Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is vulnerable with respect to nonlinear distortions caused, for example, by a nonlinear high power amplifier. In the presence of nonlinear high power amplification, OFDM suffers from crosstalk between all subcarriers. Furthermore, the power density spectrum is significantly widened.

In this paper, we simultaneously use a memoryless predistorter at the transmitter side and a nonlinear detector at the receiver side. The predistorter reduces the out-of-band power, whereas the nonlinear detector improves the bit error rate. Since maximum-likelihood detection is prohibitive, a novel reduced-state symbol detector derived from the maximum-likelihood detector is proposed. For a QPSK/OFDM system in the presence of a memoryless solid-state power amplifier it is shown that the predistorter is not only useful for spectral shaping, but also in order to reduce the computational complexity of the nonlinear detector and to provide more robustness concerning an incomplete knowledge of the characteristics of the nonlinearity at the receiver. The overall raw bit error performance is shown to be close to that of a linear QPSK/OFDM system.

Index Terms—OFDM, power amplifiers, nonlinear distortion, predistortion, nonlinear detection, maximum-likelihood estimation.

I. INTRODUCTION

OFDM is a popular multicarrier transmission technique with orthogonal subcarrier signals. It has been successfully applied in data modems, audio and video broadcasting systems, wireless local area networks, and is a suitable candidate for the next mobile radio generation (4G). Inherent advantages of OFDM include its ease of implementation (due to FFT processing), its robustness against multipath fading (due to a guard interval), and its bandwidth efficiency (due to the ability of adaptive bit loading) [1]. However, the bit error performance degrades if orthogonality can not be maintained. Reasons for this include fast fading, phase jitter, frequency offset, delay spread exceeding the guard interval, and nonlinear distortions. All these effects cause crosstalk between the subcarriers. In contrast to multicarrier transmission techniques with non-orthogonal subcarrier signals, all subcarriers interfere with each other. As a consequence, optimum reception is prohibitive if orthogonality is lost.

In this paper, we study the performance of OFDM in the presence of a high power amplifier (HPA). It is well known from numerous papers that due to nonlinear distortions (i) the bit error probability degrades and (ii) the spectral mask is difficult to maintain, see e.g. [2]-[7]. Hence, in most current OFDM systems either expensive, highly linear amplifiers are being used and/or a large power back-off is selected to maintain quasi-orthogonality. Particularly in future mobile terminals, it is desirable to apply low-cost amplifiers operating at a fairly small power back-off in order to maintain power efficiency. Therefore, the influence of nonlinear distortion is inevitable, unless some form of compensation is done.

Compensation techniques can be classified into techniques applied at the transmitter side and at the receiver side. Predistortion is a popular compensation technique applied at the transmitter side [8]-[12]. The main idea of predistortion is to shape the transmitted data symbols ("data predistortion") or the input signal of the HPA amplifier ("signal predistortion") so that the output signal of the HPA is less distorted. Predistortion does not reduce the information rate. Due to predistortion, the power density spectrum of the transmit signal improves. The bit error performance also improves, but only slightly since clipping can not be avoided. Therefore, in most publications on predistortion a large power back-off is assumed.

An alternative to predistortion are peak-to-average power reduction (PAPR) techniques applied at the transmitter side. PAPR can be achieved by channel coding, for example, or by dropping or loading some carriers, among other techniques. As opposed to predistortion, these techniques decrease the information rate, however.

At the receiver side, linear as well as nonlinear equalization/detection techniques can be applied. With nonlinear equalization/detection techniques the bit error performance can be enhanced significantly, see e.g. [13]-[15], even in the presence of a small power back-off. However, the out-of-band radiation is not affected, of course. The main challenge is the derivation of cost-efficient algorithms.

In this paper, we simultaneously use a (signal) predistorter at the transmitter side and a low-cost nonlinear detector at the receiver side. All processing is done at baseband. A related concept has been proposed in [14] for OFDM systems and in [8], [12] for single-carrier systems.

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loss with respect to the linear case can be made negligible even in the presence of a small power back-off. In Section II, the transmission model under investigation is introduced. Section III is devoted to predistortion, whereas in Section IV a reduced-state symbol detector is proposed. Numerical results are presented in Section V. Finally, conclusions are drawn in Section VI.

II. TRANSMISSION MODEL

Throughout this paper, the equivalent discrete-time channel model in complex baseband notation is used. Vectors are written in bold face. In Fig. 1, a block diagram of the transmission scheme under investigation is shown.

![Block diagram of the transmission scheme under investigation](image)

An OFDM signal can be calculated by means of an inverse discrete Fourier transform (IDFT):}

\[ s[n] = \text{IDFT}_N\{a[n]\}, \]

(1)

where \( N \) is the number of subcarriers, \( a[n] \) is the \( n \)th data vector of length \( N \), and \( n \) is the time index after serial/parallel (S/P) conversion. According to the central limit theorem, the quadrature components of the OFDM signal are Gaussian distributed, i.e., the amplitude is Rayleigh distributed. Therefore, in the presence of a nonlinear HPA a large power back-off is needed in order to avoid crosstalk between all subcarriers ("intercarrier interference"), unless predistortion, peak-to-average power reduction, or nonlinear detection is applied. Only in the linear case, the data symbols can be reconstructed without any performance loss (using a matched-filter receiver).

In this paper, a memoryless, time-invariant nonlinearity is assumed. If we denote the modulated OFDM signal as \( s[k] := A[k] \exp(j\phi[k]) \), where \( k \) is the time index before serial/parallel conversion, \( A[k] \) the amplitude of the transmit signal and \( \phi[k] \) the phase, the output signal of the HPA can be modeled as

\[ s_{HPA}[k] = g(A[k]) \exp(j[\phi[k] + \Psi(A[k])]). \]

(2)

The real-valued functions \( g(A[k]) \) and \( \Phi(A[k]) \) are usually called AM/AM and AM/PM conversion, respectively. For example, the AM/AM and AM/PM conversion of a solid-state power amplifier (SSPA) can be approximated as [2]

\[ g(A[k]) = v \left( \frac{A[k]}{A_0} \right)^p, \]

\[ \Phi(A[k]) \approx 0, \]

(3)

where \( v > 0 \) is the small signal gain, \( A_0 > 0 \) is the output saturating amplitude, and \( p > 0 \) is a parameter to control the smoothness of the transition from the linear region to the saturation level. If \( p \to \infty \), the so-called hard limiter (HL)\(^1\) is obtained. The hard limiter is defined as

\[ g(A[k]) \bigg|_{p \to \infty} = \begin{cases} vA[k] & \text{if } vA[k] \leq A_0 \\ A_0 & \text{otherwise.} \end{cases} \]

(4)

In order to provide a fair comparison of transmission schemes with different nonlinearities, the same output back-off (OBO) must be considered. The output back-off, defined as

\[ OBO := 10 \log_{10} \frac{P_{out,0}}{P_{out}} \text{ dB,} \]

determines the linear dynamic range of the HPA (and its power consumption). In (5), \( P_{out,0} \) is the maximum output power and \( P_{out} \) is the average output power.

III. PREDISTORTION

Memoryless predistortion has been investigated in many papers as a potential solution to decrease the nonlinear distortion caused by a HPA, see e.g. [8]-[12]. Naturally, this technique tries to invert the nonlinearity of the HPA. If the modulated OFDM signal is again denoted as \( s[k] = A[k] \exp(j\phi[k]) \), the output samples of the predistorter can be written as

\[ s_p[k] = f(A[k]) \exp(j\phi[k] + \Psi(A[k])), \]

(6)

where \( f(A[k]) \) and \( \Psi(A[k]) \) are the AM/AM and AM/PM conversion of the predistorter, respectively. The combination of a given memoryless HPA and the corresponding predistorter will result in

\[ s_{HPA}[k] = g(f(A[k])) \exp(j[\phi[k] + \Psi(A[k])]) + \Phi(f(A[k])). \]

(7)

Ideal predistortion is characterized as

\[ f(A[k]) = \begin{cases} \alpha A[k] & \text{if } \alpha A[k] \leq A_0 \\ A_0 & \text{otherwise,} \end{cases} \]

\[ \Psi(A[k]) + \Phi(f(A[k])) = 0, \]

(8)

where \( \alpha \) is a real-valued constant (\( \alpha > 0 \)). In this case, the combination of the HPA and the corresponding predistorter (i.e., the overall transmitter-side nonlinearity) is equivalent with the hard limiter defined in (4).

Throughout this paper we assume that the AM/PM conversion of the HPA is negligibly small and does not have to be compensated, i.e., \( \Psi(A[k]) = 0 \). The AM/AM conversion of the predistorter is modeled by a polynomial as

\[ f(A[k]) = f_1 A[k] + f_2 A^2[k] + \cdots + f_L A^L[k] := \mathbf{f} A^T[k], \]

(9)

where \( L \) is the order of the polynomial, \( \mathbf{f} := [f_1, f_2, \ldots, f_L] \), and \( A[k] := [A[k], A^2[k], \ldots, A^L[k]] \). To find the coefficient set, \( \mathbf{f} \), we apply the least mean square algorithm proposed in [10], which minimizes the mean squared error between the

\(^1\)The hard limiter is also called soft envelope limiter in some papers.
input and output amplitudes of the combined predistorter and HPA:

\[ J(f) := E \left\{ \frac{\left( g(f A^T[k] - \alpha A[k]) \right)^2}{|s_{HPA}[k]|^2} \right\}. \] (10)

In (10), averaging is done over time. The coefficient set can be calculated recursively according to

\[ f[k+1] = f[k] - \mu \nabla_f J(f[k]) \]
\[ = f[k] + \mu A[k] g(r[k] A^T[k]) \left( |s_{HPA}[k]| - \alpha A[k] \right), \]

where \( \nabla_f \) denotes the gradient, \( g(\cdot) \) is the derivative of \( g(\cdot) \), and \( \mu \) a (small) positive step size. A suitable choice for the initial coefficient set is \( f[0] := \{1, 0, \ldots, 0\} \). The steady-state coefficient set is denoted as \( f_\infty := \lim_{k \to \infty} f[k] \). Convergence is obtained after a few thousand iterations.

A drawback of this particular adaptation algorithm is the fact that \( g(\cdot) \) and hence \( g(\cdot) \) has to be known a priori. Since \( g(\cdot) \) is well-behaved, it can easily be approximated, however. At least one alternative technique exists, where \( g(\cdot) \) does not have to be calculated [11].

Predistortion can only compensate the smooth nonlinearity before the saturation point. The bit error performance in conjunction with predistortion can not be better than that of a linear transmission scheme in conjunction with a hard limiter, because the predistorter can not invert the clipping effect. Therefore, we apply an additional nonlinear detector in order to improve the bit error performance further. The proposed nonlinear detector is a simplified version of the maximum-likelihood receiver. It is particularly useful if the output backoff is small, i.e., if the power efficiency is high.

IV. NONLINEAR DETECTION

In the remainder, the transmitted signal is assumed to be distorted by additive white Gaussian noise. The received samples can be written as

\[ r[k] = s_{HPA}[k] + w[k]. \] (12)

Conceptionally, the maximum-likelihood (ML) receiver for the transmission scheme under investigation computes all possible OFDM signals. These signal hypotheses are passed through the nonlinear function \( g_{NL}(\cdot) \) representing the HPA (eventually including the predistorter). The signal hypothesis causing the smallest squared Euclidean distance with respect to the received samples is finally selected [15]:

\[ \hat{a}_{ML}[n] = \arg \min_{\tilde{a}[n]} \left\{ || r[n] - g_{NL}(\text{IDFT}_N(\tilde{a}[n])) ||^2 \right\}. \] (13)

Unfortunately, the computational complexity of the ML receiver is \( O(M^N) \), where \( M \) is the cardinality of the symbol alphabet and \( N \) the number of subcarriers. Even for a moderate number of subcarriers, the computational complexity is prohibitive.

This motivates us to derive a reduced-complexity receiver, providing an adjustable trade-off between complexity and performance. The simplest version corresponds to the conventional OFDM receiver ignoring nonlinear distortions, whereas the most complex version corresponds to the ML receiver, assuming that the nonlinearity is given.

The following two effects motivate the receiver structure under investigation:

- In the presence of severe nonlinear distortions, some subcarriers are more distorted than others, even in the absence of additive noise.
- In the case of non-binary data, it may happen that even for the same subcarrier some decisions are reliable, whereas other decisions are unreliable. For the example of QFSK, the inphase component of the received sample (after DFT) of a certain subcarrier may be close to the decision threshold, whereas the quadrature component of the same subcarrier may be more reliable.

For these reasons, we propose to identify those decisions, which are close to the corresponding decision threshold. The proposed reduced-state symbol detector (RSSD) differs from the ML receiver in the fact that only hypotheses for the “weakest” decisions (i.e., decisions near the corresponding decision threshold) are evaluated. The nonlinear detector is shown in the lower part of Fig. 1. Since a memoryless nonlinearity is assumed, the computations can be done on an OFDM symbol basis. The RSSD consists of the following steps:

1) Firstly, conventional detection is performed:

\[ y[n] = \text{DFT}_N(r[n]), \]
\[ \hat{a}[n] = \text{dec}(y[n]). \] (14)

where \( y[n] := [y_0[n], \ldots, y_N[n], \ldots, y_{N-1}[n]] \) are soft decisions, \( \hat{a}[n] := [\hat{a}_0[n], \ldots, \hat{a}_N[n], \ldots, \hat{a}_{N-1}[n]] \) are hard decisions, and \( \text{dec} \{ \} \) defines the decision thresholds. According to the principle of set-partitioning, each data symbol \( \hat{a}_n[n] \) can be decomposed into \( \log_2(M) \) bits.

2) Given the soft decisions \( y[n] \), the \( N \) bit decisions with the smallest squared Euclidean distance with respect to the decision thresholds of the conventional OFDM receiver are selected. The design parameter \( S \) may be any integer over the range \( 0 \leq S \leq N \cdot \log_2(M) \).

The corresponding set of subcarrier indices \( \eta \) is denoted by \( S \). (In the case of non-binary symbol alphabets, the \( \log_2(M) \) “weakest” decisions may occur at different subcarriers as mentioned above.)

3) Starting off from the hard decisions \( \hat{a}[n] \), we define \( H := 2^S \) new vectors \( \tilde{a}[n] := [\tilde{a}_0[n], \ldots, \tilde{a}_N[n], \ldots, \tilde{a}_{N-1}[n]] \), which are obtained by replacing the “weakest” bit decisions of all data symbols contained in \( S \) by all possible hypotheses:

\[ \tilde{a}_n[n] := \begin{cases} \hat{a}_n[n] & \text{if } \eta \not\in S \\ \text{hypothesis} & \text{if } \eta \in S. \end{cases} \] (15)

In order to obtain the hypothesis, the corresponding data symbol is decomposed into its \( \log_2(M) \) bits. The number of hypotheses, \( H = 2^S \), depends on the number of selected decisions, \( S \), but not on the cardinality of the symbol alphabet, \( M \). For \( S = 0 \), the conventional OFDM receiver is obtained. For \( S = N \cdot \log_2(M) \), the maximum-likelihood detector is obtained.
4) Reduced-state symbol detection is performed by selecting the vector \( \hat{a}[n] \) causing the smallest squared Euclidean distance with respect to the received OFDM samples given the constraint that all \( \eta \in \mathbb{S} \):

\[
\hat{a}_{\text{RSSD}}[n] = \arg \min_{\hat{a}[n], \eta \in \mathbb{S}} \left\{ \| r[n] - g_{\text{NL}}(\text{IDFT}_{N}\{\hat{a}[n]\}) \|^{2} \right\},
\]

where \( g_{\text{NL}}(.) \) is a nonlinear function which represents either the HPA alone or the HPA in conjunction with a predistorter. In the latter case, \( g_{\text{NL}}(.) \) can be well approximated by a hard limiter, as shown next.

V. NUMERICAL RESULTS

The numerical results presented in this section are based on the following set-up: In the transmitter, an OFDM signal with \( N = 128, 256, \) or 512 subcarriers is generated. All subcarriers are QPSK modulated. A solid-state power amplifier according to (3) with \( p = 2 \) is used. A predistorter according to (9) of order \( L = 5 \) with three non-zero coefficients \( f_1, f_3, \) and \( f_5 \) is applied optionally. Only the steady-state coefficient set \( f_\infty \) is considered. The channel model under consideration is an AWGN channel. The signal-to-noise ratio per information bit, \( E_b/N_0 \), shown in the following figures does not include the output back-off. At the receiver, the proposed RSSD with \( H = 4 \) or \( H = 16 \) hypotheses is applied. As a benchmark, the performance of the conventional OFDM receiver (which ignores nonlinear distortions) is shown as well.

In Fig. 2, the influence of the predistorter (PD) on the raw bit error rate (BER) is demonstrated for the conventional OFDM receiver. The important result is that the performance of a hard limiter is well approximated by means of predistortion. The predistorter is able to reduce the out-of-band radiation, but the BER performance gap with respect to the linear case is still significant.

![Fig. 2. Raw BER performance of a QPSK/OFDM system with/without nonlinear distortion (\( N = 128 \) subcarriers, \( OBO = 3.16 \) dB, conventional receiver).](image)

The performance gap can be closed by means of a predistorter at the transmitter side in conjunction with the proposed reduced-state symbol detector at the receiver side, as shown in Fig. 3. In the RSSD, for \( g_{\text{NL}}(.) \) the overall transmitter-side nonlinearity is assumed. It can again be seen that due to predistortion the overall transmitter-side nonlinearity is well approximated by a hard limiter. For RSSD with just \( H = 4 \) hypotheses, the gap with respect to the linear case is about 0.5 dB. For \( H = 16 \) hypotheses, the nonlinear distortion introduced by the SSPA is fully compensated at BERs of less than \( 10^{-2} \). Note that a maximum-likelihood receiver would have to compute \( H = 2^S = 2^{256} \) hypotheses in this example. As a reference, the performance for the conventional OFDM receiver is illustrated as well, which is much worse.

![Fig. 3. Raw BER performance of a QPSK/OFDM system with/without nonlinear distortion (\( N = 128 \) subcarriers, \( OBO = 3.16 \) dB, conventional receiver and reduced-state symbol detector).](image)

In Fig. 4, the influence of a different number of subcarriers with/without predistortion is studied. At the receiver, an RSSD with \( H = 4 \) hypotheses is used. For \( g_{\text{NL}}(.) \) the overall transmitter-side nonlinearity is assumed. As expected, the performance degrades with an increasing number of subcarriers, independently whether a predistorter is used or not. The most important result of Fig. 4 is the observation that the predistorter has a positive influence on the performance of the RSSD, particularly if the number of hypotheses (and hence the computational complexity) is small.

In the previous two figures, no modeling errors are considered in the RSSD. In order to study the influence of modeling errors, Fig. 5 displays the bit error performance of an RSSD with complete knowledge and partial knowledge of the transmitter-side nonlinearity, respectively. At the transmitter, no predistortion is applied. Complete knowledge means that an SSPA with known OBO is available in the RSSD. By partial knowledge we mean that a hard limiter with known OBO is assumed in the RSSD. Two different output back-offs and two different number of hypotheses are considered. It can be noticed that the performance difference for complete and partial knowledge is negligible. In case of predistortion there would be even less degradation, since the modeling error would be smaller. This demonstrates the robustness of the proposed nonlinear detector.
VI. CONCLUSIONS

Uncoded OFDM is known to be vulnerable with respect to nonlinear distortions. In this paper, two compensation techniques are used simultaneously: Memoryless predistortion at the transmitter and reduced-state nonlinear detection at the receiver. The polynomial-based predistorter reduces the out-of-band radiation. Moreover, in conjunction with reduced-state nonlinear detection, the predistorter improves the computational complexity and is advantageous when only incomplete information about the nonlinearity is available at the receiver. The proposed nonlinear detector is derived from the maximum-likelihood detector. Its performance/complexity trade-off is adjustable.

Numerical results are provided for an uncoded QPSK/OFDM system in conjunction with a memoryless solid-state power amplifier. Although the solid-state power amplifier is assumed to operate at a low power back-off, the raw bit error rate is shown to be close to the linear case if the number of hypotheses is sufficiently large. The proposed techniques are suitable for low-cost transceivers. The influence with respect to modeling errors is small. Further improvements are anticipated, if additional peak-to-average power reduction techniques are applied in the transmitter.

REFERENCES