereo vision system that used two vertically aligned mirrors with different curvatures ("two-biconvex lobes"). This approach is not suitable for small sensors; so, more recently, [6], proposed a single camera with nine spherical (they are easier to make) mirrors: a principal one with eight around it.

Our paper aims to estimate a quality for a stereovision system using a single camera and multi-mirrors. What happens when we increase the number of mirrors? Is it better to increase the base-line, to increase the relative height of mirrors, or to increase the number of mirrors? In [12], we proposed to study the design of this kind of sensor thanks to the global 3D reconstruction accuracy. In this paper, we propose to use three criteria (accuracy, singularities, and view field) for making comparisons between different significant categories. Each category was studied and a global comparison between best configurations proposed. About current works dedicated to a global analysis of stereovision quality one can see, for example [10] [9].

## III. SINGLE CAMERA AND MULTIPLE MIRRORS SYSTEMS

We choose paraboloidal mirrors for this study; indeed, paraboloidal mirrors are better for a stereo single camera and multi-mirrors, thanks to the orthographic projection and the invariance to the translations of the camera relatively to the positions of the different mirrors (we keep the single view point even if we translate mirrors).

### A. Camera and mirrors model

Let  $P = (X, Y, Z)$  a three dimensional point and  $p = (x, y, z)_i$  be its image on the mirror  $i$ . The projection center of mirror  $i$ , is at  $dX, dY, dZ$  as shown in "Fig. 1".

The general model for the not-centered mirrors, is given by (1):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_i = \frac{h_i}{\sqrt{\mathbf{X}^2 + \mathbf{Y}^2 + \mathbf{Z}^2 + \mathbf{Z}}} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} \quad (1)$$

where  $\mathbf{X} = X - dX, \mathbf{Y} = Y - dY, \mathbf{Z} = Z - dZ$   
 $4h_i$  is the latus rectum.

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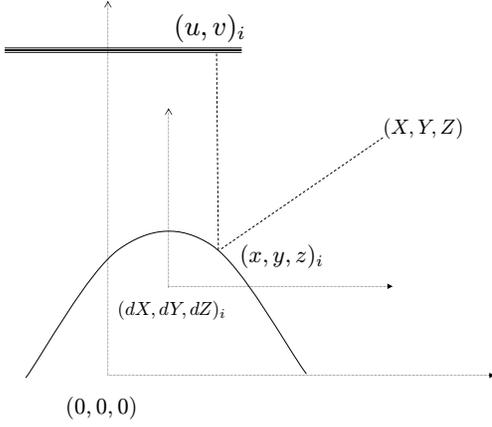


Fig. 1. Not-centered parabolic mirror and orthographic camera.

The orthographic projection of those points is given by (2):

$$\begin{pmatrix} u \\ v \end{pmatrix}_i = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \end{pmatrix} \begin{pmatrix} x + dX \\ y + dY \\ 1 \end{pmatrix}_i \quad (2)$$

$(u, v)$  are the image coordinates of this point.  $(\alpha_u, u_0, \alpha_v, v_0)$  are intrinsic camera parameters.

Now, we consider system with two mirrors "Fig. 2", to study the accuracy and the view field.

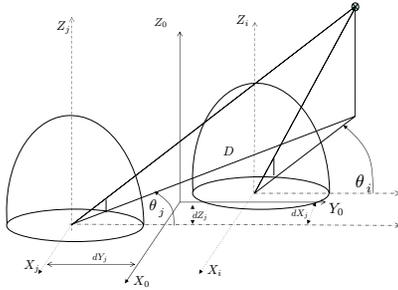


Fig. 2. Stereo system with two mirrors.

Note that, in the previous works, authors study only the special cases where the two mirrors are vertically or horizontally aligned. The equations (1) and (2) are available for this system.  $dX_i$ ,  $dY_i$  and  $dZ_i$  ( $dX_j$ ,  $dY_j$  and  $dZ_j$ ) represent translations between the general frame and mirror's  $i$  frame (respectively  $j$ ).

### B. The depth reconstruction

Depth estimation is equivalent to the 3D reconstruction. In the next development, we consider that the catadioptric image is mapped on an arbitrary cylinder like [1], to facilitate the accuracy analysis, without loss of generality.

1) *Horizontal case:* This configuration means that  $dY_i = 0$ ,  $dZ_i = 0$ ,  $dY_j = 0$ ,  $dZ_j = 0$ , and  $dX_i \neq dX_j \neq 0$ . From the general relation between the angles and the lengths of the triangle, we obtain the adapted equation (3) of the system shown in "Fig. 2":

$$D = dY \frac{\sin(\theta_i)}{\sin(\theta_i - \theta_j)} = dY \frac{\sin(\theta_i)}{\sin(\theta)} \quad (3)$$

where  $\theta = \theta_i - \theta_j$ . In the multi-mirrors case, this equation is available for the whole mirrors combinations, given lots of distances  $D_k$ . Then, by differentiate the previous equation, we can estimate the depth error (4).

$$\delta D = \frac{D \sqrt{D^2 - dY^2 \sin^2(\theta_i)}}{dY \sin(\theta)} \delta \theta \simeq \frac{D^2}{dY \sin(\theta)} \delta \theta, D \gg dY. \quad (4)$$

The equation (4) is interesting thanks to the analytical accuracy study, but the accuracy of the reconstructed 3D points it gives, is bad. Moreover, the given distances have to be translated to unique frame.

As we can see in the "Fig. 3", the accuracy is non isotropic (anisotropic). The accuracy is better when the relative angle is near to  $\frac{\pi}{2}$  or  $\frac{3\pi}{2}$ . The accuracy is bad (error  $\sim \infty$ ) when the relative angle is near to 0 or  $\pi$ .

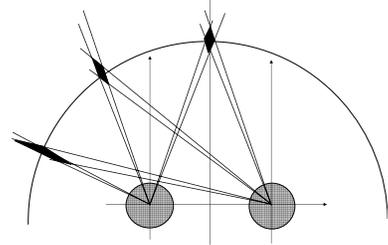


Fig. 3. Evolution of the triangulation error in the case of two horizontal mirrors.

The previous equation (3) presents a singularity when  $\theta_i = \theta_j$ . In this case and if the height of the image point on the cylinder is not equal to zero, we can, as proposed in [?], use the following relation (5):

$$D = dY \frac{v_i}{(v_i - v_j)} = dY \frac{v_i}{v} \quad (5)$$

where  $dY$  is the baseline. The accuracy remains non isotropic without singularities.

2) *Vertical case:* This configuration means that  $dX_i = 0$ ,  $dY_i = 0$ ,  $dX_j = 0$ ,  $dY_j = 0$ , and  $dZ_i \neq dZ_j \neq 0$ . To study the behavior of the accuracy, we use the equation (6) as proposed in [?].

$$D = dZ \frac{v_i}{(v_i - v_j)} = dZ \frac{v_i}{v} \quad (6)$$

where  $dZ$  is the baseline. The vertical configuration presents an isotropic accuracy and no singularities.

3) *Mixed case*: Mixed case means that mirrors are not on the same horizontal plane:  $dX \neq 0$ ,  $dY \neq 0$  and  $dZ \neq 0$ . We propose to mix the horizontal and the vertical shift, to obtain a new sensor with the advantages of these two configurations.

General equation of this system is not very easy to study. So, we propose to use another approach to model this system.

Let  $M$  be the 3D point. Its image coordinates on the mirror  $i$  are  $(x_i, y_i, z_i)$ . Equation of the line (ray) passing through the focal point and from this image point is (7), (8) and (9):

$$x_i = dx_i + \lambda_i(x - dx_i) \quad (7)$$

$$y_i = dy_i + \lambda_i(y - dy_i) \quad (8)$$

$$z_i = dz_i + \lambda_i(z - dz_i) \quad (9)$$

We can write  $\lambda$  from the equation (7):

$$\lambda_i = \frac{x_i - dx_i}{x - dx_i} \quad (10)$$

Then, we obtain the following equations:

$$-Y_i x + X_i y = -Y_i dx_i + X_i dy_i \quad (11)$$

$$-Z_i x + X_i z = -Z_i dx_i + X_i dz_i \quad (12)$$

$$-Z_i y + Y_i z = -Z_i dy_i + Y_i dz_i \quad (13)$$

Each equation represents a plane. This system of equations, represents the 3D line, which contains the 3D point. The other mirrors allow to obtain the similar system of equations, and then the 3D point  $M$ .

### C. View field

The useful field view is the set of all points in space that can be seen by the camera. In the case of one mirror, this is easy to estimate, as it is equal to the global space viewed by the mirror minus the field occluded thanks to the camera. In the case of multiple mirrors, self occlusions between mirrors must be treated; and in these cases it is interesting to study how increasing the number of mirrors remains beneficial for the sensor. In [5], the authors discussed this problem and proposed a formalization of it. However, the resulting equation for multi-mirrors is too complicated. We evaluate them by a ray-lancing based method. So, we can compare the degree of visibility of each 3D point. In the next section, we will compare different categories by using the next criteria: the accuracy, the singularities and the view field.

## IV. COMPARISON OF DIFFERENT CONFIGURATIONS

The aim of this section is to compare some configurations presented in "Fig. 4". These categories are representative of the systems based on a single camera and multi mirrors.

We can classify these configurations into four categories summarized as follows:

- Category 1 (Two): Two vertically aligned mirrors [8] or two horizontally aligned mirrors [5].
- Category 2 (Four): Four identical mirrors.
- Category 3 (OneFour): One principal mirror (in the center) and four identical secondary mirrors.

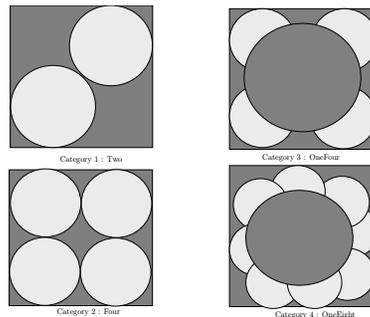


Fig. 4. Top views of the consider configurations of mirrors.

- Category 4 (OneEight): One principal mirror (in the center) and eight identical secondary mirrors (on the periphery) [6].

Two categories (Four and OneFour) are new. We realized this comparison by using the numerical simulation. To do it, we use a cylindrical environment where we put the camera. The size of this environment is characterized by its height, which is equal to 1000mm and by its radius: 1000mm. Intrinsic parameter values of the camera are:  $\alpha_u = 10$ ,  $\alpha_v = 10$ ,  $u_0 = 500$  and  $v_0 = 500$ . The maximum radius of the simulated mirrors (at  $z = 0$ ) is equal to 50mm (latus rectum= 100mm). The details of the size and the positions of the mirrors are shown in the "Fig. 5".

	(h, dX, dY, dZ) (mm)							
Two	(29, -20.5, -20.5, 0)				(29, 20.5, 20.5, dZ)			
Four	(25, 25, 25, 0)		(25, -25, 25, dZ)		(25, -25, -25, 0)		(25, 25, -25, dZ)	
OneFour	(50, 0, 0, 0)		(20, 30, 30, 0)		(20, -30, 30, dZ)		(20, -30, -30, 0)	
OneEight	(40, 0, 0, 0)	(15, 35, 5, 0)	(15, 15, 35, dZ)	(15, 15, 35, 0)	(15, -35, 15, dZ)	(15, -35, 15, 0)	(15, -15, -35, dZ)	(15, 15, -35, 0)

Fig. 5. Sizes and positions of the mirrors.

### A. The accuracy comparison results

We give a global accuracy comparison, shown in "Fig. 6". We introduced a gaussian noise (1 pixel) in the pixel point positions. The figure shows the mean value and the standard deviation of errors for the four categories described in the previous section. The accuracy was computed for all the 3D points belong to the cylindrical environment (radius=1000mm, height between(0mm and 1000mm)). Each category is evaluated with no vertical shift ( $dZ = 0$ ) between the mirror and with a vertical shift between the mirrors ( $dZ = 10mm$ ).

We computed the error (the mean of the accuracy error for the six combinations of mirrors (1-2, 1-3, 1-4, 2-3, 2-4 and 3-4), where (1..4) are the numbers of the mirrors.) accuracy

	h (mm) (radius at z=0)	Depth mean error (mm)		Depth STD error	
		dZ=0mm	dZ=10mm	dZ=0mm	dZ=10mm
Two	29 ; 29	57	54	49	47
Four	25 ; 25 ; 25 ; 25	35	36	30	31
OneFour	50 ; 20 ; 20;20;20	53	42	41	33
OneEight	40; 15 ..15	50	47	37	36

Fig. 6. Accuracy comparison for all categories.

versus the angular position for the configuration **Four**, with a fixed height equal to one meter, for the environment.

We also computed the error curve versus angular position for a shifted mirror ( $Dz = 10mm$ ). We saw, that for the categories **Two**, **OneFour**, and **OneEight**, when we increase the vertical relative position, we increase the accuracy of the depth estimation, because we increase the disparity. The great values show the critical orientations for the triangulation (near to the singularities). One can improve the accuracy by choosing the good couple of mirrors instead of making a "blind" average.

For the category **Four**, the vertical shift does not change significantly the accuracy. This is an interesting results, because this means that we can obtain the best accuracy with a minimum size of the set of mirrors.

1) *The view field discussion:* In this section we propose to discuss the view field problem. As for the other stereovision system, the problem of self occlusion occurs. This means, that a set of 3D points are not visible by all the mirrors. As we have many mirrors, we introduce the N-Visibility concept, to describe how many mirrors have seen the 3D point. For instance, if the 3D point is visible only by one mirror, we say in this case that we have: 1-Visibility. The visibility of 3D points is given in the simulated cylindrical space with a constant radius (1m). Each point is modeled by its cylindrical coordinates (height (Y axis in mm) and angle (X axis in degrees)). Because of the self occlusion, the points behind one mirror can not be seen by the other mirror. When we introduce a vertical difference of the mirror center positions, we increase the effect of the self occlusion. With the system which uses four mirrors, a big number of the 3D points in the space is visible by four mirrors (N-Visibility=4). This is an important propriety especially useful for example, in the matching process. When we introduce the vertical shift, the size of the space with a N-Visibility (equal to 4 in this case) is reduced. The category **OneFour** has the same characteristics as the **Four**. But, the principal mirror (in the center) introduce a big self-occlusion area and with the vertical shift, this area is bigger. The category **OneEight** has a N-Visibility equal to 9 for a large area of the environment. But, the size of the mirrors is small.

Thanks to its simplicity, we recommend the category **Four**. It gives the best accuracy comparing to the other categories

and it avoids the singularities. The accuracy can be improved by using the best couple of mirrors.

## V. CONCLUSIONS

In this paper, we presented a comparison between four categories of stereovision systems based on a single camera and multiple mirrors. This set of category contains two "old" configurations and two "new" configurations. We make this comparison by using the accuracy and the view field and by changing the vertical relative positions of the mirrors. The advantage of this shift is to increase the vertical disparity (and then the global disparity) and to improve the accuracy of the depth reconstruction. We showed that it is better to use more than two mirrors. We are also, already making a prototype of the category **Four** (a single camera and four mirrors). The main result of this work, is that that category is the best one and does not need a vertical shift (small size of the sensor). This new category does not have a singularity point, which is another great advantage. These preliminary results encourage us to continue our investigations, and our next work will be about the analytic formulation of the accuracy and of the view field.

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