Optimized Bit Rate Allocation for Iterative Source-Channel Decoding and its Extension towards Multi-Mode Transmission

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Abstract—Recent advancements in iterative source-channel decoding (ISCD) focus on the optimization of a single processing step of source coding (like the index assignment) or of channel coding. In this paper, we propose a new design guideline for a more general optimization of the overall ISCD scheme. In particular, we discuss how to allocate a given gross bit rate to source and channel coding.

The new bit rate allocation reveals (at least) two benefits: Firstly, the proposed ISCD scheme using a rate $r^s = 1$ channel encoder yields improved error concealing/correcting capabilities if compared to the prior ISCD approaches. Secondly, the new scheme can easily be extended to a flexible multi-mode system. While all modes use the same $r^s = 1$ channel code, the joint adaptation of quantizer resolution and index assignment allows to manage the trade-off between quantization noise and error robustness.

I. INTRODUCTION

In the past decade, the TURBO-principle has attracted high interest. At first, it had been introduced as a remarkably powerful and computationally efficient decoding technique for channel codes [1, 2]. The key element is the iterative exchange of extrinsic information between two (or more) concatenated channel decoders. The iterative refinements of the extrinsic information make it possible to approach Shannon’s information theoretic performance bound with reasonable computational complexity.

In recent years, the TURBO-principle became also very popular for other subtasks of signal transmission such as joint source-channel decoding by iterative source-channel decoding (ISCD). In the literature, there exist two different interpretations of iterative source-channel decoding (ISCD). On the one hand, ISCD denotes a system where explicit redundancy due to channel encoding as well as implicit redundancy in terms of a non-uniform distribution or correlation of the source encoded data are utilized iteratively (e.g., [3–5]). In this case, ISCD enhances the error robustness of digital speech, audio, and video communications. On the other hand, the term iterative source-channel decoding marks an iterative evaluation of variable-length source codes and channel codes (e.g., [6]). In the latter case, ISCD serves for a proper segmentation of the reconstructed bit stream after channel decoding into bit patterns of specific length. In this contribution, we restrict our considerations to digital transmission schemes belonging to the first interpretation.

This paper is structured as follows. In Section II we briefly review the literature to ISCD, and in Section III we summarize the ISCD algorithm. In Section IV we introduce

- a new bit rate allocation scheme for source and channel coding
- a new extension towards a flexible multi-mode ISCD system.

This multi-mode ISCD system comprises several source encoder modes. The error correcting/concealing capabilities are demonstrated by simulation in Section V.

II. REVIEW OF THE LITERATURE TO ISCD

In literature, there exist several examples how to jointly utilize explicit redundancy due to channel coding as well as implicit redundancy in terms of a non-uniform distribution or correlation of the source encoded data, e.g. [7–9]. The most recent approach is known as iterative source-channel decoding (ISCD) whose basic concept has been derived independently, but at the same time by N. Götz [3], by T. Hindelang et al. [4], and by M. Adrat et al. [5]. A detailed analysis of ISCD schemes and the precise formulas how to quantify extrinsic information from the natural residual source redundancy can be found in [10–12].

The simulation results presented in [3–5, 10–12] reveal that the error correcting/concealing capabilities of ISCD schemes are always superior to those of the appropriate non-iterative schemes. However, the number of profitable iterations is limited to small values (usually 2 or 3 iterations) if applied to conventionally designed ISCD schemes (e.g., using recursive systematic convolutional (RSC) codes). The fast convergence behavior has been analysed in [12–14] using the well-known EXIT-chart (extrinsic information transfer-chart) technique of S. ten Brink [15].

The error correcting/concealing capabilities of ISCD schemes can be improved if source and channel encoder are especially designed in view of the iterative evaluation at the receiver. For instance, significant improvements have been achieved in [16] by an optimization of the index assignment.
Utilizing the EXIT-chart analysis to optimize the index assignment yields even slightly higher gains in robustness [17]. Furthermore, from the ISCD system design according to the EXIT-charts it turns out [17] that it is beneficial to use recursive non-systematic convolutional (RNSC) codes instead of RSC codes. Using the optimized index assignment as well as the optimized RNSC codes permits substantial quality improvements by more than 10 iterations [17].

If the code rate $r$ of the RNSC code is raised to $r^* = 1$, then the channel encoder can be considered as a smearing filter. In [12,18,19], ISCD schemes with $r^* = 1$ have also been called TURBO error concealment.

In all the above cited papers the benefits of ISCD schemes over non-iterative source-channel decoding schemes have been demonstrated for generic source models. In addition, ISCD has also successfully been applied to speech signals [20, 21] and images [22].

### III. Iterative Source-Channel Decoding

Figure 1 illustrates a digital transmission scheme with an iterative source-channel decoder (ISCD) at the receiving end. At first, a source encoder extracts a frame $u_{\mu,\tau}$ of $M$ real-valued, but time discrete source codec parameters $u_{\mu,\tau}$ from a short segment of the input speech, audio, or video signal. The index $\mu = 1, \ldots, M$ specifies the position of $u_{\mu,\tau}$ in $z_{\tau}$, and $\tau$ denotes the time-stamp of the frame. In practice, the source codec parameters $u_{\mu,\tau}$ exhibit considerable natural residual redundancy such as a non-uniform parameter distribution or correlation. Each $u_{\mu,\tau}$ is individually quantized to one out of $Q$ quantizer reproduction levels $\tilde{x}_{\mu,\tau}(\kappa)$. The reproduction levels themselves are invariant with respect to $\tau$ and the whole set is given by $\tilde{U}_\mu$. To each index $q$ of $\tilde{u}_{\mu,\tau}^{(q)}$ specified at time $\tau$ a unique bit pattern $x_{\mu,\tau}$ is assigned. Each bit pattern $x_{\mu,\tau}$ consists of $K_\mu$ data bits $x_{\mu,\tau}(\kappa) \in \{+1, -1\}$ with $\kappa = 1, \ldots, K_\mu$. For convenience, we assume in the following that the codebooks $\tilde{U}_\mu$ of $\tilde{u}_{\mu,\tau}^{(q)}$ as well as the lengths $K_\mu$ of $x_{\mu,\tau}$ are the same for all source codec parameters $u_{\mu,\tau}$ in the frame $u_{\tau}$, i.e., $\tilde{U}_\mu = U$ and $K_\mu = K$ for all $\mu = 1, \ldots, M$. The size $Q$ of $U$ is upper bounded by $Q \leq 2^K$. The complete data stream representing the frame $u_{\tau}$ is denoted by $z_{\tau}$.

A bit interleaver $\Phi$ scrambles the data stream $z_{\tau}$ in a deterministic manner. Interleaving can also be realized on several consecutive frames, but if doing so, a delay will be introduced which might be unacceptable for real-time duplex communications. However, the interleaver has to be designed such that, at the receiving end, independent reliability gains can be extracted from source and from channel decoding. As the reliability gain of source decoding results from an evaluation of the natural residual source redundancy in the parameters $u_{\mu,\tau}$, independence will be ensured when all the mutually dependent bits $x_{\mu,\tau}(\kappa)$ with $\kappa = 1, \ldots, K$ are spread over the interleaved data stream $\tilde{x}_{\tau}$ as far as possible.

In order to improve the error resistance in the initial iterations some other initializations for the extrinsic information of source decoding are discussed in [10–12]. However, the best possible error correcting/concealing capabilities after the final iteration will not be enhanced by these improved initial settings for $L^{\text{SD}}_{\tilde{x}_{\mu,\tau}}(x_{\mu,\tau}(\kappa))$. 

![Fig. 1. Digital transmission scheme with an iterative source-channel decoder (ISCD) at the receiving end](image-url)
2) Decode Channel Code:
- quantify the extrinsic information \( L_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) of soft-output channel decoding

The channel decoder (CD) in Figure 1 restores extrinsic information from the artificial redundancy which has explicitly been introduced by channel encoding. For this purpose, the channel-related \( L \)-values according to Eq. (1) as well as (in iterations \( i > 0 \)) the extrinsic information \( L_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) of source decoding are evaluated. Since the determination rules for \( L_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) are well-known from conventional TURBO-channel decoding, we refer the reader to the literature, e.g., [2].

3) Decode Source Code:
- quantify the extrinsic information \( L_{SD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) of source decoding, i.e., utilize the natural residual source redundancy

The source decoder (SD) determines extrinsic information mainly from the bit patterns \( x_{\mu,\tau} \) after source encoding. Such residual redundancy appears on parameter-level, e.g., in terms of a non-uniform distribution \( P(x_{\mu,\tau}) \), in terms of correlation, or in any other possible mutual dependency in time \( \tau \). The latter terms of residual redundancy are usually approximated by a first order Markov chain, i.e., by the conditional probability distribution \( P(x_{\mu,\tau} | x_{\mu,\tau-1}) \). These source statistics can usually be measured once in advance for a representative signal data base.

The technique how to combine this a priori knowledge \( P(x_{\mu,\tau} | x_{\mu,\tau-1}) \) on parameter-level with the soft-input values \( L_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) on bit-level is also well-known in the literature. The algorithm how to compute the extrinsic \( L \)-value \( L_{SD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \) has been detailed in, e.g., [3–5, 10–14, 16–19].

4) Determine a posteriori \( L \)-value \( L(x_{\mu,\tau}(\kappa) | z_{\tau}) \):
- quantify the extrinsic \( L \)-values by summation of both extrinsic \( L \)-values \( \sum_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) + L_{CD}^{[\text{ex}]}(x_{\mu,\tau}(\kappa)) \)

The summation of both terms of extrinsic information yields a posteriori \( L \)-values \( L(x_{\mu,\tau}(\kappa) | z_{\tau}) \) for single data bits \( x_{\mu,\tau} \) given the (entire history of) observations \( z_{\tau} \). The history is implicitly been taken into account throughout the utilization of the natural residual redundancy. The a posteriori information serves as input for the final parameter estimation process (see [3–5, 10–14, 16–19]). Usually, the minimum mean squared error (MMSE) is used as fidelity criterion for parameter estimation. With this, the mean squared error between the originally extracted source codec parameter \( u_{\mu,\tau} \) and its reconstruction \( \hat{u}_{\mu,\tau} \) is minimized.

IV. NEW BIT RATE ALLOCATION

Recently, several improvements of the error correcting/concealing capability of ISCD have been proposed. Most of these advancements focus on the optimization of a single processing step at the transmitter. For instance, in [16, 17] concepts to optimize the index assignment have been introduced, and in [17] it was demonstrated that recursive non-systematic convolutional (RNSC) codes are superior to the appropriate systematic counterparts.

In the following, we will incorporate the results of [16, 17] in a new, more general design guideline for ISCD schemes. In this respect, we analyse how to optimally allocate a given channel gross bit rate (number of bits per frame) to source and channel coding.

The gross bit rate is given by the number \( N \) of code bits \( y(\lambda) \) which represent a frame \( y_{\mu} \) of source code parameters \( u_{\mu,\tau} \) at specific time \( \tau \). In the following, the numbers of \( N \) and \( M \) shall be fixed for all possible allocations under consideration. Thus, from the relation \( N = (M \cdot K + J)/r \) it follows that we are free to dimension the code rate \( r \) and the memory \( J \) of the RNSC code as well as the number \( K \) of data bits per bit pattern \( x_{\mu,\tau} \). Notice, a higher number \( K \) of data bits needs not to come along with a higher number \( Q \) of quantizer reproduction levels because \( Q < 2^K \). Moreover, the impact of the RNSC code memory \( J \) on the gross bit rate becomes negligibly small for large numbers of \( M \cdot K \). Thus, in the following we restrict our considerations to a proper dimensioning of the code rate \( r \), of the number \( K \) of bits per \( x_{\mu,\tau} \) and of the number \( Q \) of quantizer reproduction levels.

In conventional designs to ISCD [3–5, 10–14, 16, 17] the code rate usually amounts to \( r = 1/2 \). Half of the gross bit rate is spent for error protection by channel coding and the other half is used for quantization and index assignment. Given the product of \( r \cdot N \) as well as fixed values for \( J \) and \( M \), the number of bits per \( x_{\mu,\tau} \) results in \( K = (r \cdot N - J)/M \). Finally, it is most common to use \( Q = 2^K \) in order to reduce the quantization noise for any given \( K \) as much as possible.

As an alternative, inspired by the TURBO error concealment schemes [12, 18, 19], we propose to use channel codes with code rates near \( r^* = 1 \). The search for appropriate channel codes with \( r^* = 1 \) for ISCD can still follow the design guidelines given in [17]. Proposing channel codes of \( r^* = 1 \) agrees with recent considerations of A. Ashikhmin et al. [23] who have analytically shown that (in case of a binary erasure channel) the inner component(s) of a serially concatenated TURBO scheme must be of \( r^* = 1 \). Otherwise, if this code rate is \( r < 1 \), the overall TURBO scheme will suffer from an inherent capacity loss. Notice, the analytical considerations of B. Hochwald and K. Zeger [24] to the optimal bit rate allocation to (lossy) source and channel coding do not hold in case of an iterative TURBO-like source-channel decoding scheme.

As a consequence of \( r^* = 1 \), a higher bit budget is available for source coding. This enables us to increase either the number \( Q \) of quantizer reproduction levels, or the number \( K \) of bits per \( x_{\mu,\tau} \), or even both1. A higher value for \( Q \) reduces the quantization noise, and a higher value for \( K \) will

1Note, if \( K \) is increased while keeping \( Q \) constant, then the index assignment can be considered as a (non-linear) block code which introduces some artificial redundancy (e.g. \( K = 4 \) but \( Q = 10 < 2^4 \)).
introduce artificial redundancy to the bit patterns $x_{\mu,\tau}$. This redundancy helps to improve the error robustness. The search for an appropriate index assignment for ISCD can also still follow the design guidelines given in [17].

In Section V, we will perform two experiments. On the one hand, we will assign the complete spare bit budget to the index assignment. Therewith, the error robustness in noisy channel conditions is expected to be improved while the baseline performance in case of error-free channels remains unchanged (due to the same quantizer). On the other hand, the close relation between the number $Q$ of quantizer reproduction levels and the number $K$ of bits per $x_{\mu,\tau}$ is exploited to construct a multi-mode system. This multi-mode system is based on the fact that several realizations of $Q \leq 2^K$ are possible for a given $K$. The lower the number of $Q$ is, the higher the error robustness can be. Of course, at the same time higher quantization noise has to be accepted in good channel conditions. Thus, adaptive mode-switching according to the channel condition will be beneficial.

Notice, in contrast to prior multi-mode joint source-channel coding standards like the GSM-AMR (global system for mobile communications - adaptive multi-rate codec) the channel coding component with $\mu, \tau$ needs not to be adapted in case of a dynamic dimensioning of $Q$. The number $M \cdot K$ of input data bits $x_{\mu,\tau}(n)$ to channel encoding is the same for all realizations of $Q$.

V. SIMULATION RESULTS

To demonstrate the improved error correcting/concealing capabilities of the proposed ISCD schemes, $M = 250$ source codec parameters $u_{\mu,\tau}$ are modelled by $1^st$ order Gauss-Markov processes with correlation $\rho = 0.7$. Such or even higher terms of correlation can be found in real-world applications. The parameters are individually quantized using a $Q$-level Lloyd-Max quantizer. In the first experiment, the number of levels is fixed to $Q = 8$. The index assignment is optimized in view of ISCD according to the design guidelines given in [17]: in the conventional approach to ISCD with $K = 3$ and $r = 1/2$ the optimized index assignment reads $q = \{0, \ldots, 7\} \rightarrow \{3, 4, 7, 2, 1, 5, 6, 0\}_{10}$ in case of the proposed ISCD scheme with $K^* = 6$ and $r^* = 1$ the optimized mapping is $q^* = \{0, \ldots, 7\}_{10} \rightarrow \{41, 3, 16, 51, 63, 34, 6, 8\}_{10}$.

Bit interleaving $\Phi$ is realized by pseudo-random interleavers of size $M \cdot K = 750$ resp. $M \cdot K^* = 1500$. For channel encoding terminated, memory $J = 3$ RSC codes with generator polynomials $G = (1 + D + D^3, 1 + D + D^2 + D^3)$ resp. $G^* = (1 + D + D^2 + D^3, 1 + D^2 + D^3 + D^4)$ are used. Thus, the gross bit rate is $N = 1506$ resp. $N^* = 1503$.

The simulation results in the left part of Figure 2 show the parameter signal-to-noise ratio (SNR) of the originally extracted source codec parameter $u_{\mu,\tau}$ and the corresponding estimate $\hat{u}_{\mu,\tau}$ as a function of the channel quality $E_s/N_0$ (note, even if $r \ll r^*$ we can use the $E_s/N_0$ for a fair comparison because $N \approx N^*$).

The dashed curves depict the results of conventional ISCD using $r = \frac{750}{1500} \approx 1/2$, and the solid curves depict the respective results of the proposed approach with $r^* = \frac{1500}{1503} \approx 1$. While the new approach is inferior in the initial iteration (curves labelled Iteration 1), it becomes superior with higher numbers of iteration (e.g. Iteration 10). The baseline parameter SNR of 14.6 dB is preserved down to significantly lower $E_s/N_0$ of $\Delta E_s/N_0 \approx 2.61$ dB.

Each EXIT-chart [12–15,17] in the right part of Figure 2 comprises two EXIT characteristics, one for channel coding and one for source coding. Both curves describe an envelope for the attainable region of the decoding trajectory (see the step-curve). A higher error robustness is achievable if the intersection of the EXIT characteristics gets closer to the upper right corner. The new ISCD scheme (right subplot) is obviously better qualified to fulfill this constraint. Moreover, the number of steps of the decoding trajectory determines the number of reasonable iterations. For a channel condition of $E_s/N_0 = -2.5$ dB about 3 iterations are necessary for the conventional ISCD approach (left subplot). Up to 25 iterations make sense for the new ISCD scheme. For additional details on EXIT-charts in ISCD we refer the reader to [12–15,17,25].

In the second experiment, we simulate a multi-mode system with several different values for $Q$ given a fixed value for $K$. The simulation results for $K^* = 6$ and $G^* = (1 + D + D^2 + D^3, 1 + D^2 + D^3 + D^4)$ resp. $(1 + D + D^2 + D^3)$ are shown in Figure 3. The curves for $Q = 8$ are taken from Figure 2.

![Figure 2](image-url)  
**Fig. 2.** Parameter SNR (left) and EXIT-charts for $E_s/N_0 = -2.5$ dB (right) of the conventional resp. proposed approach to ISCD
Obviously, selecting a specific number $Q$ of quantizer reproduction levels in combination with an appropriately optimized index assignment [17] allows to manage the trade-off between error robustness and quantization noise. The particular benefits of each mode can easily be exploited by an adaptive mode-switching. Remember, the overall multi-mode ISCD scheme is characterized by the fact that the channel code $G^*$ is the same for all modes.

We have obtained similar results for various other parameter settings of correlation $\rho$, number of codec parameters $M$, bit numbers $K$ resp. $K^*$ and channel codes $G$ resp. $G^*$. However, the appropriateness of the new bit rate allocation for the application to a real-world speech, audio, or video codec remains to be shown. Moreover, a comparison of the novel multi-mode transmission scheme with known standardized multi-mode schemes like the GSM-AMR remains to be done.

VI. CONCLUSIONS

In this paper, we presented a new, optimized bit rate allocation for iterative source-channel decoding. Instead of using an inner channel code of code rate $r = 1/2$, we propose to use codes with $r^* = 1$. Our proposal agrees with recent research findings to serially concatenated channel codes. The spare bit budget can either be used to increase the error robustness (due to artifical redundancy in the index assignment) or to decrease quantization noise (due to a higher quantizer resolution). The new ISCD scheme provides substantially improved error correcting/concealing capabilities if compared to all formerly known approaches. Moreover, it constitutes the basis for a very efficient multi-mode system.

REFERENCES


