

# Joint ranging and clock synchronization for a satellite array

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**Abstract**—OLFAR is Orbiting Low Frequency Antennas for Radio astronomy, a study to investigate the feasibility of a untethered satellite array ( $\geq 10$  satellites) in space for ultra low frequency observations (0.3MHz-30MHz). Synchronization and localization are two key aspects for the coherent functioning of satellite nodes. Recently, various estimators have been proposed for pairwise synchronization between two nodes based on time stamp exchanges via two way communication. In this paper, we propose a closed form centralized Global Least Squares (GLS) estimator, which exploits two way communication information between all the nodes in the network. The fusion center based GLS uses a single clock reference node and estimates all the unknown clock offsets, skews and pairwise distances in the network. The GLS estimate for clock offsets and skews is shown to outperform prevalent estimators. Furthermore, a new Cramer Rao Lower Bound (CRLB) is derived for the entire network and the proposed GLS solution is shown to approach the theoretical limits. To illustrate the applicability of the GLS solution despite missing communication links, a few network topologies are presented.

**Index Terms**—OLFAR, joint estimation, clock synchronization, skew, offset, distance, wireless network, anchorless

## I. INTRODUCTION

The Orbiting Low Frequency Antennas for Radio astronomy (OLFAR) [1] is a Dutch funded program which aims to design and develop a detailed system concept for an interferometric array ( $\geq 10$ ) of identical, scalable and autonomous nano satellites in space to be used as a scientific instrument for ultra low frequency observations (0.3KHz - 30MHz). The OLFAR cluster could either orbit the moon, whilst sampling during the Earth-radio eclipse phase, or orbit the Earth-moon L2 point, sampling almost continuously or Earth-trailing and leading orbit. Due to its distant deployment location (far from the earth orbiting global positioning systems) and the large number of satellites, autonomous network synchronization and localization is one of the key issues in OLFAR.

The coherent functioning of OLFAR network relies heavily on time synchronization among satellite nodes. All nodes in the network must be synchronized to a global reference, to facilitate accurate time stamping of data and synchronized communication of processed information [2]. Such global time synchronization is achieved by estimating all clock offsets and clock skews of the nodes and compensating the respective clocks aptly. Furthermore, autonomous position estimation is equally critical as time synchronization. The intermediate distances between all the nodes in the network is one of the key inputs for almost all localization techniques [3].

For a pair of nodes capable of two way communication with each other, Freris et al. proposed a model [4], which is based on exchanging time stamps between the nodes. The model contains 2 clock offsets, 2 clock skews and the distance between the nodes, which results in an unsolvable five dimensional problem. However, traditionally, one clock is assumed to be the reference clock which reduces the cardinality to 3 and the absolute clock offset and clock skew of the second node can be estimated. Maximum likelihood estimates for various noise profiles are presented for joint estimation of clock offset and clock skew in [5], such as the GMLL estimate

for gaussian noise and unknown delay. A step further, Leng et al. presented a Low Complexity Least Squares (LCLS) solution [6] for the joint estimate, however the distance was ignored as a nuisance parameter. In similar lines, but towards localization, Zheng et al. [7] showed if 3 nodes are completely synchronized and their positions known, then the clock offset, skew and position of an unknown node can be estimated for a 2D scenario. In this paper, we propose a centralized Global Least Squares (GLS) estimator which exploits the two way communication information between all the nodes and estimates all the unknown clock offsets, skews and pairwise distances in the network using a single clock reference. Although the presented algorithm is in the context of the OLFAR project, they are readily applicable to terrestrial networks as well.

*Notation:* The element wise matrix Hadamard product is denoted by  $\odot$ , element wise Hadamard division by  $\oslash$ ,  $(\cdot)^{\odot N}$  denotes element-wise matrix exponent. The Kroneker product is indicated by  $\otimes$  and the transpose operator by  $(\cdot)^T$ .  $\mathbf{1}_N = [1, 1, \dots, 1] \in \mathbb{R}^{N \times 1}$  is a vector of ones and  $\mathbf{I}_N$  is a  $N \times N$  identity matrix.

## II. PROBLEM FORMULATION

Consider a network of  $N$  nodes equipped with independent clock oscillators which, under ideal conditions, are synchronized to the global time. However, in reality, due to various oscillator imperfections and environment conditions the clocks at the nodes vary independently. Clocks are inherently Let  $t_i$  be the local time at node  $i$ , then its divergence from the ideal global time  $t$  is to first order given by the affine clock model,

$$t_i = \omega_i t + \phi_i \quad (1)$$

where  $\omega_i \in \mathbb{R}_+$  and  $\phi_i \in \mathbb{R}$  are the clock skew and clock offset of node  $i$ . The clock skew and clock offset parameters for all  $N$  nodes are represented by  $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_N]^T \in \mathbb{R}_+^{N \times 1}$  and  $\boldsymbol{\phi} = [\phi_1, \phi_2, \dots, \phi_N]^T \in \mathbb{R}^{N \times 1}$  respectively. Alternatively, the translation from local time  $t_i$  to the global time  $t$  is written as a function of local time,

$$\mathcal{F}_i(t_i) \triangleq t = \beta_i t_i - \alpha_i \quad (2)$$

where

$$[\beta_i, \alpha_i] \triangleq [\omega_i^{-1}, \omega_i^{-1} \phi_i] \quad (3)$$

are the calibration parameters needed to correct the local clock of node  $i$ . Following immediately, for all  $N$  nodes in the network, we have  $\boldsymbol{\beta} \triangleq \mathbf{1}_N \oslash \boldsymbol{\omega} \in \mathbb{R}_+^{N \times 1}$  and  $\boldsymbol{\alpha} \triangleq \boldsymbol{\phi} \oslash \boldsymbol{\omega} \in \mathbb{R}^{N \times 1}$ . All  $M = \binom{N}{2}$  unique pairwise distances between  $N$  nodes are given by  $\mathbf{d} = [d_{11}, d_{12}, \dots, d_{(N-1)(N)}] \in \mathbb{R}^{M \times 1}$  and subsequently the propagation delay between nodes is given by  $\boldsymbol{\tau} = \mathbf{d}c^{-1} \in \mathbb{R}^{M \times 1}$ , where  $c$  is the speed of the electromagnetic wave in the medium.

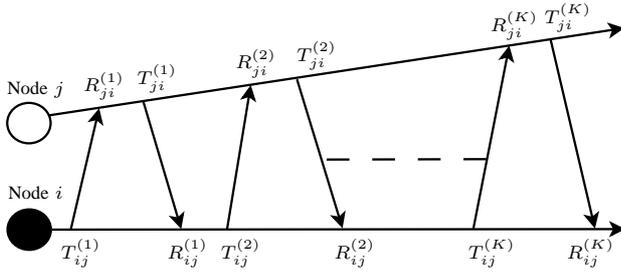


Fig. 1. Figure shows the classical two way communication between a node pair  $(i, j)$ . Node  $i$  is the reference with  $[\omega_i, \phi_i] = [1, 0]$  and also initiates the communication with node  $j$  whose clock skew ( $\omega_j$ ), clock offset ( $\phi_j$ ) and distance ( $d_{ij}$ ) from node  $i$  are unknown. There are  $K$  two way communications between the node pair during which  $2K$  time markers are recorded at the respective nodes

Given a single reference node (providing the reference clock) and two way communication between all nodes, we intend to efficiently estimate all the absolute clock skews ( $\omega$ ), clock offsets ( $\phi$ ) and pairwise distances ( $d$ ) in the network.

### III. NETWORK SYNCHRONIZATION AND RANGING

Consider the classical two way communication between a sensor pair  $(i, j)$  as shown in Figure 1. Node  $i$  initiates the communication and up-links a message to node  $j$  and node  $j$  responds by downlinking a message back to node  $i$ . Both the nodes communicate messages back and forth to each other, and the transmission and reception times are recorded independently by the clocks at the respective nodes. For the uplink,  $T_{ij}^{(k)}$  denotes the local time recorded at node  $i$  for the  $k$ th message departing to node  $j$  and  $R_{ji}^{(k)}$  is the corresponding local time marker recorded by the Node  $j$  on receiving the message from node  $i$ . Similarly during down linking,  $T_{ji}^{(k)}$  and  $R_{ij}^{(k)}$  are the local timings recorded at node  $j$  and  $i$  respectively. There are  $K$  such two way communications between the sensor pair, during which we assume that the propagation delay between the two nodes  $\tau_{ij} = d_{ij}/c \equiv d_{ji}/c$  is fixed. The transmission and reception markers are then related as [2] [4]

$$\begin{aligned} T_{ij}^{(k)} + q_1^{(k)} &= \omega_i(\mathcal{F}_j(R_{ji}^{(k)} + q_2^{(k)}) - \tau_{ij}) + \phi_i, \\ R_{ij}^{(k)} + q_3^{(k)} &= \omega_i(\mathcal{F}_j(T_{ji}^{(k)} + q_4^{(k)}) + \tau_{ij}) + \phi_i \end{aligned} \quad (4)$$

where  $\{q_1^{(k)}, q_2^{(k)}, q_3^{(k)}, q_4^{(k)}\} \sim \mathcal{N}(0, 0.5\sigma^2)$  are gaussian i.i.d noise variables plaguing the timing measurements. Rearranging the terms and from (1), (2) and (3) we have

$$\begin{aligned} \beta_i T_{ij}^{(k)} &= \beta_j R_{ji}^{(k)} + \alpha_i - \alpha_j - \tau_{ij} - \beta_i q_1^{(k)} + \beta_j q_2^{(k)}, \\ \beta_i R_{ij}^{(k)} &= \beta_j T_{ji}^{(k)} + \alpha_i - \alpha_j + \tau_{ij} - \beta_i q_3^{(k)} + \beta_j q_4^{(k)} \end{aligned} \quad (5)$$

For all  $K$  two way communications, a generalized model for a pair of sensors is

$$[\mathbf{t}_{ij} \quad -\mathbf{t}_{ji} \quad -\mathbf{1}_{2K} \quad \mathbf{1}_{2K} \quad -\mathbf{e}] \begin{bmatrix} \beta_i \\ \beta_j \\ \alpha_i \\ \alpha_j \\ \tau_{ij} \end{bmatrix} = \mathbf{q}_{ij} \quad (6)$$

where  $\mathbf{t}_{ij}, \mathbf{t}_{ji} \in \mathbb{R}^{2K \times 1}$  are time markers recorded at node  $i$  and node  $j$  respectively while communicating with each other and are given by

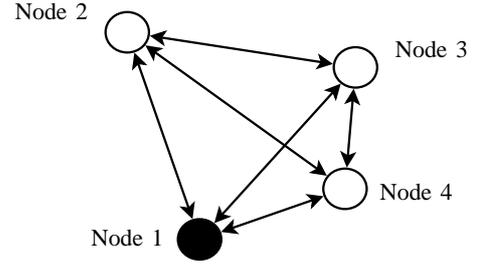


Fig. 2. An illustration of a network with  $N = 4$  nodes, each capable of two way communication. Node 1 (shaded in black) is the clock reference with  $[\omega_1, \phi_1] = [1, 0]$ . The clock skews and clock offsets of node 2, 3 and 4 are unknown and are to be estimated, in addition to all the pairwise distances.

$$\begin{aligned} \mathbf{t}_{ij} &= [T_{ij}^{(1)}, R_{ij}^{(1)}, T_{ij}^{(2)}, \dots, R_{ij}^{(K)}]^T, \\ \mathbf{t}_{ji} &= [R_{ji}^{(1)}, T_{ji}^{(1)}, R_{ji}^{(2)}, \dots, T_{ji}^{(K)}]^T \end{aligned} \quad (7)$$

$\mathbf{e} = [-1, +1, \dots, +1]^T \in \mathbb{R}^{2K \times 1}$  and  $\mathbf{q}_{ij}$  is the i.i.d noise vector, which is modeled as  $\mathbf{q}_{ij} \sim \mathcal{N}(0, 0.5\sigma^2(\beta_i^2 + \beta_j^2)) \in \mathbb{R}^{2K \times 1}$ . In reality, the clock skews  $\omega_i, \omega_j$  are very close to 1 and the errors are of the order of  $10^{-4}$ . Hence the noise vector could be approximated by

$$\mathbf{q}_{ij} \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}^{2K \times 1} \quad (8)$$

Such an approximation is satisfactory and is implicitly employed in various cases such as [2], [5], [6] and [7]. The pairwise model in (6) is not solvable as is, since the measurement matrix is rank deficient. However by asserting one node as the clock reference and extending this model to the entire network and we propose to find a global optimal solution for all unknown clock parameters and distances using a single reference. As an illustration, Figure 2 shows a network consisting of  $N = 4$  nodes, all capable of two way communication with each other. Without loss of generality, we assume node 1 is the reference node in this sensor network and that all the links are present. Rearranging the terms in (6), for all  $\{i, j\}$ , we have

$$[\mathbf{T} \quad \mathbf{E}_1 \quad \mathbf{E}_2] \begin{bmatrix} \beta \\ \alpha \\ \tau \end{bmatrix} = \mathbf{q} \quad (9)$$

where  $\mathbf{T} \in \mathbb{R}^{2KM \times N}$  contains all the timing vectors from all the  $N$  nodes,  $\mathbf{E}_1 \in \mathbb{R}^{2KM \times N}$ ,  $\mathbf{E}_2 = -\mathbf{I}_M \otimes \mathbf{e} \in \mathbb{R}^{2KM \times M}$ . For  $N = 4$ ,  $\mathbf{T}$  and  $\mathbf{E}_1$  are of the form

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_{12} & -\mathbf{t}_{21} & & & \\ \mathbf{t}_{13} & & -\mathbf{t}_{31} & & \\ \mathbf{t}_{14} & & & -\mathbf{t}_{41} & \\ & \mathbf{t}_{23} & -\mathbf{t}_{32} & & \\ & \mathbf{t}_{24} & & -\mathbf{t}_{42} & \\ & & \mathbf{t}_{34} & -\mathbf{t}_{43} & \end{bmatrix} \quad (10)$$

$$\mathbf{E}_1 = \begin{bmatrix} -\mathbf{1}_{2K} & +\mathbf{1}_{2K} & & & \\ -\mathbf{1}_{2K} & & +\mathbf{1}_{2K} & & \\ -\mathbf{1}_{2K} & & & +\mathbf{1}_{2K} & \\ & -\mathbf{1}_{2K} & +\mathbf{1}_{2K} & & \\ & -\mathbf{1}_{2K} & & +\mathbf{1}_{2K} & \\ & & -\mathbf{1}_{2K} & +\mathbf{1}_{2K} & \end{bmatrix} \quad (11)$$

and a similar structure can be generalized for  $N \geq 4$ . The global noise vector is  $\mathbf{q} = [\mathbf{q}_{12}, \mathbf{q}_{13}, \dots, \mathbf{q}_{(N-1)(N)}] \in \mathbb{R}^{2KM \times 1}$  where each  $\mathbf{q}_{ij}$  is given by (8). Since node 1 is the reference node, i.e.,

$[\beta_1, \alpha_1] = [1, 0]$ , rearranging the terms in (9) we have

$$\bar{\mathbf{A}}\boldsymbol{\theta} = -\bar{\mathbf{t}}_1 + \mathbf{q} \quad (12)$$

where

$$\begin{aligned} \bar{\mathbf{A}} &= [\bar{\mathbf{T}} \quad \bar{\mathbf{E}}_1 \quad \mathbf{E}_2] \in \mathbb{R}^{2KM \times L} \\ \boldsymbol{\theta} &= [\bar{\boldsymbol{\beta}} \quad \bar{\boldsymbol{\alpha}} \quad \boldsymbol{\tau}]^T \in \mathbb{R}^{L \times 1} \end{aligned}$$

where  $L = 2N + M - 2$  and  $\bar{\mathbf{T}}, \bar{\mathbf{E}}_1 \in \mathbb{R}^{2KM \times (N-1)}$  are submatrices of  $\mathbf{T}$  and  $\mathbf{E}_1$  respectively, excluding the corresponding first columns.  $\bar{\boldsymbol{\beta}}, \bar{\boldsymbol{\alpha}} \in \mathbb{R}^{(N-1) \times 1}$  represent the unknown clock parameters of all the nodes excluding node 1.  $\bar{\mathbf{t}}_1 \in \mathbb{R}^{2KM \times 1}$  is the first column of matrix  $\mathbf{T}$  which contains the timing markers recorded at node 1, whilst communicating with the other nodes in the network. Analyzing the components of matrix  $\bar{\mathbf{A}}$ , both  $\bar{\mathbf{T}}, \bar{\mathbf{E}}_1$  are full rank, since the respective first  $2K(N-1)$  rows are formed by block diagonal matrices. Note that all columns of  $\mathbf{E}_2$  are also independent. In addition, if  $K \geq 2$ , then a Global Least Squares (GLS) solution is feasible and is obtained by minimizing the least squares norm, i.e.,

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg \min_{\boldsymbol{\theta}} \|\bar{\mathbf{A}}\boldsymbol{\theta} - \bar{\mathbf{t}}_1\|_2^2 \\ &= (\bar{\mathbf{A}}^T \bar{\mathbf{A}})^{-1} \bar{\mathbf{A}}^T \bar{\mathbf{t}}_1 \end{aligned} \quad (13)$$

Hence, the unknown clock skews ( $\bar{\boldsymbol{\omega}} \triangleq \mathbf{1}_N \otimes \bar{\boldsymbol{\beta}}$ ), the unknown clock offsets ( $\bar{\boldsymbol{\phi}} \triangleq \bar{\boldsymbol{\alpha}} \otimes \boldsymbol{\beta}$ ) of the nodes and the pairwise distances ( $\bar{\mathbf{d}} \triangleq \boldsymbol{\tau} \mathbf{c}$ ) in the network can be estimated by solving (13). Note that in the two way communication model, there is no assumption that the messages have to be alternating regularly. Hence the measured time stamps are valid as long as the distance between the nodes and the clock parameters are stable within reasonable limits during the estimation process. Secondly, if the two way link is replaced with one way communication then matrix  $\bar{\mathbf{A}}$  is rank deficient and hence there is no optimal solution to jointly estimate the clock parameters and pairwise distances.

#### IV. CRAMER RAO LOWER BOUND

The Cramer Rao Lower Bound (CRLB) on the error variance for any unbiased estimator states [8]

$$\varepsilon\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})\} \geq \frac{1}{L} \mathbf{F}^{-1} \quad (14)$$

where  $\mathbf{F}$  is the Fisher information matrix and  $L$  is the length of the estimated vector  $\hat{\boldsymbol{\theta}}$ . The error vector  $\mathbf{q}$  in (12) is gaussian by assumption and the corresponding Fisher information matrix is [8]

$$\mathbf{F} = \frac{1}{\sigma^2} \mathbf{J}^T \mathbf{J} \quad (15)$$

where  $\mathbf{J} \in \mathbb{R}^{2KM \times L}$  is the Jacobian matrix. For jointly estimating the clock skew  $\bar{\boldsymbol{\omega}}$ , clock offset  $\bar{\boldsymbol{\phi}}$  and all the pairwise distances  $\bar{\mathbf{d}}$ , we have

$$\mathbf{J} = \left[ \frac{\partial \bar{\mathbf{A}} \boldsymbol{\theta}}{\partial \boldsymbol{\theta}^T} \right] \triangleq \begin{bmatrix} \mathbf{J}_{\bar{\boldsymbol{\omega}}} & \mathbf{J}_{\bar{\boldsymbol{\phi}}} & \mathbf{J}_{\boldsymbol{\tau}} \end{bmatrix} \quad (16)$$

where the independent components can be shown as

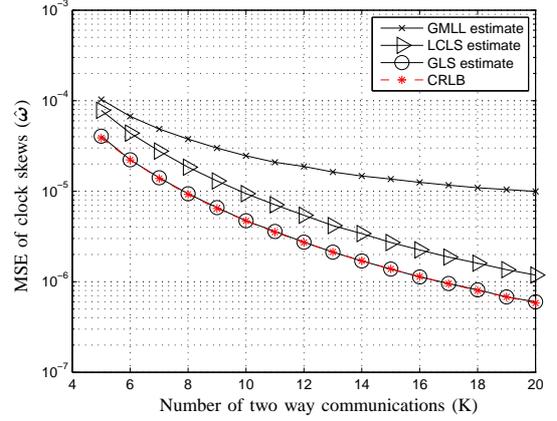


Fig. 4. Mean Square Error (MSE) plot of estimated clock skews ( $\hat{\boldsymbol{\omega}}$ ) for a network of  $N = 4$  nodes, where noise is gaussian with  $\sigma = 0.1$

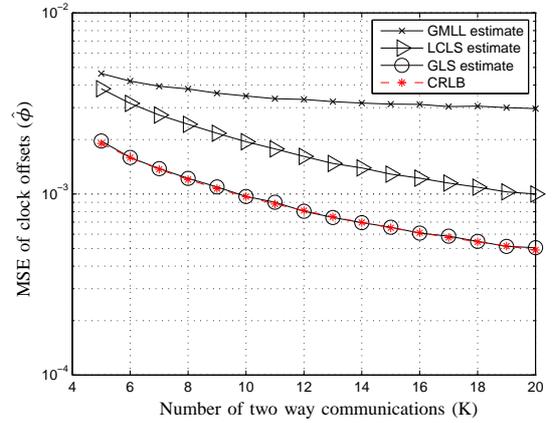


Fig. 5. Mean Square Error (MSE) plot of estimated clock offsets ( $\hat{\boldsymbol{\phi}}$ ) for a network of  $N = 4$  nodes, where noise is gaussian with  $\sigma = 0.1$

$$\begin{aligned} \mathbf{J}_{\bar{\boldsymbol{\omega}}} &= -(\bar{\mathbf{T}} + \bar{\mathbf{E}}_1 \odot \mathbf{1}_{2KM} \bar{\boldsymbol{\phi}}^T) \odot (\mathbf{1}_{2KM} \bar{\boldsymbol{\omega}}^T)^{\odot 2} \\ \mathbf{J}_{\bar{\boldsymbol{\phi}}} &= \bar{\mathbf{E}}_1 \odot \mathbf{1}_{2KM} \bar{\boldsymbol{\omega}}^T \\ \mathbf{J}_{\boldsymbol{\tau}} &= \mathbf{E}_2 \end{aligned} \quad (17)$$

#### V. SIMULATIONS

Simulations are conducted to evaluate the performance of the proposed estimator. We consider a network of  $N = 4$  nodes, as shown in Figure 2, wherein all the nodes are located within 100 meters of each other and consequently  $\bar{\mathbf{d}}$  is a random vector in the range  $(0, 100\text{m}]$ . The clock offsets ( $\bar{\boldsymbol{\phi}}$ ) and clock skews ( $\bar{\boldsymbol{\omega}}$ ) are uniform randomly distributed in the range  $[-1, 1]$  seconds and  $[0.998, 1.002]$  respectively. The transmission time markers  $\bar{\mathbf{t}}_{ij}$  are linearly distributed between 1 to 100 seconds, for number of two way communication  $K$  from 5 to 20. The noise variance on the timing markers is  $\sigma = 0.1$  and all results presented are averaged over 10,000 independent monte carlo runs.

Figures 4 and 5 show the Mean Square Errors (MSE) of clock skews and offsets against the number of two way communications  $K$  for various estimators. The Low Complexity Least Squares (LCLS) [6] and the Maximum likelihood GMLL [5] algorithms are independently applied, pairwise from node 1 to every other node, to estimate

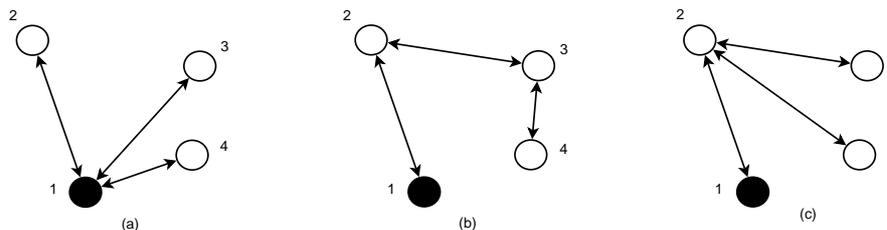


Fig. 3. An illustration of 3 networks with  $N = 4$  nodes each capable of two way communication. The node shaded in black is the clock reference. The 3 networks are illustrative examples where GLS algorithm can be applied for network wide clock synchronization, despite missing communication links.

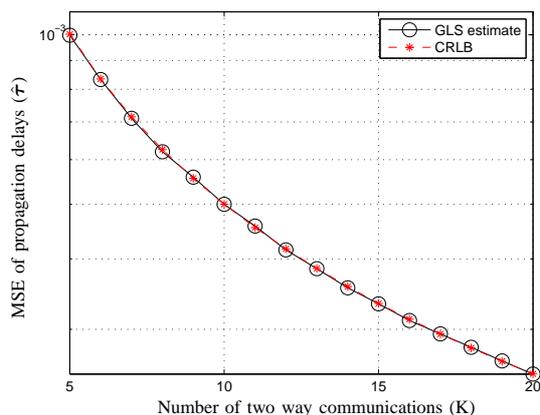


Fig. 6. Mean Square Error (MSE) plot of estimated propagation delays ( $\hat{\tau}$ ) for a network of  $N = 4$  nodes, where noise is gaussian with  $\sigma = 0.1$

all the unknown skews  $\bar{\omega}$  and offsets  $\bar{\phi}$ . Secondly, the proposed Global Least Squares (GLS) solution, which exploits information from all the pairwise two way communications, outperforms the (LCLS) for both clock skew and clock offset estimation and achieves the theoretical Cramer Rao Lower Bound. In addition to clock parameters, the pairwise distances  $\mathbf{d}$  are also estimated in terms of propagation delays  $\tau$ . Figure 6 shows the proposed Global Least Squares (GLS) solution for  $\tau$  which achieves the Cramer Rao Lower Bound. Note that there are no prevalent estimators available to all the pair wise distances along with the clock parameters.

### VI. TOPOLOGIES

In the OLFAR network, each nano satellite will be equipped with a high quality Rubidium clock oscillator for accurate time stamping of observed data and for communication. The centralized GLS algorithm can readily be applied to estimate the clocks parameters and the intermediate distances between the satellites. As an extension, Multi-dimensional scaling (MDS) [3] can be applied on the estimated distances to obtain all relative positions of the nodes, thereby achieving absolute clock synchronization and relative localization for an anchorless sensor network. The data observed by each satellite is distributed to all the other satellites using a frequency distributed correlator architecture [1]. Since the data rates between the satellite nodes is  $\geq 10$  MBits/sec, the measured time stamp information form a relatively trivial part of the house keeping information.

The closed form GLS solution (13) is for a full mesh network *i.e.*, all nodes are are connected to each other. However, more in general, if some pairwise communications links are missing then corresponding rows in matrix  $\bar{\mathbf{A}}$  are dropped. Consequentially, the pairwise distances between those particular nodes cannot be optimally estimated.

However, despite missing links network wide synchronization is still feasible using (13) if and only if  $\bar{\mathbf{A}}$  is full rank and every node in the network communicates at least with one other node. As an example, Figure 3 shows three topologies with missing communication links, where the GLS algorithm is still applicable. Hence far away satellite nodes can still be synchronized even if there is no direct link with the reference node.

### VII. CONCLUSIONS

Autonomous synchronization and localization are key requirements of the OLFAR network. In this paper, an efficient and novel closed form Global Least Squares (GLS) estimator for network wide synchronization is proposed. The GLS utilizes a single reference node and exploits all two way communication information between nodes in the network. The performance of the proposed estimator is shown to better available solutions for clock skews and offsets, in addition to estimating the pairwise distances between all nodes in a closed form. To the best of author’s knowledge, all the prevalent estimators has been presented, for the given data model. A new CRLB has been derived and the proposed solution is shown to achieve the bound. The applicability of GLS solution despite missing links has been discussed with the help of few network topologies.

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