An Interpretation of Channel Overloading and Construction of Some Novel Schemes

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Abstract—Channel overloading, which refers to accommodating a larger number of users on a channel than can be achieved with orthogonal signaling, is currently a very hot topic for future wireless systems. A number of approaches have been described in the literature, and the most efficient of them use several sets of orthogonal signal waveforms. Multiuser interference, which is inherent to channel overloading, is handled using maximum-likelihood detection or iterative interference cancellation. In this paper, channel overloading is interpreted as multilevel coding of an expanded signal constellation. This interpretation gives further insight into the performance and limitations of existing channel overloading techniques and allows us to construct some novel schemes.

Index Terms—Channel overloading, combined TDMA/OCDMA, multilevel coding, multiple access, set partitioning.

I. INTRODUCTION

Efficient use of the available radio spectrum is a major requirement for future wireless systems. In a cellular system based on code-division multiple access (CDMA), the number of sequences assigned to users in a cell is typically less than the spreading factor (spreading sequence length), and cells are underloaded. To make a better use of the radio spectrum, it is of considerable interest to assign more sequences than the spreading factor, i.e., to overload the channel. This kind of channel overloading has been actually provisioned in third-generation (3G) wireless standards [1]. In a series of recent papers, one of the present authors and his co-authors analyzed the general problem of channel overloading and proposed to use two sets of orthogonal signal waveforms (see, e.g. [2] and [3]). Maximum likelihood (ML) detection, which has a computational complexity that increases exponentially with the number of users, is intractable for these schemes. For this reason, the authors used a two-stage iterative interference cancellation instead.

An ad-hoc channel overloading technique was described by Ross and Taylor in [4]. This technique, which can be interpreted as a special case of channel overloading using multiple sets of orthogonal signal waveforms, makes it possible to increase the number of users on a channel by approximately 33% at best. A low-order polynomial complexity (in $O(K^{2.5})$) optimal detector has been shown to exist for such a construction in [5]. The channel overloading concept described in [2], [3], and other papers of the same authors generally do not have the same limitation in the number of users, but performance degradation becomes significant when the number of users is increased.

In this paper, we give a new interpretation to channel overloading. Various channel overloading techniques are interpreted as a multilevel coding of an expanded signal constellation. Not only this interpretation gives further insight into the performance limitations of existing channel overloading schemes, but it also allows constructing several new schemes, following the rate-splitting multiple access technique presented in [6]. For simplicity, we will limit our study to real signals using binary phase-shift keying (BPSK) modulation. Throughout the paper, we assume perfect timing synchronization and power control and consider an additive white Gaussian noise (AWGN) channel.

The paper is organized as follows: First, in Section II, we describe the concepts proposed in [2]–[4] and give an interpretation of these concepts in terms of multilevel coding of an expanded signal constellation. Section III describes some novel schemes with higher capacity and performance compared to existing schemes. Finally, we give a summary of our results and present our conclusions in Section IV.

II. AN INTERPRETATION OF CHANNEL OVERLOADING

A. The Principle of Channel Overloading

In multiuser communications, channel overloading refers to accommodating a larger number of users than can be achieved with orthogonal signaling. A simple way to describe it is as follows: Suppose that all users need the same data rate which requires a bandwidth of $W$ Hz in the single-user case. Channel overloading consists of accommodating more than $N$ users on a multiple access channel which has a bandwidth of $N W$ Hz.

In the sequel, $K$ denotes the number of users. Hence, the case of channel overloading corresponds to $K \geq N$, that is $K = N + M$, the overload factor (OF) being given by $OF = M / N$. Furthermore,

- $a = (a_1, a_2, ..., a_K)^T$ is the set of BPSK symbols associated to the $K$ users,
- $s = (s_1, s_2, ..., s_K)^T$ is the transmitted signal block.

The minimum Euclidean distance $d_{min}$ is defined as:

$$d_{min} = \min_{a^*} d(s(a^*), s(a))$$  \hspace{1cm} (1)
It represents the minimum Euclidean distance between the transmitted sequences \( s^0 \) and \( s^{0,0} \) corresponding to two different sets of symbols \( a^0 \) and \( a^{0,0} \) associated to the \( K \) users.

Also, the asymptotic gain of a scheme refers to the savings attainable in the transmitted energy per information bit to the noise spectral density ratio \((E_b/N_0)\) required to achieve a given bit error probability when this scheme is used in comparison with the BPSK performance.

**B. Channel Overloading Using Two Sets of Orthogonal Signal Waveforms**

In [2], [3] and subsequent papers on the subject by the same authors, a general channel overloading concept was described which uses two sets of orthogonal signal waveforms to accommodate \( K = N + M \) users \((OF = M/N)\). The waveforms of the first signal set in this scheme are assigned to the first \( N \) users, and \( M \) waveforms from the second set are assigned to the next \( M \) users.

For instance, using TDMA (time-division multiple access) and OCDMA (orthogonal CDMA) for the first set and the second set of users respectively, the transmitted signal block can be written as follows:

\[
s_k = a_k + \frac{1}{\sqrt{N}} \sum_{m=1}^{N} a_{N+k,m} w_m \quad k = 1, 2, \ldots, N \tag{2}
\]

where \( w_m \) designates the \( k \)th chip of the Walsh-Hadamard sequence assigned to the \( m \)th OCDMA user \((w_m\) taking its values from \((-1, +1))\).

If we restrict the \( M \) parameter to 1, the transmitted signal can be written as:

\[
s_k = a_k + \frac{1}{\sqrt{N}} a_{N+1,k} \quad k = 1, 2, \ldots, N \tag{3}
\]

Suppose that the data symbols take their values from the alphabet \((-1, +1]\). Clearly, the alphabet of the transmitted signal samples will be \( A_4 = \{\pm 1, \pm \sqrt{2}/2\}\). This is a quaternary alphabet with non-uniformly spaced constellation points. Following Ungerboeck’s set partitioning concept, this constellation can be partitioned into two subsets \( B_0 = \{\pm 1 + \sqrt{3}/2\}\) and \( B_1 = \{\pm 1 - \sqrt{3}/2\}\). From (3), it is clear that all signal samples of a block take their values either from the subset \( B_0 \) (if \( a_{N+1,k} = +1 \)) or from \( B_1 \) (if \( a_{N+1,k} = -1 \)). That is, this channel overloading technique can be interpreted as a block coded modulation (BCM) based on one-step partitioning of a quaternary non-uniform signal constellation, in which the bit that selects the \( B \)-subset is a simple repetition code. The transmitted \( N \)-dimensional signal thus takes its values from \( B_0 B_0 \ldots B_0 \) or \( B_1 B_1 \ldots B_1 \). Provided that \( N \geq 4 \), the minimum Euclidean distance \( d_{min} \) of this multilevel code is 2, and channel overloading is achieved without any asymptotic performance loss. However, it is worth noting that when \( M > 1 \) and depending on the value of \( N \), the minimum distance \( d_{min} \) can decrease and reach 0.

**C. Construction Proposed by Ross and Taylor**

A particular channel overloading scheme was proposed by Ross and Taylor in [4]. This scheme aims at increasing the number of users while keeping \( d_{min} \) at the minimum distance of the single user system, if the users are constrained to be of equal energy (implying no asymptotic performance loss). It uses a block length of \( N = 4^j \), with \( n \) integer. The maximum number of additional users which can be accommodated is equal to \( M = \sum_{j=1}^{n} 4^{n-j} = \frac{1}{3}(N-1) \). This provides a maximum overload equal to \( OF = \sum_{j=1}^{n} 4^{n-j} = \frac{1}{3}(1 - \frac{1}{N}) \). As a consequence, 1/3 is an upper bound to the overload factor that can be achieved with this channel overloading technique.

This scheme uses \( n+1 \) orthogonal sets of users such that the \( j \)th set \((j = 0, \ldots, n)\) comprises \( 4^j \) users. In a fully loaded system, the transmitted signal block can be expressed as:

\[
s_k = a_k + \frac{1}{\sqrt{N}} a_{16, k/4} + \frac{1}{4} a_{21} \quad k = 1, 2, \ldots, N \tag{4}
\]

Thus, for \( N = 16 \), 5 symbols are transmitted per block in addition to the original 16 symbols as follows:

\[
s_k = a_k + \frac{1}{\sqrt{N}} a_{16, k/4} + \frac{1}{4} a_{21} \quad k = 1, \ldots, 16 \tag{5}
\]

This scheme can be interpreted similarly as the combined TDMA/OCDMA scheme. From (5), the transmitted signal samples take their values from the alphabet \( A_0 = \{\pm 1, \pm \sqrt{2}/2, \pm \sqrt{3}/2\} \) for \( N = 16 \). This signal set can be partitioned into two subsets \( B_0 = \{\pm 1 \pm \sqrt{2}/2 \pm \sqrt{3}/2\} \) and \( B_1 = \{\pm 1 \pm \sqrt{2}/2 \mp \sqrt{3}/2\} \). A further partitioning step gives \( C_0 = \{\pm 1 \pm \sqrt{2}/2 \pm \sqrt{3}/2\} \), \( C_2 = \{\pm 1 \pm \sqrt{2}/2 \mp \sqrt{3}/2\} \), \( C_1 = \{\pm 1 \pm \sqrt{2}/2 \pm \sqrt{3}/2\} \), and \( C_3 = \{\pm 1 \pm \sqrt{2}/2 \mp \sqrt{3}/2\} \). Clearly, selection of a \( B \)-subset for the transmitted signal samples is made by symbol \( a_{21} \), which forms a repetition code over the length-16 data block. In addition, the \( C \)-subsets from which they take their values are also determined by a coded sequence. Specifically, the transmitted block takes its values from \( C_{a_1}, C_{a_2}, C_{a_3}, \ldots C_{a_{15}} \), where the indices of the \( C \)-subsets satisfy the following repetition codes: \( a_4 = a_3 = a_2 = 1, a_5 = a_6 = a_7 = 3, a_{12} = a_{11} = a_{10} = a_9, \) and \( a_{15} = a_{14} = a_{13} \).

**III. Construction of Some New Schemes**

**A. One-Level Partitioning**

Following the previous interpretations of existing channel overloading schemes in terms of multilevel coding of an expanded signal constellation, we now investigate a transmission scheme which uses one level of signal superposition and a one-level partitioning of the resulting signal constellation. If we scale by \( \lambda \) the pulse which carries
the excess bits, the transmitted signal levels can be expressed as:

\[ s_k = a_k + \Lambda b_k \quad k = 1, 2, ..., N \]  \hfill (6)

where the \( a_k \)'s are the primary (uncoded) bits after BPSK mapping, and the \( b_k \)'s are the excess (coded) bits after BPSK mapping.

The alphabet of the transmitted signal samples is \( A_B = \{ \pm 1 \pm \Lambda \} \), which can be partitioned into two subsets \( B_0 = \{ \pm 1 \} \) and \( B_1 = \{ \pm 1 - \Lambda \} \). The minimum Euclidean distance within each of these subsets is 2, and the minimum distance between the \( B \)-subsets is \( d_{\text{min}}(B_0, B_1) = \min(2\Lambda, 2|1 - \Lambda|) \), which is maximized for \( \Lambda = 1/2 \). This gives \( d_{\text{min}}(B_0, B_1) = 1 \). The group of additional bits selects a \( B \)-subset for each symbol, and the primary bits determine a point in each subset. The additional bits are encoded using a Reed-Muller (RM) code with a Hamming distance \( M_{\text{min}} \).

To verify, let us consider the case corresponding to \( N = 8 \). We call \( (i_1, i_2, ..., i_8) \) the \( N = 8 \) primary information bits and \( (b_1, i_{10}, i_{11}, i_{12}) \) the \( M_{\text{min}} = 4 \) additional information bits (\( i_k \in \{0, 1\} \) for \( k = 1, 2, ..., 12 \)). Bits \( (i_1, i_2, ..., i_8) \) are transmitted serially using BPSK mapping, resulting in \( (a_1, a_2, ..., a_8) \) (with \( a_j \in \{-1, 1\} \) for \( j = 1, 2, ..., 8 \)). The additional bits \( (b_1, i_{10}, i_{11}, i_{12}) \) are coded with an RM code of Hamming distance 4. The resulting sequence \( (x_1, x_2, ..., x_{12}) \) is given by:

\[
\begin{align*}
x_1 &= i_1 \\
x_2 &= i_9 \oplus i_{12} \\
x_3 &= i_{10} \\
x_4 &= i_{10} \oplus i_{12} \\
x_5 &= i_{11} \\
x_6 &= i_{11} \oplus i_{12} \\
x_7 &= p \\
x_8 &= p \oplus i_{12}
\end{align*}
\]  \hfill (7)

with \( p = i_9 \oplus i_{12} \). After BPSK mapping of \( (x_1, x_2, ..., x_{12}) \) \( (x_j \in \{0, 1\} \) for \( j = 1, 2, ..., 8 \)), we obtain the symbols \( (b_1, b_2, ..., b_8) \) (with \( b_j \in \{-1, 1\} \) for \( j = 1, 2, ..., 8 \)). Hence, the transmitted sequence is given by:

\[ s_k = a_k + \frac{1}{2} b_k \quad k = 1, 2, ..., 8 \]  \hfill (8)

Using this design, we obtain \( OF = 50\% \). Furthermore, the transmitted energy per bit is equal to \( 5/6 \), which implies an improvement of 0.8 dB of the asymptotic performance with respect to BPSK.

The trellis associated to this scheme is shown in Fig. 1 and Fig. 2 for \( N = 8 \) and \( N = 16 \), respectively. In both cases, the upper half-trellis corresponds to \( i_2 = 0 \), and the lower half-trellis to \( i_2 = 1 \). Each transition corresponds to the transmission of two consecutive samples in the trellis of Fig. 1 and of four samples in the trellis of Fig. 2. Generally, the trellis becomes more and more complex as \( N \) is increased.

We now investigate the mean BER performance of TDMA/OCDMA, Ross and Taylor’s construction and one-level partitioning scheme after a ML decoding, performed in the latter case by the Viterbi algorithm. We consider an AWGN channel. The BER results, evaluated by means of Monte-Carlo simulations, are given as a function of the ratio \( E_b/N_0 \). They can be confronted with the theoretical BPSK performance.

![Fig. 1. State trellis for one-level partitioning with \( N = 8 \)](image)

![Fig. 2. State trellis for one-level partitioning with \( N = 16 \)](image)

![Fig. 3. Mean BER performance](image)

Fig. 3 shows that for \( (N = 8, M = 4) \), the one-level partitioning scheme gives much better results than the combined TDMA/OCDMA scheme, whose minimum Euclidean distance is smaller. In particular, one-level partitioning avoids the phenomenon of BER floors, which is due to a minimum Euclidean distance equal to 0 and which can be encountered in the TDMA/OCDMA scheme (e.g. with \( N = 16, M = 4 \)). Fig. 3 also shows the BER performance obtained with Ross and Taylor’s construction and with the proposed one-level partitioning scheme, when the \( N \) parameter is fixed to 16 and the number of excess users is taken equal to the maximum allowed by each construction (that is \( M = 5 \) for Ross and Taylor’s construction and \( M = 11 \) for the one-level partitioning scheme).
The asymptotic mean probability of error can be derived as:

\[
\text{P}_{\text{mean}} = C_{\text{mean}} \cdot \Phi \left( \frac{12}{5N_0} \right),
\]

with \( C_{\text{mean}} = \begin{cases} \frac{3989}{192} & \text{if } N = 8 \\ \frac{135N^2 - 368N + 324}{96N} & \text{if } N > 8 \end{cases} \)

and \( \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du \).

Similarly, the asymptotic probabilities of error for set 1 and set 2 users can be written:

\[
P_{\text{e1}} = C_1 \cdot Q \left( \frac{12}{5N_0} \right) \quad \text{and} \quad P_{\text{e2}} = C_2 \cdot Q \left( \frac{12}{5N_0} \right),
\]

with \( C_1 = \begin{cases} \frac{205}{16} & \text{if } N = 8 \\ \frac{9073}{256} & \text{if } N > 8 \end{cases} \)

\( C_2 = \begin{cases} \frac{27N - 22}{32} & \text{if } N = 8 \\ \frac{81N^2 - 324N + 1}{32N} & \text{if } N > 8 \end{cases} \)

Because of these coefficients, the asymptotic gain cannot be observed on Fig. 5: The plot of the asymptotic probabilities of error gives clear evidence that this gain is only visible for high SNR values.

![Fig. 5. Mean BER performance of 1-level partitioning scheme with 50% overload and N as a parameter](image)

**C. Two-Level Partitioning**

A more powerful scheme consists in using two levels of signal superposition and a two-level partitioning of the resulting signal constellation. The signal levels transmitted are:

\[
s_k = a_k + \lambda b_k + \mu c_k \quad k = 1, 2, ..., N
\]

where \( \lambda \) and \( \mu \) denote the two pulses carrying the excess symbols. The \( a_k \)'s are the primary bits after BPSK mapping, and the \( b_k \)'s and \( c_k \)'s are the excess (coded) bits after BPSK mapping.

In this case, the signal set is \( A_N = \{ \pm 1 \pm \lambda \pm \mu \} \), which can be partitioned into two subsets \( B_0 = \{ \pm 1 \pm \lambda + \mu \} \) and \( B_1 = \{ \pm 1 \pm \lambda - \mu \} \). A further partitioning step gives
\[ C_0 = \{ \pm 1 + \lambda + \mu \}, \quad C_1 = \{ \pm 1 - \lambda + \mu \}, \quad C_2 = \{ \pm 1 + \lambda - \mu \}, \quad \text{and} \quad C_3 = \{ \pm 1 - \lambda - \mu \}. \]

Clearly, we have:
\[
\begin{align*}
    d_{\min}(C_i) &= 2 \quad \text{for } i = 1, 2, 3, 4, \\
    d_{\min}(B_0) &= d_{\min}(B_1) = \min(2\lambda, 2|1 - \lambda|), \\
    \text{and} \quad d_{\min}(A_0) &= \min(2\mu, 2|\lambda - \mu|, 2|1 - \lambda - \mu|).
\end{align*}
\]

The distance \( d_{\min}(B_1) \) is maximized for \( \lambda = 1/2 \), giving 
\( d_{\min}(B_1) = 1 \). Next, given this value of \( \lambda \), \( d_{\min}(A_0) \) is 
maximized for \( \mu = 1/4 \), providing 
\( d_{\min}(A_0) = 1/2 \). The \( c_i \)'s select a \( B \)-subset for each symbol, the \( b_i \)'s select a \( C \)-subset from the 
\( B \)-subset selected by the \( c_i \)'s, and finally the \( a_i \)'s determine a point in the 
\( C \)-subset determined by the \( b_i \)'s. The group of bits giving the \( b_i \)'s is encoded using an RM code with a Hamming distance \( d_H \) 
of 4, in order to verify: \( d_H d_{\min}(B_i) = d_{\min}(C_i) = 4 \). As for the group of bits giving the 
\( c_i \)'s, it is encoded using an RM code with a Hamming distance of 16, in order to have: 
\( d_H d_{\min}(A_i) = d_{\min}(C_i) = 4 \). This condition implies a block length of at least 16. The number of 
information bits per code word is thus
\[
M_2 = \left( \frac{2^{q} - q - 1}{6} \right) \left[ \frac{2^{q} - 5}{6} q - 1 \right] = \frac{2^{q+1} - 11 q^2 - 2 q - 2}{6}.
\]

For \( N = 16 \), the code related to the \( c_i \)'s is in fact a repetition code of length 16. As for the spread energy per bit, it is 
equal to 21/28 for \( N = 16 \), which represents an asymptotic performance gain of 1.25 dB.

**D. Comparison of the Different Proposed Schemes**

The achievable overloads using one-level (OF1) and two-level partitioning (OF2) are indicated in Table I. Table I also shows the corresponding asymptotic gains \( G_1 \) and \( G_2 \), given by:
\[
G_1 = \frac{5 N}{4(N + M_1)} \quad \text{and} \quad G_2 = \frac{21 N}{16(N + M_2)}.
\]

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We observe that the acceptable overload allowed by two-level partitioning becomes very significant with increasing \( N \). However, this is accompanied by an increasing complexity of the trellis, causing an important decoding complexity.

**IV. SUMMARY AND CONCLUSIONS**

In this paper, we interpreted various channel overloading techniques as multilevel coding of an expanded signal constellation. The first technique examined, which uses a combination of TDMA and OCDMA, does not have any limitation in terms of overloading, but it can suffer from a decrease of the minimum Euclidean distance, causing some performance degradation. The second technique, described by Ross and Taylor, does not affect the minimum Euclidean distance, but the overload factor is limited to 33%. The interpretation of these techniques as multilevel codes enabled us to construct some new channel overloading schemes making use of one or two-level signal superposition. The proposed schemes offer the combined advantages of combined TDMA/OCDMA and of the construction technique of Ross and Taylor. They achieve high channel overloads while significantly improving asymptotic performance.

**REFERENCES**


