

# Limited Feedback Precoding for Wireless MIMO Broadcast Channels

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**Abstract**—Recently, a number of techniques have been introduced to exploit multiuser diversity of a wireless multiple input multiple output (MIMO) broadcast channel (BC) that consists of a base station (BS) with  $t$  transmit antennas and  $K$  mobile stations (MS) with multiple antennas. However, prior works have ignored the rate overhead associated with feedback of MIMO BC channel state information (CSI), which is roughly  $K$  times larger than single-user MIMO CSI (i.e., it is  $\mathcal{O}(tr)$  where  $r = \sum_{k=1}^K r_k$  and  $r_k$  is the number of antennas at the  $k$ th MS). Considering the amount of feedback signaling, quantization is a necessity for effective feedback transmission as a form of partial CSI. In this paper, we propose the greedy multi-channel selection diversity (greedy MCSD) scheme based on block MMSE QR decomposition with dirty paper coding (block MMSE-DP), where partial CSI is almost sufficient. The sum-rate performance of our novel scheme approaches extremely close to the sum capacity of MIMO BC as the number of users increases, whereas the feedback overhead is reduced by a factor of  $2t^3/L(t^2-t)$ , in which  $L$  is the number of active channel vectors. Simulation results validate the expectation from the analysis.

## I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have been one of key techniques to achieve high rate and high reliability over wireless downlink (broadcast) channels. The investigation of the capacity region has been of concern in a MIMO broadcast channel (BC), where the base station (BS) has multiple transmit antennas and each mobile user has possibly multiple receive antennas [1]. In [2], it was shown that an achievable rate region for the multiple-input single-output (MISO) BC is obtained by applying dirty paper coding (DPC) [3], [4], or known interference cancellation, at the transmitter. Sum-power iterative water-filling (SP-IWF) provides the optimum transmission policies for MIMO BC, whereas reducing the computational complexity and feedback overhead is still an ongoing research area (refer to [5] and references therein). As a low computational complexity approach, the greedy-type user selection [6] and the joint-channel decomposition [7] are utilized instead of the optimal power allocation policy, respectively. For feedback reduction, the random beamforming technique is introduced in [8], where a significant number of users is, however, required to achieve sum capacity compared to the case of deterministic beamforming. As another approach for feedback reduction, the efficient vector quantization [9] is developed to represent the partial channel state information (CSI) [10], while most attention has been focused on single-user MIMO systems. However, prior works on sum-rate near-

optimal transmit schemes do not take into account the feedback overhead, to our best knowledge.

In this paper, we find the cost-effective scheme in terms of complexity and feedback overhead that obtains near the maximum sum-rate of the wireless MIMO BC. As a solution to this problem, we propose the greedy multi-channel selection diversity (greedy MCSD) scheme based on novel block MMSE QR decomposition with DPC (block MMSE-DP). Simulation results indicate that the sum-rate performance of our scheme approaches extremely close to the sum capacity of MIMO BC with a few users (e.g., a gap of 0.4 bps/Hz from SP-IWF), whereas the feedback overhead is significantly reduced.

The remainder of this paper is organized as follows. In Section II, the system model is introduced. In Section III, SP-IWF, greedy-type user selection, and time-division multiple-access (TDMA) are analyzed. Our novel scheme is investigated in Section IV. Section V provides the analysis of sum-rate performance. Numerical results are presented in Section VI and concluding remarks are contained in Section VII.

## II. SYSTEM MODEL

Consider a  $K$  user wireless downlink communications system with multiple transmit antennas at the base station, as shown in Fig. 1, and multiple receive antennas for each user. We assume that the base station has  $t$  transmit antennas, the user  $k$  has  $r_k$  receive antennas, and the number of all receive antennas in the system is  $r = \sum_{k=1}^K r_k$ . Also, we model the channel as a frequency-flat block fading channel. Interference from neighboring cells is modeled as additive Gaussian noise, as we concentrate on the single cell model. The received signal of user  $k$  is expressed as<sup>1</sup>

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k \quad (1)$$

where  $\mathbf{x}$  is the  $t \times 1$  sum transmit signal vector, i.e.,  $\mathbf{x} = \sum_{k=1}^K \mathbf{x}_k$  in which  $\mathbf{x}_k$  is the transmit signal vector of user  $k$ . The total sum transmit power of all users is constrained by  $P$ , i.e.,  $tr(\sum_{k=1}^K \mathbf{\Sigma}_k) \leq P$  where  $\mathbf{\Sigma}_k = E[\mathbf{x}_k \mathbf{x}_k^H]$  is the transmit covariance matrix of user  $k$ . The  $t \times 1$  vector  $\mathbf{z}_k$  represents the random additive noise for user  $k$  where  $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ . The channel  $\mathbf{H}_k$  is a  $r_k \times t$  matrix, whose entries are assumed to be independent and identically distributed (i.i.d.)

<sup>1</sup>The superscripts  $T$  and  $H$  stand for transpose, conjugate transpose, respectively. The cardinality of the set  $S$  is notated as  $|S|$ .

circularly symmetric complex Gaussian random variables with zero-mean and unit variance. Also,  $\mathbf{H}_k$  is independent of  $\mathbf{H}_j$  for all  $j \neq k$ .

In general, it is difficult for the base station to have the perfect knowledge of downlink CSI because the feedback link has delayed lossy feedback characteristics. Hence, the problem at hand is to find the transmit and receive structure that minimizes the feedback rate subject to the performance constraint such that the data throughput is kept as close as possible to the sum capacity.

### III. SP-IWF, MMSE-DP, AND TDMA

For performance comparison with the proposed multiuser MIMO scheme, as explained in the next section, we describe the following approaches: SP-IWF, greedy MMSE QR decomposition with DPC (MMSE-DP), and TDMA. The sum-rate maximization can be solved by using SP-IWF, which achieves the sum capacity of a MIMO BC at the expense of higher complexity. For the comparison purpose, we present the algorithm of greedy MMSE-DP for MIMO, which is the updated version of greedy (ZF) QR decomposition with DPC (ZF-DP) for MISO in [6] so as to apply MMSE filtering and to consider the receivers equipped with multiple receive antennas. However, in this case the receive array gain is not exploited (see Lemma 2). The algorithm for greedy MMSE-DP is outlined as follows.

- 1) BS selects the index pairs of user and its antenna,  $\{k_j, i_j\}_j$ ,  $j \leq \frac{1}{2}t(t+1)$  [11], on the basis of the downlink channel matrices  $\{\mathbf{H}_k\}_{k=1}^K$  where  $\mathbf{H}_k^H = [\mathbf{h}_{k,1}, \dots, \mathbf{h}_{k,M_r}]$ . Before starting the procedure, we take as initial value  $m = 0$ .
- 2) Let  $m = m + 1$ , and if  $m > K$ , stop the procedure. Otherwise, BS calculates the received SNR as function of  $m$  for all  $(k, i)$ , as follows

$$\gamma_{k,i}(m) = \mathbf{h}_{k,i}^H \left( \sum_{j=1}^{m-1} \mathbf{h}_{k_j, i_j} \mathbf{h}_{k_j, i_j}^H + mP^{-1} \mathbf{I} \right)^{-1} \mathbf{h}_{k,i}, \quad (2)$$

determine the index pair of user and its antenna as the  $m$ th selected index pair, as follows

$$(k_m, i_m) = \arg \max_{k \in \{1, \dots, K\}, i \in \{1, \dots, r_k\}} \gamma_{k,i}(m), \quad (3)$$

and calculates the sum-rate achievable with  $\gamma_{k_1, i_1}, \dots, \gamma_{k_m, i_m}$ , as follows

$$R_m = \sum_{j=1}^m \log_2(1 + \gamma_{k_j, i_j}(m)). \quad (4)$$

Repeat the algorithm until  $R_m \leq R_{m-1}$ . Note that the total throughput of this scheme is give by  $R_{\text{MMSE-DP}} = R_{m-1}$ .

In the case of TDMA, the base station transmits to only a single-user at a time by using all transmit antennas, which is a suboptimal solution when the base station has multiple transmit antennas, denoted as TDMA-MIMO, whereas the

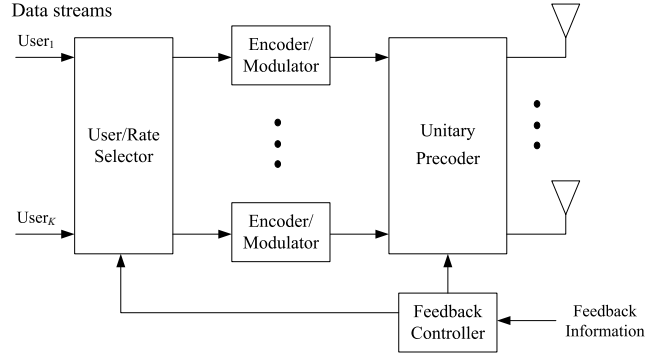


Fig. 1. Schematic of PU<sup>2</sup>RC transmitter

optimality holds if and only if the number of transmit antennas is equal to one. It is shown that the maximum sum-rate of TDMA-MIMO is the largest single-user capacity of the  $K$  users, which is given by

$$C_{\text{TDMA-MIMO}} \triangleq \max_{i=1, \dots, K} C(\mathbf{H}_i, P) \quad (5)$$

where  $C(\mathbf{H}_i, P)$  denotes the single-user capacity of the  $i$ th user subject to power constraint  $P$ .

### IV. GREEDY MCSD WITH BLOCK MMSE-DP

We propose a multiuser MIMO scheme, i.e., greedy MCSD with block MMSE-DP, based on per-user unitary FB/BF and rate control (PU<sup>2</sup>RC) [12]. FB and BF abbreviate feedback and beamforming, respectively. The transmitter structure of the proposed system is shown in Fig. 1. Beamforming is employed at the transmitter using unitary matrix  $\mathbf{W}$  that is a function of unitary matrix  $\mathbf{V}_k$  and diagonal matrix  $\mathbf{D}_k$ , which are obtained by singular value decomposition (SVD) such that  $\mathbf{H}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^H$ . Based on space-time multiple access (STMA), the data stream of each user is allocated to each beam vector of the unitary transform matrix  $\mathbf{W}$ . Also, the transmitter adjusts the data rate per-stream independently. Note that by this structure, PU<sup>2</sup>RC uses spatial multiplexing to transmit simultaneously to multiple users, and multiple streams are transmitted to multiple users. Transmissions are beamformed using a unitary matrix based on SVD of the MIMO channels.

In [12], it was proposed that PU<sup>2</sup>RC performs unitary beamforming with a finite set in a predetermined way, where combining with the user and beam selection leads to additional throughput improvement due to interference reduction between users. The amount of feedback information can be reduced by using a predetermined finite set, and furthermore, applying a codebook design such as Grassmannian line packing to the finite set improves the throughput performance much further [13]. However, performance is severely degraded when there are smaller number of receive than transmit antennas. To mitigate the performance degradation in such cases, in this paper we investigate the enhanced PU<sup>2</sup>RC scheme that is combined with DPC. A detailed discussion of this scheme will be found in the following subsections. In brief, our

proposed scheme uses known interference cancellation and non-predetermined beamforming at the transmitter.

#### A. Block MMSE-DP

As the first stage of block QR decomposition, the channel is rotated using the left unitary matrix obtained by SVD of the each user channel (see Lemma 1), which reduces feedback rate overhead. This is equivalent to the process that sets the receiver spatial filtering as  $\mathbf{W}_{\text{rx}} = \mathbf{U}_k^H$  where  $\mathbf{H}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^H$ . By doing so, as described in [10], the overhead can be reduced by a factor of  $2t^2/(t^2 - t)$ . MIMO channel is hence decomposed into multiple MISO channels  $\mathbf{F}_k$ , which is referred to as the effective channel matrix

$$\mathbf{F}_k = \mathbf{U}_k^H \mathbf{H}_k = \mathbf{D}_k \mathbf{V}_k^H. \quad (6)$$

We also denote each row of  $\mathbf{F}_k$  as the effective channel vector. In the transmitter, controlled beamforming is implemented by applying (ZF) QR decomposition to the combination of the effective channels, i.e.,  $\mathbf{F} = [\mathbf{F}_1^T, \dots, \mathbf{F}_K^T]^T$ , which represents the effective BC. Since  $\mathbf{F}^H = [\mathbf{f}_1, \dots, \mathbf{f}_r]$  is also seen as the combination of the effective channel vectors where  $\mathbf{f}_i \in \mathbb{C}^{t \times 1}$ ,  $\mathbf{F}$  can be treated as the multiple MISO channels, which is well discussed in [14]. As in the algorithm of [6] for MISO, the QR decomposition is obtained using the Gram-Schmidt orthogonalization procedure to the rows of  $\mathbf{F}$ . That is, geometrical projection is performed using SVD decomposition of each  $\mathbf{H}_k$ , and then the finite dimensional subspace is determined by QR process with  $\mathbf{F}$ . Mathematically, QR decomposition of  $\mathbf{F}$  is represented as  $\mathbf{F} = \mathbf{R}_f \mathbf{Q}_f$ , where  $\mathbf{R}_f$  is a  $r \times t$  lower triangular matrix and  $\mathbf{Q}_f$  is a  $t \times t$  matrix with orthonormal rows. The unitary matrix  $\mathbf{Q}_f^H$  is used for transmit beamforming, i.e.,  $\mathbf{W}_{\text{tx}} = \mathbf{Q}_f^H$ , and hence is applied to the transmitted signal

$$\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{z} = \mathbf{R}_f \mathbf{Q}_f \mathbf{W}^H \mathbf{s} + \mathbf{z} = \mathbf{R}_f \mathbf{s} + \mathbf{z} \quad (7)$$

where  $\mathbf{y} = [\mathbf{y}_1^T, \dots, \mathbf{y}_r^T]^T$  and  $\mathbf{z} = [\mathbf{z}_1^T, \dots, \mathbf{z}_r^T]^T$ .

Throughout this paper, we denote the combination of block MMSE QR decomposition with known interference cancellation, or DPC, as block MMSE-DP. The optimality of block QR decomposition and the actual implementation of this technique are treated in the following two subsections, respectively.

#### B. Optimality of Block QR Decomposition

In order to derive the procedure employing known interference cancellation, the congrate interfering channel matrix is defined as

$$\bar{\mathbf{H}}_k = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_{k-1}^T]^T. \quad (8)$$

**Theorem 1:** The objective of the transmit covariance matrix design is to find a covariance matrix set that maximizes the system throughput, subject to the sum power constraint and the unknown-interference free constraint. The transmit covariance matrix satisfying this objective is obtained by QR decomposition of  $\mathbf{F}$ .

*Proof:* The problem may be formulated as finding  $\{\Sigma_i^*\}_{i=1}^K$ , such that

$$\{\Sigma_i^*\}_{i=1}^K = \arg \max_{\Sigma_i \in \mathbb{C}^{t \times r_i}} R_{\text{BC}}(\{\Sigma_i\}_{i=1}^K) \quad (9)$$

subject to the sum power constraint

$$\sum_{i=1}^K \text{tr}(\Sigma_i) \leq P, \quad \Sigma_i \geq 0 \quad (10)$$

and the unknown-interference free constraint

$$\mathbf{H}_i \Sigma_j = \mathbf{F}_i \Sigma_j = \mathbf{0}, \quad j, i = 1, 2, \dots, K, \quad j < i \quad (11)$$

where

$$R_{\text{BC}}(\{\Sigma_i\}_{i=1}^K) = \sum_{i=1}^K \log_2 \frac{|\mathbf{I} + \mathbf{H}_i(\Sigma_1 + \dots + \Sigma_i)\mathbf{H}_i^H|}{|\mathbf{I} + \mathbf{H}_i(\Sigma_1 + \dots + \Sigma_{i-1})\mathbf{H}_i^H|}. \quad (12)$$

The constraint in (11) ensures that no interference to user  $k$  is caused by the users from 1 to  $k-1$ , noting that interference from other users (i.e.  $k = k+1, \dots, K$ ) is subtracted by known interference cancellation. Let  $\bar{\mathbf{V}}_k^0$  consist of the right singular vectors corresponding to the zero singular values of  $\bar{\mathbf{H}}_k$ , which is equivalently obtained by SVD of  $\bar{\mathbf{F}}_k^T = [\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_{k-1}]$  as follows

$$\bar{\mathbf{F}}_k = \begin{bmatrix} \bar{\mathbf{D}}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_k^1 & \bar{\mathbf{V}}_k^0 \end{bmatrix}^H \quad (13)$$

where  $\bar{\mathbf{D}}_k$  is a diagonal matrix and  $\bar{\mathbf{V}}_k^1 = [\mathbf{V}_1^T, \mathbf{V}_2^T, \dots, \mathbf{V}_{k-1}^T]^T$ . Thus, constructing  $\Sigma_k$  with  $\bar{\mathbf{V}}_k^0$  satisfies the constraint in (11), i.e.,  $\Sigma_k = \bar{\mathbf{V}}_k^0 \bar{\Sigma}_k \bar{\mathbf{V}}_k^{0H}$  where  $\bar{\Sigma}_k$  is designed as follows. When the constraint in (11) is satisfied, the system throughput is simply derived as

$$R_{\text{BC}}(\{\Sigma_i\}_{i=1}^K) = \sum_{i=1}^K \log_2 |\mathbf{I} + \mathbf{H}_i \bar{\mathbf{V}}_i^0 \bar{\Sigma}_i \bar{\mathbf{V}}_i^{0H} \mathbf{H}_i^H| \quad (14)$$

where  $R_{\text{BC}}$  is upper bounded by the sum capacity of the MIMO BC.

The optimization of (14) is done by the SVD of  $\mathbf{H}_i \bar{\mathbf{V}}_i^0 = \mathbf{U}_i \mathbf{D}_i \mathbf{V}_i^H$ , and hence

$$\bar{\Sigma}_i^* = \bar{\mathbf{V}}_i^{0H} \mathbf{V}_i (\mathbf{P}_i \mathbf{D}_i) \mathbf{V}_i^H \bar{\mathbf{V}}_i^0. \quad (15)$$

where the diagonal matrix  $\mathbf{P}_i$  is designed to satisfy the sum power constraint in (10). For example, optimal  $\{\mathbf{P}_i\}_i$  can be found by performing SP-IWF with total multiple MISO channels  $\mathbf{F}$ . Moreover, the procedure described above, i.e., (13) – (15), is mathematically equivalent to (ZF) QR decomposition of  $\mathbf{F}$ , i.e., equivalent to block QR decomposition of  $\{\mathbf{H}_k\}_k$ . When MMSE filtering is considered, MMSE QR decomposition [15] also can be used instead.  $\square$

#### C. Greedy Multi-Channel Selection Diversity

Multiuser diversity is the promising solution to achieve the sum capacity of the multiuser channel. In the proposed scheme, we select the strong effective channel vectors among available multi-user effective channel vectors. Greedy MCSD is processed through the greedy-type ordering and selection of the channel vectors for active users. When channel vectors are selected and ordered, diversity gain is achieved with the increase of the number of users and antennas therein. The similar approach for MISO case was greedy ZF-DP as described

in [6]. In our proposed method, we present two key functions. Firstly, the effective channel vectors  $\{\mathbf{f}_i\}_i$  are exploited instead of the channel vectors  $\{\mathbf{h}_i\}_i$ , since we consider multi-channel section diversity but not just multiuser selection. Secondly, to further reduce the feedback overhead, a part of the channel selection process is performed at the MS side as well (see Theorem 3).

Let  $A \subset \{1, \dots, r\}$ ,  $|A| \leq \frac{1}{2}t(t+1)$  be a subset of the effective channel vector indices that the base station selects for transmission using MCSD, and  $\mathbf{F}(A)^H = [\mathbf{f}_1^A, \dots, \mathbf{f}_{|A|}^A]^T$  be the corresponding submatrix of  $\mathbf{F}$ . The  $t \times |S|$  unitary beamforming matrix  $\mathbf{W}^H(A)$  is now obtained by MMSE QR decomposition of the submatrix  $\mathbf{F}(A)$ . The maximum sum-rate of this system is then given by

$$R_{\text{MCSD}} = \max_A f_a(A) \leq R_{\text{BC}}(\{\Sigma_i^*\}_{i=1}^K). \quad (16)$$

In (16), the cost function  $f_a(A)$  is defined as

$$\begin{aligned} f_a(A) &= \log_2 |\Phi_a| + \\ &\log_2 \left| \mathbf{I} + \frac{P}{|A|} \Phi_a^{-1/2} \mathbf{f}_k^A \mathbf{f}_k^{AH} \Phi_a^{-1/2} \right| \\ &= \log_2 \left| \mathbf{I} + \frac{P}{|A|} \sum_{i \in A} \mathbf{f}_i^A \mathbf{f}_i^{AH} \right| \end{aligned} \quad (17)$$

where  $\Phi_a = \log_2 \left| \mathbf{I} + \frac{P}{|A|} \sum_{i \in A, i \neq k} \mathbf{f}_i^A \mathbf{f}_i^{AH} \right|$ . The equality in (16) holds if and only if  $|A|$ , ordering, and power allocation of the selected channels are optimized. Moreover, we also investigate the approach that MS can select and feeds back  $L$  active channel vectors corresponding to the  $L$  largest eigenmodes out of  $\min(t, r_k)$  effective channel vectors, resulting that the feedback amount is further reduced by a factor of  $\min(t, r_k)/L$ .

## V. PERFORMANCE ANALYSIS

In this section, the performance analysis is presented. Throughout this paper, the entries of  $\mathbf{H}_k$  are assumed to be i.i.d. zero-mean complex-Gaussian random variables.

**Lemma 1:** Assume that CSI of all user  $j$  ( $\neq k$ ), i.e.,  $\{\mathbf{H}_j\}_{j \neq k}$ , is not known to user  $k$ . That is, CSI of all user  $j$  is not delivered to user  $k$  from the transmitter as well as not exchanged between users. In this case, each user  $k$  can estimate the achievable throughput by using  $\mathbf{F}_k = \mathbf{U}_k^H \mathbf{H}_k$ , assuming that interference from all user  $j$  is averaged out.

*Proof:* The proof is given in [16]. ■

**Lemma 2:** We compare the performance of block MMSE-DP with (non-block) MMSE-DP in terms of the achievable throughput. In block MMSE-DP, receive beamforming is performed by the left unitary matrix of the corresponding channel, whereas MMSE-DP is known to be optimal for non-cooperative reception across receive antennas. For average throughput, block MMSE-DP outperforms MMSE-DP in a MIMO BC.

*Proof:* The proof is given in [16]. ■

**Theorem 2:** In MIMO BC, the system with beamforming at each receiver using the left singular matrix offers the average

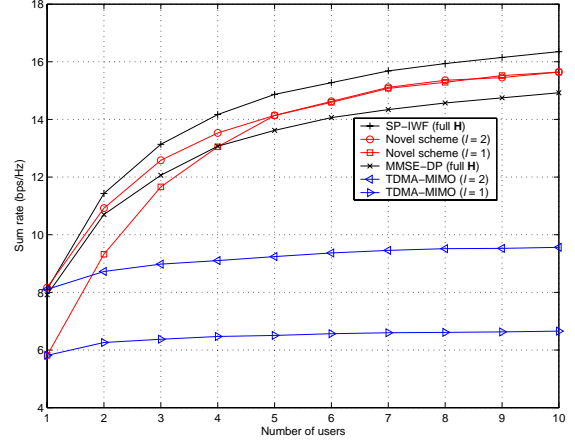


Fig. 2. Ergodic sum-rate comparison when  $t = 4$  and all  $r_k = 2$

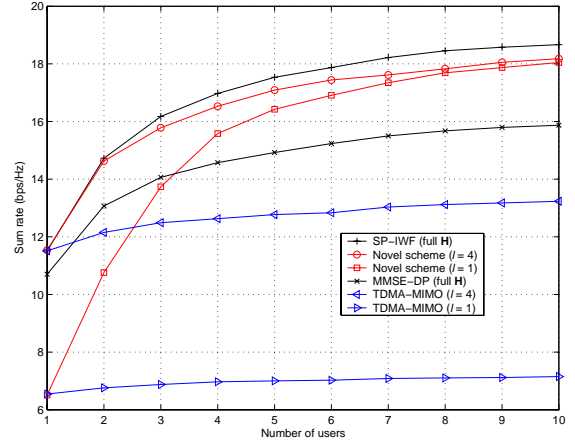


Fig. 3. Ergodic sum-rate comparison when  $t = 4$  and all  $r_k = 4$

throughput that is no worse than using any fixed unitary matrix beam at all receivers.

*Proof:* The proof is given in [16]. ■

**Theorem 3:** In the proposed MCSD scheme, we select the strong effective (sub) channels, of which the number is larger or equal to one. The sum-rate achievable with the selected channels is almost equal to the sum capacity of total multiple MISO channels  $\{\mathbf{F}_k\}_k$  in (6).

*Proof:* The proof is given in [16]. ■

## VI. NUMERICAL RESULTS

In this section, numerical results are presented. In Figs. 2 and 3, we compare the ergodic sum-rate performance of different MIMO downlink strategies. The SNR is assumed to be 10 dB. Given the number of users, TDMA-MIMO achieves the maximum sum-rate corresponding to the largest single-user capacity, which shows relatively a small gain in proportion to the number of users. When the number of the active channel vectors is equal to the number of the effective channel vectors and one user is assumed, the performance of the proposed novel scheme is the same as that of TDMA-MIMO since in

both cases receivers feed back the effective channel matrix  $\mathbf{F}_k = \mathbf{D}_k \mathbf{V}_k^H$ , instead of the full channel matrix  $\mathbf{H}_k$ . In both figures, the performance of the novel scheme with full effective channel vectors (i.e.  $L = 2, 4$ , respectively) is extremely close to the sum capacity driven by SP-IWF, while the performance with the active (partial effective) channel vector (i.e.  $L = 1$ ) reaches the sum-rate with full vectors when the number of users is equal to 5 and 10, respectively. Both figures show sum-rate improvement of 2 bps/Hz over MMSE-DP scheme with *full* channel feedback and the gap of 0.4 bps/Hz from SP-IWF.

Furthermore, both figures illustrate the behaviors of sum-rate corresponding to different feedback overheads. In Fig. 2, each user has two eigenmodes, i.e., two effective channel vectors, available since four transmit and two receive antennas are assumed. The sum-rate of the novel scheme with feedback of one active channel vector (one eigenvector multiplied by the corresponding eigenvalue that is the largest one) gets tightly close to the performance having feedback of two active channel vectors when the number of users is five. Contrastingly, TDMA-MIMO with one vector never becomes close to TDMA-MIMO with two vector. Four transmit and four receive antennas are considered in Fig. 3, where two feedback signaling (i.e. one, four active channel vectors) are examined for the novel and TDMA-MIMO schemes. Both figures show that the novel scheme with reduced feedback, i.e., with the fewer active channel vectors, achieves slightly lower rate performance with small number of users compared to the scheme with full effective channel vector, and however, the performance approaches extremely close to the upper bound as the number of users increases. Note that when the feedback bandwidth is fixed, the reduction of active channel vectors may improve the quality of feedback signaling, of which the level can be represented based on the Shannon distortion-rate function  $D_k(R) \propto 2^{-2\alpha_k R}$  where  $R$  is the given feedback rate and  $\alpha_k = L / \min(t, r_k)$  [9]. Therefore, in the proposed scheme feedback of active channel vectors is shown to have the equivalent sum-rate performance with feedback of full effective channel vectors, resulting in the outstanding feedback robustness. That is, the feedback signaling per user can be significantly reduced with the increase of the number of users.

## VII. CONCLUSION

In this paper, we have proposed a multiuser MIMO transmission scheme that is efficient in terms of computational complexity and feedback overhead while obtaining near the maximum sum-rate of BC. Our novel scheme has employed *block* MMSE-DP at the transmitter, which reduces the computational complexity of designing transmit covariance matrices. Using MCS in combination with block MMSE-DP, the proposed scheme with partial CSI at the transmitter has still achieved the near-optimal sum capacity, which was not observed in TDMA-MIMO. Numerical results have shown that the gain of sum-rate is 2 bps/Hz over the conventional MMSE-DP scheme with *full* channel feedback and the gap from SP-IWF is negligibly small (i.e. 0.4 bps/Hz).

## ACKNOWLEDGMENT

The authors would like to thank Seung-Yong Park (Samsung), Chang Soon Park, Marios Kountouris and Antonio Forenza for their valuable technical comments and discussions.

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