Theoretical bounds for ranging with Multi Band OFDM and Direct Sequence UWB signals

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Abstract — **The definition of Ultra Wide Band (UWB) signals set by the Federal Communications Commissions (FCC) opened the way to both impulse and non-impulse UWB signal formats. This possibility is reflected within the IEEE 802.15.3a TG, aiming at the definition of a standard for UWB-based high bit rate WPANs. The two main proposals considered in this group are in fact a Multi Band OFDM approach, based on the transmission of non-impulse OFDM signals combined with Frequency Hopping (FH), and the Direct-Sequence (DS) UWB approach, based on impulse radio transmission of UWB DS-coded pulses. In this paper, the ranging capabilities of these two proposals are analyzed by determining the Cramer-Rao Lower Bound (CRLB) for the distance estimation error. The CRLB is evaluated under the condition of both ideal and real, multipath-affected, channel models. Results show that DS-UWB is in general best suited for ranging, thanks to its larger bandwidth and its higher frequencies of operation, and also quantify the degree to which multipath may affect ranging accuracy.**

Index Terms—**UWB, ranging, localization, Cramer–Rao Lower Bound**

I. Introduction

ULTRA Wide Band (UWB) radio has gained popularity world-wide thanks to its promise of providing very high bit rates at low cost. The interest towards this transmission technique led, yet in 2001, to the creation of the IEEE 802.15.3a Study Group, devoted to the definition of a novel standard for Wireless Personal Area Networks (WPANs) based on a UWB physical layer capable of bit rates in the order of 500 Mb/s.

The activity of the IEEE Group (now referred to as IEEE 802.15.3aTG) further intensified after the release of the first world-wide official UWB emission masks by the US Federal Communication Commission (FCC) in February 2002 [1]. This release officially opened the way to the development of commercial UWB products. The application scenarios suitable for UWB communications naturally emerged from the strong emitted power limits set by the FCC, that is either high bit rates over short ranges, dealt with in the IEEE 802.15.3aTG, or low bit rates over medium-to-long ranges, that is the typical framework of the IEEE 802.15.4TG.

The several different UWB PHY proposals originally submitted to the 802.15.3a Task Group converged into two main proposals: the Multi Band OFDM solution, based on the transmission of non-impulse OFDM signals combined with Frequency Hopping (FH) over instantaneous frequency bandwidths of 528 MHz, and the Direct-Sequence (DS) UWB proposal, based on impulse radio transmission of UWB DS-coded pulses.

Technical discussions and evaluation of such proposals focused on the priority of the IEEE 802.15.3a Task Group, that is the achievement of a high bit rate. As a consequence, proposals overlooked one of the most appealing features of UWB radio: the capability of estimating distance between terminals with high accuracy, and providing thus joint communications and ranging. The UWB ranging capability is particularly attractive as a support for location-aware applications in ad-hoc and sensor networks, that is the focus of the IEEE 802.15.4a Working Group, specifically aimed at low bit rate networks with location and tracking.

Although not specifically designed for ranging support, both MB-OFDM and DS-UWB proposals adopt UWB emissions with bandwidths exceeding 500 MHz, in compliance with the UWB definition given by the FCC, and can thus potentially provide high accuracy in ranging.

The goal of this work is to determine and compare ranging accuracy of MB-OFDM and DS-UWB proposals in an indoor environment. We will first carry out the analysis in an ideal case by determining the Cramer-Rao Lower Bound (CRLB) in presence of an ideal channel. The CRLB establishes in fact the lower bound on the ranging accuracy that can be obtained given a signal format characterized by a given bandwidth and energy. Next, we will introduce a real channel model that takes into account multipath as well as frequency selectivity, and evaluate its impact on the ranging accuracy that can be obtained with the two proposals.

The paper is organized as follows. Section II reports the signal definition for both 802.15.3a proposals, while Section III reviews and fixes the notation for the CRLB. In Sections IV and V the CRLB is derived for the impulse vs. non-impulse UWB 802.15.3a proposals, for an ideal vs. a real channel. Conclusions are included in Section VI.

II. Signal Definitions

Notations for the two UWB signal formats under discussion within the IEEE 802.15.3a Task Group are given in this section.

A. Multi Band Orthogonal Frequency Division Multiplexing (MB-OFDM)

An OFDM modulated signal consists in the parallel transmission of N signals that are modulated at N frequency carriers f_m (m=0,…,N-1). All sub-carriers f_m are equally spaced by Δf. The binary sequence is usually mapped on a QPSK

constellation, and each QPSK symbol ($c_m = a_m + jb_m$) modulates a different sub-carrier f_m.

The frequency carriers (f_p) used in the 802.15.3a MB-OFDM format [2] occupy the frequency interval between 3.1 GHz and 10.6 GHz, that is in the frequency interval where the FCC has allocated a transmission power of -41.3 dBm/MHz [1].

TABLE I
MAIN PARAMETERS OF THE MB-OFDM PROPOSAL

Parameter	Value
N_{SD}: Number of data subcarriers	100
N_{SDP}: Number of defined pilot subcarriers	12
N_{SG}: Number of guard carriers	10
N_{ST}: Total number of subcarriers	122 (=N_{SD}+N_{SDP}+N_{SG})
Δf: Subcarrier frequency spacing	4.125 MHz (=528MHz/128)
T_{FFT}: IFFT/FFT period	242.2 ns (1/ Δf)
T_{CP}: Cyclic prefix duration	60.61 ns (=32/528MHz)
T_{GI}: Guard interval duration	9.47 ns (=5/528MHz)
T_{SYM}: Symbol interval	312.5 ns (=T_{CP}+T_{FFT}+T_{GI})

In the 802.15.3a MB-OFDM format, the available frequency interval is divided into 13 frequency intervals. Each interval corresponds to one band of the MB-OFDM, and is 528 MHz wide. The center frequency of each band and the band number are related according to the following rule:

$$\text{Center frequency for band } n_b = \begin{cases} 2904 + 528 \times n_b & n_b = 1 \ldots 4 \\ 3168 + 528 \times n_b & n_b = 5 \ldots 13 \end{cases} \quad (MHz) \qquad (1)$$

The MB-OFDM proposal foresees two different modes of transmission: a mandatory Mode 1 and an optional Mode 2. Mode 1 uses three bands of operation: Band 1 [3.168 GHz, 3.696 GHz], Band 2 [3.696 GHz, 4.224 GHz], and Band 3 [4.224 GHz, 4.752 GHz]. Mode 2 considers seven bands: Band 1, 2, 3 (same as Mode 1), Band 6 [6.072 GHz, 6.60 GHz], Band 7 [6.60 GHz, 7.128 GHz], Band 8 [7.128 GHz, 7.656 GHz], and Band 9 [7.656 GHz, 8.184 GHz]. The four unmentioned bands have been reserved for future use. Table I reports the main parameters values, such as the number of the subcarriers, the duration of the waveform, the time of the FFT. The parameters set also includes a guard interval, which is introduced to mitigate Inter-Symbol Interference ISI. The pilot carriers are used for channel estimation.

In time, the signal is divided into two parts: the useful signal, of duration 242.4 ns (T_{FFT}), and the cyclic prefix, of duration 70.1 ns (T_{GI}), for an overall duration of 312.5 ns (T_{SYM}=T_{FFT}+T_{GI}). The cyclic prefix, located at the onset of the transmitted signal, is a replica of the final interval of the transmitted signal, and it is used for synchronization and for channel estimation purposes.

Under the above conditions the transmitted signal can be written as follows:

$$x(t) = g_T(t) \cdot \sum_{m=0}^{N-1} \left(a_m \cos\left(2\pi\left(f_p + f_m\right)t + \phi\right) - b_m \sin\left(2\pi\left(f_p + f_m\right)t + \phi\right)\right) \qquad (2)$$

where $g_T(t)$ is the impulse response of the pulse shaper and ϕ is the phase at t=0.

B. Direct Sequence UWB (DS-UWB)

A DS-UWB signal consists in the transmission of a binary sequence coded with a pseudorandom sequence, and which modulates the amplitudes of a train of short pulses. The bandwidth of such a signal depends on the width of the pulse. The adoption of a pseudorandom sequence guarantees a close to flat PSD (Power Spectral Density).

The transmitter is composed of four main blocks: a repeater, a transmission coder, a PAM (Pulse Amplitude Modulation) modulator, and a pulse shaper.

Each bit of the binary sequence is repeated N_s times, so that the output of the repeater is a sequence of N_sN_b bits, where N_b is the number of bits of the input sequence. The repeater introduces thus redundancy in the transmitted sequence.

The transmission coder applies a binary code of period N_p to the output sequence of the repeater. Most commonly, N_p is a multiple of N_s.

The output sequence of the transmission coder enters the PAM modulator, which generates a train of Dirac pulses, located at multiples of T_s.

The output of the PAM modulator enters the pulse shaper filter with impulse response p(t). The impulse response is a pulse with duration smaller than T_s.

The output signal of the transmission cascade is expressed as follows:

$$s(t) = \sum_{j=-\infty}^{+\infty} d_j \, p\left(t - jT_s\right) \qquad (3)$$

where the symbols d_j are the symbols of the output sequence of the PAM modulator.

As for the OFDM-MB transmission, the frequency interval occupied by the transmission signal is between 3.1 GHz and 10.6 GHz, where a transmission power of -41.3 dBm/MHz is allowed [1].

The DS-UWB proposal uses two different carrier frequencies for transmission located at 4.104 GHz (Low Band) and 8.208 GHz (High Band). For the low (high) frequency band the filter cutoff frequency (-3 dB point) is 684 MHz (1368 MHz) leading to a bit duration of 1/57 ms (1/114 ms).

III. CRAMER-RAO LOWER BOUND (CRLB)

CRLB allows establishing achievable performance using an ideal unbiased estimator. This performance is not attainable using a real estimator, but the bound allows understanding the trend of the estimator. The Cramer-Rao lower bound provides the minimal achievable error variance for an unbiased estimator σ_t^2.

The first step is finding the characteristics of an UWB signal that minimize CRLB. We can hypothesize that signal $s(t\,;\{a_k\})$ depends on time t and some unknown set of

parameters $\{a_k\}$. Overlapped to the signal is thermal noise $w(t)$. The overall frequency occupation is B, and therefore the power of thermal noise can be defined as follows:

$$\sigma_w^2 = F\,k\,T\,B/2 \tag{4}$$

The received signal is $r(t) = s(t\,;\{a_k\}) + w(t)$. At the receiver the signal is sampled at frequency f_s=B, and thus the sampling period is T_s=1/B. The sequence of transmitted samples is $s_n = s(nT_s\,;\{a_k\})$, while the corresponding noise and received samples are $w_n = w(nT_s)$ and $r_n = s_n + w_n$ respectively.

The Cramer-Rao theorem indicates that for any unbiased estimator, the minimal achievable error variance σ_t^2 is:

$$\sigma_t^2 \geq F_n^{-1} \tag{5}$$

where F_n is the Fisher information matrix, defined as follows:

$$F_n = -E\left\{\left(\frac{\partial}{\partial\theta}\ell(X;\theta)\right)^2\right\} = -E\left\{\left(\frac{\partial^2}{\partial\theta^2}\ell(X;\theta)\right)\right\} \tag{6}$$

where $\ell(X;\theta)$ is the log likelihood function with respect to parameter θ. The log likelihood function is the logarithm of the probability of the estimation error, conditioned to the knowledge of τ and $\{a_k\}$, which is defined as follows:

$$p(\mathbf{r} \mid \tau;\{a_k\}) = \frac{1}{(2\pi)^{N/2}}\exp\left\{-\frac{\sum_n [r_n - s(nT_s - \tau;\{a_k\})]^2}{\sigma_w^2}\right\} \tag{7}$$

To evaluate CRLB we need the first and second derivatives of log likelihood function, that is:

$$\frac{\partial}{\partial\tau}\ln[p(\mathbf{r} \mid \tau;\{a_k\})] = -\frac{2}{\sigma_w^2}\sum_n [r_n - s(nT_s - \tau;\{a_k\})]\dot{s}(nT_s - \tau;\{a_k\}) \tag{8}$$

$$-\frac{\partial^2}{\partial\tau^2}\ln[p(\mathbf{r} \mid \tau;\{a_k\})] = \frac{2}{\sigma_w^2}\sum_n \left\{\dot{s}^2(nT_s - \tau;\{a_k\}) - [r_n - s(nT_s - \tau;\{a_k\})]\ddot{s}(nT_s - \tau;\{a_k\})\right\} \tag{9}$$

The next step is the evaluation of the average value of the second derivative of the log likelihood function, which corresponds to the Fisher information matrix:

$$F_n = \frac{4}{N_0}\int \dot{s}^2(t;\{a_k\})\,dt \tag{10}$$

The minimal achievable variance for any unbiased estimator (CRLB) is thus:

$$\sigma_t^2 = \frac{1}{F_n} = \frac{N_0}{4\int \dot{s}^2(t;\{a_k\})dt} = \frac{1}{2\left(\frac{2E}{N_0}\right)\beta^2} \tag{11}$$

where:

$$\beta^2 = \frac{\int \dot{s}^2(t;\{a_k\})dt}{\int s^2(t;\{a_k\})dt} = -4\pi^2\frac{\int f^2\,S^2(f;\{a_k\})df}{\int S^2(f;\{a_k\})df} \tag{12}$$

The maximum theoretical ranging accuracy achievable with the proposed UWB signal formats can be obtained by considering the corresponding *s(t)*.

IV. CRLB WITH AN IDEAL CHANNEL

The first step is the comparison between the two proposals under the hypothesis of an ideal channel. In this case, the CRLB is:

$$\sigma_t^2 = \frac{N_0 \cdot D^2}{16\pi^2 T\int f^2 PSD(f)\partial f} \tag{13}$$

For a given PSD(f), eq. (13) shows that σ_t^2 depends on D^2 and T, where D is the distance between transmitter and receiver, and T is the observation interval. In the following, we will analyze the accuracy in terms of distance estimation. Variances of time estimation error (σ_t^2) is in fact related to the distance estimation error (σ_x^2) as follows:

$$\sigma_x^2 = c^2 \cdot \sigma_t^2 \tag{14}$$

where *c* is the propagation speed of the signal.

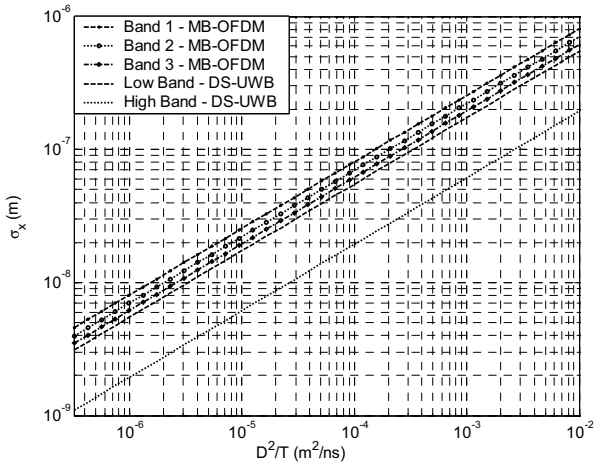

Figure 1 - Standard deviation of distance estimation error in logarithmic scale for MB-OFDM and DS-UWB signals

Figure 1 plots σ_x for the three bands used by MB-OFDM Mode 1 and for the two bands used by DS-UWB as a function of D^2/T, and shows that a similar trend for all, although different degrees of accuracy are achievable by different

signals. The performance differences are mainly due to two factors: the difference in the width of occupied frequencies and in the value of the center frequency. The High Band of DS-UWB has best ranging performance, thanks to the large bandwidth (about 1.3 GHz vs. about 600 MHz of the Low Band) and the higher frequency carrier. As an example, at D = 1 m, with an observation time T = 312.5 ns, the expected σ_x is about 10^{-7} m. The other signals lead to an error that is almost one degree of magnitude larger with, as expected, a slightly better performance for Band 3 of MB-OFDM.

It should be also noted that the better ranging performance obtained for High Band of the DS-UWB proposal is obtained at the price of a shorter communication range, due to the higher propagation loss at high frequencies.

V. CRLB WITH A REAL CHANNEL

We adopt the channel model proposed in Batra et al. [2] within the IEEE 802.15.3a Channel Model subcommittee. This model hypothesizes the presence of strong multipath, which causes several overlapped replicas of a transmitted signal. The model assumes that all channel parameters are random variables with specific, well defined distributions.

Given this model, we can consider some realizations of the channel impulse response, and evaluate the corresponding CRLB.

The channel model introduces N replicas of the signal that are equally spaced in time and with amplitudes depending on both distance and delay. The channel impulse response can be expressed as follows:

$$h(t) = \sum_{n=1}^{N} \alpha_n(D,\tau_n)\delta(t-\tau_n) \tag{15}$$

$$\alpha_n(D,\tau_n) = k \cdot \frac{e^{-D}}{D} e^{-\frac{\tau_n}{\tau_0}} \tag{16}$$

For comparing the two proposals we refer to a the set of channel parameters identified by the scenario A in Table II.

Figure 2 shows CRLB for both ideal channel and scenario A for MB-OFDM and DS-UWB as a function of D^2/T. Note that for low distances, losses are contained, while they dramatically increase for higher distances. Note that performance depends exponentially upon distance, since α_n depends on distance as e^{-D}.

Figure 2 shows that in scenario A the MB-OFDM in Band 2 leads to the lowest estimation error with a variance of the estimation error that, for low distances, is close to the CRLB achievable with the ideal channel. A similar result is obtained for both bands used in the DS-UWB proposal.

These results are due to the particular transfer function of the channel considered in scenario A.

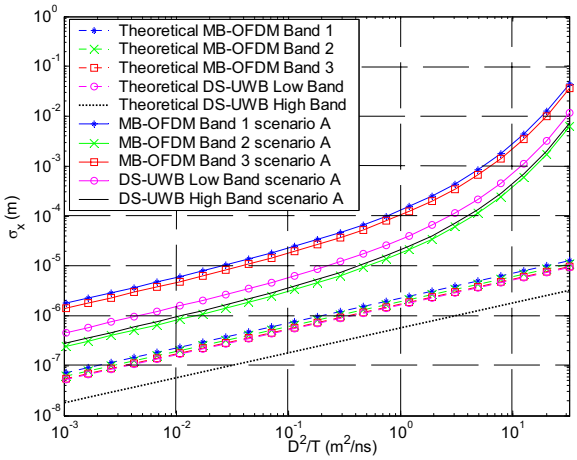

Figure 2 - Standard deviation of distance estimation error in logarithmic scale for different signals: MB-OFDM and DS-UWB for a non ideal channel (scenario A – see Table II)

Figure 3 shows the transfer function $|H(f)|^2$ of the channel, where we can see peaks, i.e. smaller losses, in the frequency ranges corresponding to Band 2 used in MB-OFDM and to the DS-UWB bands.

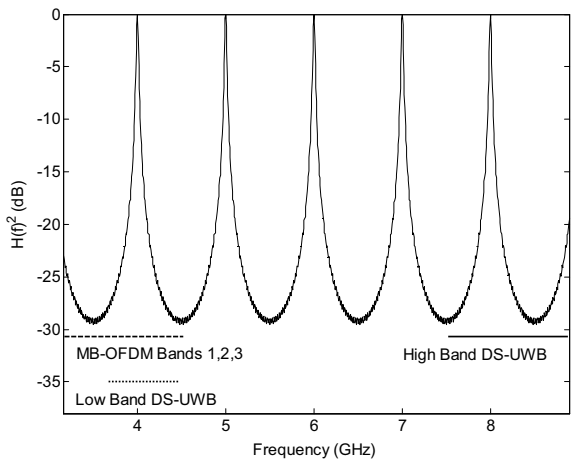

Figure 3 - Transfer function of the channel H(f) for scenario A in Table II

The transfer function of the channel depends upon the particular choice of the parameters. As a consequence, the CRLB for MB-OFDM and DS-UWB in the different bands will vary in a different way as a function of such parameters.

In order to evaluate the effect of the parameters of the channel model on the achievable performance, let us consider a test case, in particular the MB-OFDM using Band 2 in order to analyze the effect of different channel realizations on the corresponding CRLB. Note that considerations made in the remaining part of this section apply to the DS-UWB case as well.

We will focus in particular on the effect of the variations of parameters τ_n and N, representing the delay between two consecutive replicas of the signal and the number of replicas, respectively. Table II reports the values assumed for τ_n and N

in six test scenarios labeled from A to F, and the constant values assumed for the parameters k and τ_0 in all the six scenarios. Note that the values of τ_n and N were selected in order to keep constant the duration of channel impulse response.

TABLE II
CHANNEL PARAMETERS CONSIDERED IN SECTION V

Scenario Identification	τ_n *(ns)*	N	k	τ_0 *(ns)*
A	From 0 to 50 ns spaced 1 ns	50	0.1	15
B	From 0 to 50 ns spaced 1.25 ns	40	0.1	15
C	From 0 to 50 ns spaced 1.67 ns	30	0.1	15
D	From 0 to 50 ns spaced 2.5 ns	20	0.1	15
E	From 0 to 50 ns spaced 5 ns	10	0.1	15
F	From 0 to 50 ns spaced 50 ns	2	0.1	15

Figure 4 shows the CRLB for the MB-OFDM using Band 2 in the six scenarios and in the ideal case considered in section IV. It can be observed that, by increasing the delay between two replicas, moving from scenario A to F, the CRLB obtained approaches the CRLB obtained using an ideal channel.

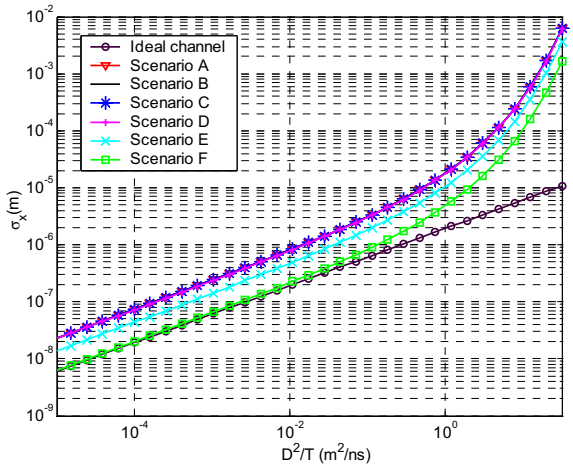

Figure 4 - CRLB obtained for the MB-OFDM signal using Band 2 in scenarios defined in Table II

This effect can be explained by considering the effect of the delay τ_n on the channel transfer function, presented in Figure 5. Figure 5 shows in fact that, as τ_n increases, the multipath effect decreases and the channel transfer function resembles the transfer function of an ideal channel. Note that the value of τ_n affects the positions and the number of the peaks in the transfer function. The two limit cases are the presence of only two replicas of signal (Scenario F) and the presence of 50 replicas (Scenario A). The transfer function of the Scenario F is almost flat, while the Scenario A has a big peak in correspondence of Band 2 used in the MB-OFDM.

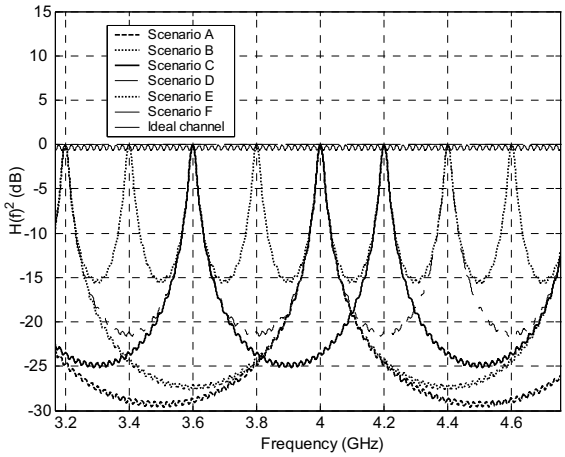

Figure 5 - Transfer function of the different realizations of channel H(f) reported in Table III for the MB-OFDM signal using Band 2

VI. CONCLUSION

In this paper, we analyzed the ranging capabilities of the two UWB signal formats proposed within the IEEE 802.15.3a TG, that is the impulsive DS-UWB and the non-impulsive MB-OFDM. The analysis was carried out by evaluating the Cramer-Rao Lower Bound for the two proposed UWB signals, taking into account the emission limits set by the FCC for indoor UWB emissions. The CRLB was first evaluated considering an ideal channel, and the results highlighted that the DS-UWB signal using the High Band is potentially the best solution to perform ranging, thanks to its larger bandwidth and higher operative frequencies. Next, the CRLB was evaluated in presence of a real channel model with multipath. In this case the results showed that DS-UWB and MB-OFDM are affected differently by the channel, and that the degree of multipath dramatically changes the behavior of the two signals in terms of ranging accuracy. This effect was analyzed in detail in the case of the MB-OFDM signal operating in Band 2, showing that strong multipath significantly reduces the accuracy in distance estimation.

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