Reduced Link Correlation Using Multiple Antennas

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Abstract - **High-frequency backscatter radio systems operate in the backscatter channel, a channel whose envelope probability density function (PDF) and bit-error-rate (BER) performance are strongly affected by the relationship between small-scale fading in the reader-to-tag and tag-to-reader propagation links – i.e., link correlation. This paper shows that backscatter radio systems using co-located reader transmitter and receiver antennas and a single RF transponder antenna can have the highest link correlation which results in the worst communication performance. It is shown that using separate reader antennas and multiple RF transponder antennas will decrease the detrimental effects of link correlation. Results show that if the correlation of the envelopes of the reader-to-tag and tag-to-reader propagation links is below approximately 0.6, near maximum backscatter communication performance can be achieved.**

I. INTRODUCTION

Backscatter radio has been the subject of intensive research because of its potential applications in radio frequency identification (RFID), backscatter sensors, and passive data storage retrieval. Designers of backscatter radio systems face a myriad of challenges in order to design backscatter transponders (RF tags) that boast both long range and reliable communication. Though RF tags commonly operate in a line-of-sight (LOS) channel, their range and reliability are hindered by small-scale fading caused by indoor operation, a cluttered reader environment, and the inhomogeneous nature of the tagged objects. Research has shown that the statistical properties of the backscatter channel are significantly different than those found in a one-way channel and result in much deeper fades [1]. In an effort to reduce fading in the backscatter channel, Ingram, et. al., [2] were the first to propose using multiple reader transmitter and receiver antennas along with RF tags that use multiple antennas for transmit diversity and spatial multiplexing. Kim, et. al., [1] reported the first measured cumulative density function (CDF) of the backscatter channel consisting of a single reader transmitter, reader receiver, and RF tag antenna. Kim, et. al., found that the measured CDF closely matched that of the product of two independent Rician distributions. It has also been shown that the envelope probability density function (PDF) of the received backscatter signal at the nth reader receiver antenna can be improved if multiple antennas are used on each RF tag to modulate backscatter [3]. While using multiple RF tag antennas does decrease small-scale fading by improving the received envelope PDF, the severity of fading can also be reduced by minimizing the statistical relationship of fading in the forward and backscatter links of the backscatter channel. In a conventional one-way channel, spatial fading correlation will hinder communication and limit available diversity gains. In a pinhole channel, such as the backscatter channel, the relationship of fading between the links – link correlation [4] – can have the same effect. This can occur even if small-scale fading in each link is uncorrelated. Previous work on realistic pinhole channels has focused on situations in which the two links of the pinhole channel are likely dissimilar (e.g. outdoor propagation [5] or amplify-and-forward channels [6]) justifying the assumption of independent links. In many backscatter radio systems, however, reader transmitter and receiver antennas may be closely spaced or even co-located resulting in high link correlation. This paper demonstrates that the distribution of the nth received signal improves as link cor-

relation is reduced resulting in a decreased bit-error-rate (BER). After briefly describing the general dyadic backscatter channel and envelope PDF in Section II, Section III shows the effect of link correlation on the envelope fading distribution and BER.

II. BACKGROUND

2.1 The Dyadic Backscatter Channel

The $M \times L \times N$ dyadic backscatter channel [3], the most general backscatter channel, consists of M transmitter, L RF tag, and N receiver antennas, shown in Figure 1. This channel is a pinhole chan-
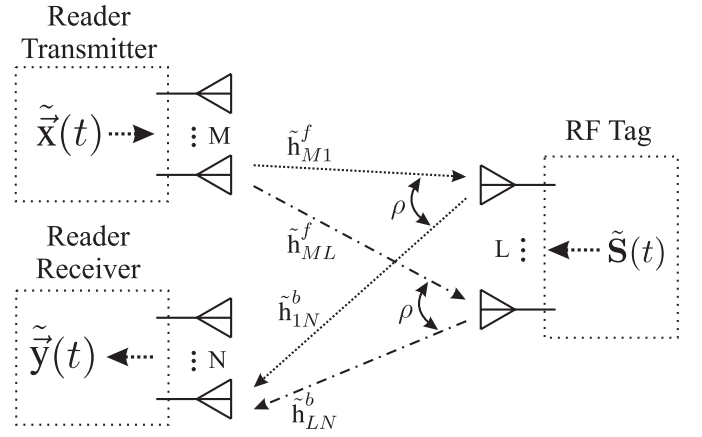

FIGURE 1 - THE GENERAL $M \times L \times N$ DYADIC BACKSCATTER CHANNEL WITH M READER TRANSMITTER ANTENNAS, L RF TAG ANTENNAS, AND N READER RECEIVER ANTENNAS.

nel [7] which allows the overall channel to be divided into a forward and backscatter link. This is shown in Figure 1 where $\tilde{h}_{ml}^{f}$ denotes the propagation path from the mth reader transmitter antenna to the lth RF tag antenna and $\tilde{h}_{ln}^{b}$ denotes the path from the lth RF tag antenna to the nth reader receiver antenna. The collection of these paths compose the forward and backscatter links, $\tilde{\mathbf{H}}^{f}$ (an $M \times L$ baseband impulse response matrix) and $\tilde{\mathbf{H}}^{b}$ (an $L \times N$ baseband impulse response matrix), respectively. In this channel, the time-varying modulation applied to backscatter by the RF tags is governed by $\tilde{\mathbf{S}}(t)$, the narrowband $L \times L$ signaling matrix, while the signals transmitted from the reader are described by $\tilde{\vec{x}}(t)$, an $M \times 1$ vector.

2.2 The Envelope PDF

In this paper, the envelope PDF of the signal received at the nth reader receiver antenna through a non-line-of-sight (NLOS) backscatter channel will be developed. Consequently, the forward and backscatter links are assumed to experience uncorrelated Rayleigh fading. While many backscatter systems operate in the LOS backscatter channel, described by a Rician distribution, the NLOS backscatter channel is worthy of investigation because it represents the extreme, worst-case fading scenario for a backscatter system. Since both the forward and backscatter links experience uncorrelated Rayleigh fading, the elements of $\tilde{\mathbf{H}}^{b}$ and $\tilde{\mathbf{H}}^{f}$ can be modeled as independent, identically distributed (i.i.d.), zero-mean, complex Gaussian random

variables. Though the elements of each link are statistically independent, elements of $\tilde{\mathbf{H}}^b$ and $\tilde{\mathbf{H}}^f$ that are associated with a common RF tag antenna are correlated. This is shown in Figure 1 where the level of correlation is given by ρ – the normalized link correlation coefficient which describes the level of correlation between the real components of $\tilde{\mathbf{H}}^f$ and $\tilde{\mathbf{H}}^b$ and the imaginary components of $\tilde{\mathbf{H}}^f$ and $\tilde{\mathbf{H}}^b$ ($-1 \leq \rho \leq 1$). The real and imaginary components of each link are statistically independent. The physical significance of ρ will be explored in Section III. Returning to the channel matrix elements, each element of the overall channel matrix, $\tilde{\mathbf{H}}$, is,

$$\tilde{h}_{ij} = \tilde{h}^b \tilde{h}^f \tag{1}$$

where $\tilde{h}^b = \left(w_b + jv_b\right)$, $\tilde{h}^f = \left(w_f + jv_f\right)$, and $w_{f,b}$ and $v_{f,b}$ are $\sim \mathcal{N}\left(0, \sigma_{f,b}^2/2\right)$. The signal received at the nth receiver antenna has the distribution of the sum of ML complex Gaussian products,

$$\tilde{\mathbf{y}}_n(t) \sim \sum_{j=1}^{L} \underbrace{(\tilde{h}_{1j}^f + \ldots + \tilde{h}_{Mj}^f)}_{\tilde{g}_j^f} \tilde{h}_{jn}^b. \tag{2}$$

Since, in this model of the backscatter channel, matrix elements (or propagation paths) that originate or terminate on a common RF tag antenna are correlated, only L of the products in (2) are statistically independent. The M forward link terms in (2) are i.i.d., complex Gaussian random variables which can be represented by a single, complex Gaussian random variable, $\tilde{g}_j^f$, with variance $M\sigma_f^2$. With this view, the signal received at the nth reader receiver antenna has the distribution of the sum of L i.i.d., dependent, complex Gaussian products. Since the envelope of the nth received signal is of concern, the distribution of the nth received envelope is that of the sum of L i.i.d., dependent Rayleigh random variable products. From Simon [8], the PDF of the product of two dependent Rayleigh random variables is[1]

$$f_\alpha\left(\alpha, \rho\right) = \frac{4\alpha\left(1 - |\rho|^2\right)}{\sigma_b^2 \sigma_f^2 M \gamma^2}$$
$$\times \ \text{I}_0\left(\frac{2\alpha|\rho|}{\sigma_b \sigma_f \sqrt{M}\gamma}\right) \text{K}_0\left(\frac{2\alpha}{\sigma_b \sigma_f \sqrt{M}\gamma}\right), \tag{3}$$

where α is the envelope of the signal, $\gamma = 1 - \rho^2$, I_0 is a zeroth order modified bessel function of first kind, and K_0 is a zeroth order modified bessel function of the second kind. The PDF of the sum of L independent random variables can be found from the multiplication of their characteristic functions (CF). Applying the Hankel transform to (3) yields its CF which, raised to the Lth power, is the CF of the signal received at the nth receiver antenna [3], given as follows:

$$\Phi(\nu; \rho) = \left[\frac{\sigma_b^4 \sigma_f^4 M^2}{16} \frac{\gamma^4}{\left(1 - |\rho|^2\right)^2} \right.$$
$$\left. \times \left(\nu^2 + \frac{4(|\rho| - 1)^2}{\sigma_b^2 \sigma_f^2 M \gamma^2}\right)\left(\nu^2 + \frac{4(|\rho| + 1)^2}{\sigma_b^2 \sigma_f^2 M \gamma^2}\right) \right]^{-L/2}, \tag{4}$$

where ν is the index of the characteristic function. The PDF of the signal envelope received at the nth reader receiver antenna, $f(\alpha, \rho)$, can be found by applying the inverse-Hankel transform to (4); however, an analytical solution can only be found for the case of independent forward and backscatter links ($\rho = 0$) and fully correlated forward and backscatter links ($\rho = 1$). Fortunately, since these are the two extreme cases of link correlation, they are of primary interest. The PDF for the

case of independent links is [3]

$$f_\alpha(\alpha, 0) = \alpha^L \left(\frac{2}{\sqrt{M}\sigma_b \sigma_f}\right)^{1+L}$$
$$\times \frac{2^{1-L}}{\Gamma(L)} \text{K}_\nu\left(\frac{2\alpha}{\sqrt{M}\sigma_b \sigma_f}\right), \tag{5}$$

where α is the channel envelope, $\Gamma(\cdot)$ is the gamma function, and $\text{K}_\nu(\cdot)$ is a modified bessel function of the second kind with order $\nu = 1 - L$. The PDF for the case of fully correlated links is [3]

$$f_\alpha(\alpha, 1) = \alpha^{L/2} \left(\frac{1}{\sqrt{M}\sigma_b \sigma_f}\right)^{1+L/2}$$
$$\times \frac{2^{1-L/2}}{\Gamma(L/2)} \text{K}_\nu\left(\frac{\alpha}{\sqrt{M}\sigma_b \sigma_f}\right), \tag{6}$$

where the order of the modified bessel function of the second kind is $\nu = 1 - L/2$. For reference, plots of (5) and (6) are given in Figure 2(a) along with the PDF of a conventional one-way Rayleigh fading channel, for comparison. The random variables that correspond with these PDFs have been normalized to unit power (i.e., $\mathcal{E}\{\alpha^2\} = 1$ where $\mathcal{E}\{\cdot\}$ denotes the ensemble average). It can be shown that as $L \to \infty$, (5) and (6) approach a Rayleigh distribution.

2.3 Notes on the Correlation

In this model of the backscatter channel, several assumptions regarding the correlation between the real parts and between the imaginary parts (both denoted by ρ) of the forward and backscatter links have been made. First, propagation paths in the forward or backscatter links are statistically independent. Second, correlation between propagation paths of the forward and backscatter links may occur only for paths that that originate or terminate on a single RF tag antenna. Third, the level of correlation between these links is the same for each path and given by ρ. So, while the envelope CF described by (4) is general in that it holds for any value of M, L, or ρ, care must be taken in the choice of ρ in order to realize a physically meaningful result. Each element of the correlation matrix, $\mathbf{V}$, can be calculated as

$$V_{ij} = \frac{\text{cov}(A_i, A_j)}{\sigma_i \sigma_j} \tag{7}$$

where A_i is the ith element of

$$\vec{A} = [\tilde{h}_{11}^f, \tilde{h}_{21}^f, \cdots, \tilde{h}_{ml}^f, \tilde{h}_{11}^b, \tilde{h}_{21}^b, \cdots, \tilde{h}_{ln}^b]^{\text{T}}$$

(a column vector formed from the entries of the forward and backscatter link matrices), σ_i is the standard deviation of the ith element of $\vec{A}$, and $\text{cov}(x, y)$ is the covariance between the scalars x and y. Since $\mathbf{V}$ is a correlation matrix, it must be positive semi-definite. This constraint places a limit on ρ that is a function of M. In this paper, only the $1 \times L \times 1$ channel is presented which allows the correlation, ρ, to vary between zero and one ($0 \leq \rho \leq 1$) while satisfying the positive semi-definite criteria.

To this point, only the correlation between the real and imaginary components of the links has been discussed. It will prove useful in the following section to discuss the correlation between the envelopes of the forward and backscatter links. To this end, it can be shown that $|\rho|^2 \approx \rho_e$ where ρ_e denotes the correlation between the envelopes of the forward and backscatter links and the argument of ρ is usually assumed to be zero [9, 10]. In the remainder of this paper, results will be presented in terms of link envelope correlation, ρ_e.

III. LINK ENVELOPE CORRELATION EFFECTS
3.1 Distribution Improvement

Figure 2 and Figure 3 plot the PDF and CDF of several backscatter channels as a function of ρ_e. In each of the figures, the random variables that correspond to the plotted distributions have been normalized to unit power (i.e., $\mathcal{E}\{\alpha^2\} = 1$). From these figures, several observations can be made:

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[1]Equation (3) differs from that given by Simon [8] in that it has been normalized to satisfy $\int_0^\infty f_\alpha(\alpha, \rho)d\alpha = 1$.

1) The probability of the signal envelope having a very small value decreases as ρ_e is reduced, as can be seen in Figure 3. The reason for this improvement is intuitive. As the envelope correlation between the forward and backscatter links is reduced, the likelihood that both links will fade simultaneously decreases; therefore, the probability of receiving a very small envelope is lessened.

2) The probability of receiving a small envelope value also decreases as antennas are added to the RF tag, which is seen most clearly in Figure 2(b). Modulating backscatter with multiple antennas on the RF tag causes multiple signals, whose fading is uncorrelated, to be summed at each reader receiver antenna reducing the probability of a signal fade. In addition, each RF tag antenna increases the effective scattering aperture of the RF tag. These effects combine to form an effective pinhole diversity gain [3].

3) As additional antennas are added to the RF tag, the overall level of fading decreases, but the envelope distribution becomes less sensitive to ρ_e. This can be seen by comparing Figure 3(a) and Figure 3(b); the CDF of the $1 \times 2 \times 1$ channel does not change with ρ_e as much as the $1 \times 1 \times 1$ channel. This is because RF tags modulating backscatter with multiple antennas provide more statistically independent pinholes through which signals may propagate [4].

4) Figure 2 shows that, when normalized to equal power, the $1 \times L \times 1$ channel with independent links has the same distribution as the $1 \times 2L \times 1$ channel with fully correlated links [4]. This is because, in the $1 \times 2L \times 1$ channel with fully correlated links, link correlation reduces the number of statistically independent propagation paths by a factor of 2. Analysis of (5) and (6) shows that this is a general result.

3.2 BER Improvement

The changes in the envelope distribution as link envelope correlation is decreased result in a BER improvement of the signal received at the nth reader receiver antenna. Plots of the BER are shown in Figure 4 as a function of the number of RF tag antennas, L, and link envelope correlation, ρ_e. In these Monte Carlo simulations, Rayleigh fading forward and backscatter links, uncoded BPSK modulation, and noise and interference that is additive, white, and Gaussian were assumed. Each curve represents the average BER of the signal received at the nth reader receiver antenna with no diversity combining. Each BER curve is plotted against the signal-to-noise plus interference ratio (SINR) at the nth reader receiver antenna in the $1 \times 1 \times 1$ channel. From Figure 4, it can be seen that, like the distribution improvements discussed in Section 3.1, the BER decreases as additional antennas are used to modulate backscatter on each RF tag and as link correlation is reduced. Furthermore, the sensitivity of the BER to ρ_e decreases as more antennas are added becoming almost negligible for $L \geq 3$. The source of these BER changes is the improved distribution of the nth received signal along with increased RF tag scattering aperture as additional RF tag antennas are used. Therefore, the underlying causes of the BER improvements are the same as those listed in Section 3.1 for the distribution improvements. For the $1 \times 1 \times 1$ and $1 \times 2 \times 1$ channels, shown in Figure 4, the greatest BER increase occurs between $\rho_e = 0.6$ and $\rho_e = 1$. A similar trend can be seen in the CDF improvement shown in Figure 3. Therefore, if ρ_e is kept at or below approximately 0.6 in these channels, backscatter communication with a near minimum BER is possible. This agrees with the generally accepted rule that envelope correlation between diversity branches in a one-way channel is acceptable below 0.5 to 0.7 [10].

3.3 Discussion

A high degree of link envelope correlation will occur when the dominant mechanism of wave propagation (i.e., NLOS propagation in this paper) and the angles of arrival/departure at the reader are similar.
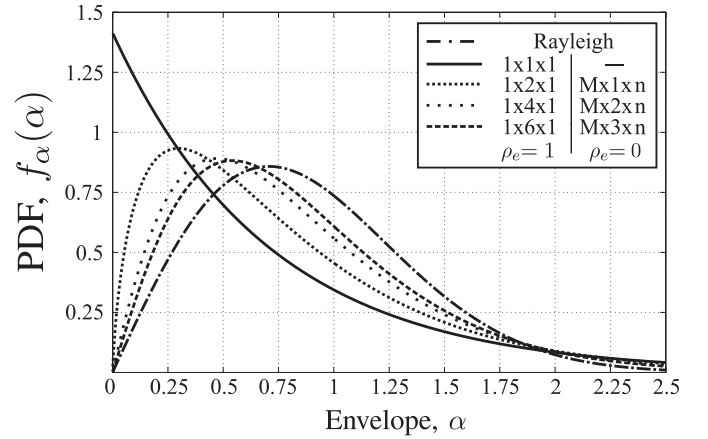
(a) Normalized dyadic backscatter channel PDF
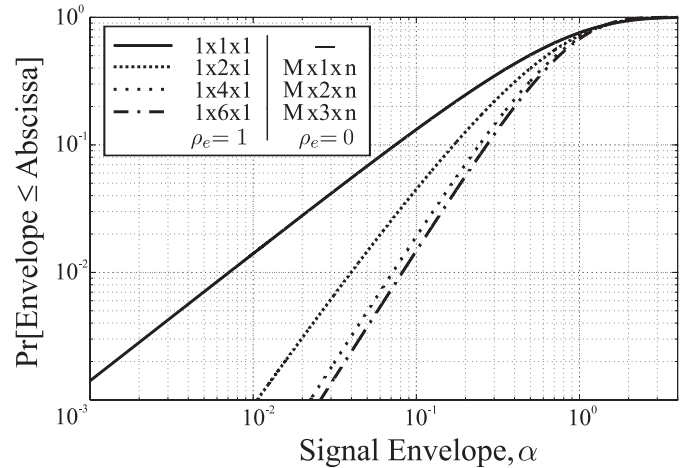
(b) Normalized dyadic backscatter channel CDF

FIGURE 2 - THE (A) PDF AND (B) CDF OF THE SIGNAL RECEIVED AT THE nth READER RECEIVER ANTENNA FOR DIFFERENT VALUES OF LINK ENVELOPE CORRELATION, ρ_e.
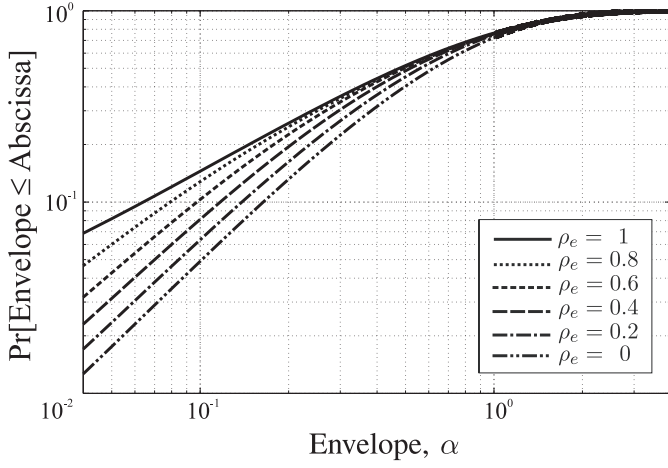
Since a high level of link envelope correlation implies that the propagation environment of the forward and backscatter links are similar, fully correlated links can **only** occur when the reader transmitter and receiver antennas are co-located and have the same antenna patterns. If the antennas are spatially separated and/or the antenna patterns are different, ρ_e will be reduced. The separation distance and pattern required to reduce ρ_e to an acceptable level, which this paper shows is approximately $\rho_e \leq 0.6$, will vary depending upon the channel.

This discussion of link envelope correlation gives added motivation for following the two backscatter radio system design guidelines proposed previously [3]:

- **Multiple RF tag antennas:** Modulating backscatter with multiple antennas on each RF tag will decrease the BER and reduce link envelope correlation effects allowing more reliable backscatter communication.

- **Separate reader antennas:** Using spatially separated reader transmitter and receiver antennas will reduce link envelope correlation and decrease the BER of the received backscatter signal.

IV. CONCLUSION

Small-scale fading in the dyadic backscatter channel, which can be more severe than that found in a one-way channel, is worsened by link envelope correlation. This paper has shown that using multiple antennas on each RF tag to modulate backscatter will improve the distribution and lower the BER of the nth received signal as well as reduce the

(a) CDF of the $1 \times 1 \times 1$ dyadic backscatter channel
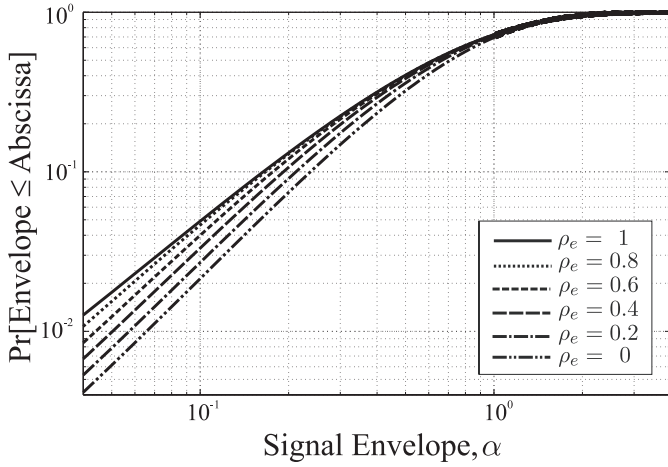
(b) CDF of the $1 \times 2 \times 1$ dyadic backscatter channel

FIGURE 3 - PLOTS OF THE CDF OF THE (A) $1 \times 1 \times 1$ AND (B) $1 \times 2 \times 1$ CHANNELS AS A FUNCTION OF LINK ENVELOPE CORRELATION, ρ_e.
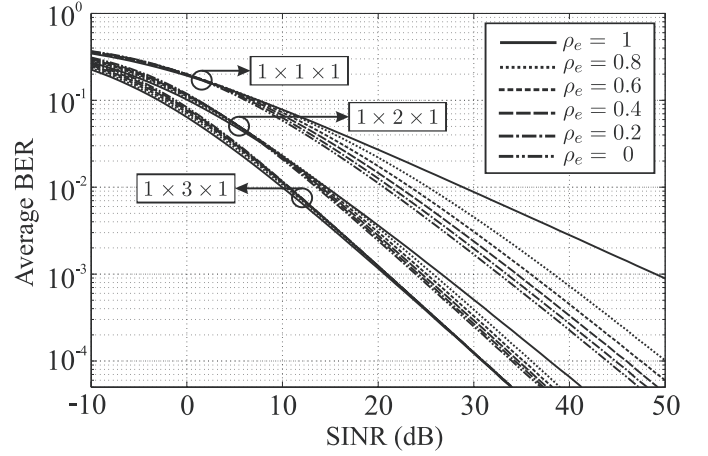

FIGURE 4 - AVERAGE BER PLOTS OF THE $1 \times 1 \times 1$, $1 \times 2 \times 1$, AND $1 \times 3 \times 1$ DYADIC BACKSCATTER CHANNELS FOR SEVERAL VALUES OF LINK ENVELOPE CORRELATION, ρ_e.

effect of link envelope correlation. Link envelope correlation can also be reduced by using spatially separated reader transmitter and receiver antennas and, if link envelope correlation is kept below an acceptable level (approximately $\rho_e \leq 0.6$), RF tags will communicate with a near minimum BER.

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