Sparse MIMO Radar with Random Sensor Arrays and Kerdock Codes

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***Abstract*—We derive a theoretical framework for the recoverability of targets in the azimuth-range-Doppler domain using random sensor arrays and tools developed in the area of compressive sensing. In one manifestation of our theory we use Kerdock codes as transmission waveforms and exploit some of their peculiar properties in our analysis. Not only is our result the first rigorous mathematical theory for the detection of moving targets using random sensor arrays, but also the transmitted waveforms satisfy a variety of properties that are very desirable and important from a practical viewpoint.**

I. Introduction

In recent years, radar systems employing multiple antennas at the transmitter and the receiver (also referred to as MIMO radar, where MIMO stands for multiple-input multiple-output) have attracted enormous attention in the engineering and signal processing community. Existing theory focuses mainly on the detection of a single target. Only very recently, in the footsteps of compressive sensing, do we see the emergence of a rigorous mathematical theory for MIMO radar that addresses the more realistic and more interesting case of multiple targets [13]. However, for the widely popular case of randomly spaced antennas, the mathematical theory is still in its infancy.

On the other hand, mathematicians and engineers have devoted substantial efforts to the design of radar transmission waveforms that satisfy a variety of desirable properties. The vast majority of this research has focused on single antenna radar systems, and it is a priori not clear whether and how these waveforms can be utilized for MIMO radar. In this paper we bring together these two independent areas of research, MIMO radar with random antenna arrays and radar waveform design, by developing a rigorous mathematical framework for accurate target detection via random arrays, which at the same time utilizes some of the most attractive radar waveforms, such as Kerdock codes.

In radar processing we are interested in a given area, which is usually called the radar scene. We would like to detect the location and the strength of the objects of interest, as well as the velocity if there is relative motion between the radar and the objects. Usually the radar scene is divided into a grid of range-azimuth-Doppler (distance, direction and speed) resolution cells. In many practical cases the radar scene is sparse in the sense that only a small fraction of the grid points is occupied by the targets of interest.

While the conventional radar processing techniques do not take advantage of the fact that the radar scene is often sparse, the recent development of compressive sensing (CS) provides us the possibility to utilize this structure. In fact recent works (such as [8], [12], [13] and the reference therein) created important linkage between radar processing and CS. As in CS, we also have to solve the following inverse problem in radar processing:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{y}$ is a vector of measurements collected by the receiver antennas over an observation interval, $\mathbf{A}$ is a measurement matrix whose columns correspond to the signal received from a single unit-strength scatterer at a particular range-azimuth-Doppler grid point, $\mathbf{x}$ is a vector whose elements represent the complex amplitudes of the scatterers, and $\mathbf{w}$ is the unknown noise vector. Note that this is an under-determined equation (if $\dim(\mathbf{y}) < \dim(\mathbf{x})$) and in general it has infinitely many solutions. But given that $\mathbf{x}$ is sparse from our assumption, this problem can have a satisfactory solution.

One of the algorithms that can be used to solve (1) is as follows:

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_1, \tag{2}$$

which is also known as lasso. Here $\lambda > 0$ is a regularization parameter that trades off goodness of fit with sparsity. [3] showed that if we assume $\mathbf{x}$ is drawn from a generic S-sparse target model (i.e. the support of $\mathbf{x}$ is selected uniformly at random and the phases of the non-zero entries of $\mathbf{x}$ are random and uniformly distributed in $[0, 2\pi)$) then with a particular choice of λ, (2) will recover the support of $\mathbf{x}$ correctly with high probability given that the coherence and the operator norm of $\mathbf{A}$ can be well controlled.

Our paper provides two main contributions: (i) We derive the first rigorous mathematical theory for the detection of moving targets in the azimuth-range-Doppler domain for random sensor arrays. (ii) The transmitted waveforms satisfy a variety of properties that are very desirable and important from a practical viewpoint. In particular, we show that Kerdock sequences, which would perform very poorly in single-antenna radar, are nearly ideally suited for MIMO radar with randomly spaced antennas. Thus, our framework does not just lead to useful theoretical insights, but also has a very strong practical appeal.

A. Connections with prior work and innovations

Random sensor arrays have been around for decades [11]. Recently, [4] made an explicit connection between random sensor arrays and the CS. The setup in [4] is quite different from ours, since the author is only concerned with angular resolution, while it is often crucial in practice to be able to estimate range and Doppler as well. Moreover, the theoretical analysis in [4] follows more an engineering style and places less emphasis on mathematical rigor.

On the other hand, [13] is closest to this paper. [13] considers a MIMO radar setting with a very specific (non-random) choice for the antenna locations, but random waveforms, while the current paper deals with randomly spaced antennas, but very specific, deterministic waveforms. In practice, the random waveforms are much harder to implement on a digital device and they exhibit a larger peak-to-average-power ratio compared to carefully designed deterministic waveforms. On the other hand it makes no difference from the viewpoint of physics or hardware, if we place the antennas at random or at deterministic locations.

B. Notation

For a matrix $\mathbf{A}$, we use $\mathbf{A}^*$ to denote its adjoint matrix, which is its conjugate transpose. The operator norm of $\mathbf{A}$ is the largest singular value of $\mathbf{A}$ and is denoted by $\|\mathbf{A}\|_{\text{op}}$.

For $\mathbf{x} \in \mathbb{C}^n$, let $\mathbf{T}_\tau$ denote the circulant translation operator, defined by $\mathbf{T}_\tau \mathbf{x}(l) = \mathbf{x}(l-\tau)$, for $\tau = 1, \dots, n$, where $l-\tau$ is understood modulo n, and let $\mathbf{M}_f$ be the modulation operator defined by $\mathbf{M}_f \mathbf{x}(l) = \mathbf{x}(l)e^{2\pi i f l/n}$.

II. Problem Setup

We consider a MIMO radar employing N_T antennas at the transmitter and N_R antennas at the receiver. We assume for convenience that transmitter and receiver are co-located. Furthermore, we assume a coherent propagation scenario, i.e., the element spacing is sufficiently small so that the radar return from a given scatterer is fully correlated across the array. The arrays and all the scatterers are assumed to be in the same 2-D plane. The extension to the 3-D case is straightforward.

The array manifolds $\mathbf{a}_T(\beta)$, $\mathbf{a}_R(\beta)$ with randomly spaced antennas are given by

$$\mathbf{a}_T(\beta) = \left[e^{2\pi i p_1 \beta}, e^{2\pi i p_2 \beta}, \dots, e^{2\pi i p_{N_T} \beta}\right]^T, \tag{3}$$

and

$$\mathbf{a}_R(\beta) = \left[e^{2\pi i q_1 \beta}, e^{2\pi i q_2 \beta}, \dots, e^{2\pi i q_{N_R} \beta}\right]^T, \tag{4}$$

where we assume that the relative antenna spacings p_j's and q_j's are i.i.d. uniformly on $[0, \frac{N_R N_T}{2}]$. The j-th transmit antenna repeatedly transmits the signal $s_j(t)$ and the receive antennas take N_s samples of the signal. Let $\mathbf{Z}(t;\beta,\tau,f)$ be the $N_R \times N_s$ noise-free received signal matrix from a unit strength target at direction β, delay τ, and Doppler f (corresponding to its radial velocity with respect to the radar). Then

$$\mathbf{Z}(t;\beta,\tau,f) = \mathbf{a}_R(\beta)\mathbf{a}_T^T(\beta)\mathbf{S}_{\tau,f}^T,$$

where $\mathbf{S}_{\tau,f}$ is a $N_s \times N_T$ matrix whose columns are the circularly delayed and Doppler shifted signals $s_j(t-\tau)e^{2\pi i f t}$.

We let $\mathbf{z}(t;\beta,\tau,f) = \text{vec}\{\mathbf{Z}\}(t;\beta,\tau,f)$ be the noise-free vectorized received signal. We set up a discrete azimuth-range-Doppler grid $\{\beta_l, \tau_j, f_k\}$ for $1 \le l \le N_\beta$, $1 \le j \le N_\tau$ and $1 \le k \le N_f$, where $\Delta_\beta, \Delta_\tau$ and Δ_f denote the corresponding discretization stepsizes. Using vectors $\mathbf{z}(t;\beta_l,\tau_j,f_k)$ for all grid points (β_l,τ_j,f_k) we construct a complete response matrix $\mathbf{A}$ whose columns are $\mathbf{z}(t;\beta_l,\tau_j,f_k)$ for $1 \le l \le N_\beta$ and $1 \le j \le N_\tau$, $1 \le k \le N_f$. In other words, $\mathbf{A}$ is a $N_R N_s \times N_\tau N_\beta N_f$ matrix with columns

$$\mathbf{A}_{\beta,\tau,f} = \mathbf{a}_R(\beta) \otimes \mathbf{S}_{\tau,f}\mathbf{a}_T(\beta). \tag{5}$$

Assume that the radar illuminates a scene consisting of S scatterers located on S points of the (β_l,τ_j,f_k) grid. Let $\mathbf{x}$ be a sparse vector whose non-zero elements are the complex amplitudes of the scatterers in the scene. The zero elements corresponds to grid points which are not occupied by scatterers. We can then define the radar signal $\mathbf{y}$ received from this scene by (1) where $\mathbf{y}$ is an $N_R N_s \times 1$ vector, $\mathbf{x}$ is an $N_\tau N_\beta N_f \times 1$ sparse vector and $\mathbf{w}$ is an $N_R N_s \times 1$ complex Gaussian noise vector. Our goal is to solve for $\mathbf{x}$, i.e., to locate the scatterers (and their reflection coefficients) in the azimuth-delay-Doppler domain.

As for the signal matrix $\mathbf{S}$, for our main results we choose the Kerdock waveforms, as described in Section III, as discrete transmission waveforms.

Remark: The assumption that the targets lie on the grid points, while common in compressive sensing, is certainly restrictive. A violation of this assumption will result in a model mismatch, sometimes dubbed *gridding error*, which can potentially be quite severe [9], [5]. Recently some interesting strategies have been proposed to overcome this gridding error [6], [15]. But these methods are not directly applicable to our setting. This model mismatch issue is beyond the scope of this paper and will be addressed in our future research.

III. Kerdock Codes

We briefly review the construction of Kerdock codes and some of their fundamental properties. A simple way to construct these Kerdock codes is the following, in which they arise as eigenvectors of time-frequency shift operators. Let p be an odd prime number and consider the translation operator $\mathbf{T}$ and the modulation operator $\mathbf{M}$ on $\mathbb{C}^p$. For each $k = 0, \dots, p-1$ we compute the eigenvector decomposition of $\mathbf{TM}_k$ (which always exists, since $\mathbf{TM}_k$ is a unitary matrix)

$$U_{(k)}\Sigma_{(k)}U_{(k)}^* = \mathbf{TM}_k, \tag{6}$$

where the unitary matrix $U_{(k)}$ contains the eigenvectors of $\mathbf{TM}_k$ and the diagonal matrix $\Sigma_{(k)}$ the associated eigenvalues[1]. Furthermore, we define $U_{(p)} := \mathbf{I}_p$. Now, let $u_{k,j}$ be the j-th column of $U_{(k)}$. The set consisting of the p^2+p vectors $\{u_{k,j}, k = 0,\dots,p; j = 0,\dots,p-1\}$ forms a $\mathbb{Z}_p$-Kerdock code. There are numerous equivalent ways to derive this Kerdock code, but, as pointed out earlier, not *all* Kerdock codes over $\mathbb{Z}_p$ are equivalent (see also the comment following

[1]The attentive reader will have noticed that $U_{(0)}$ is just the $p \times p$ DFT matrix.

Corollary 11.6 in [2]). But we will be a bit sloppy, and simply refer to the Kerdock code constructed above as *the* Kerdock code.

In the following theorem we collect those key properties of Kerdock codes that are most relevant for radar. These properties are either explicitly proved in [2], [10] or can be derived easily from properties stated in those papers.

Theorem 3.1: Kerdock codes over $\mathbb{Z}_p$, where p is an odd prime, satisfy the following properties:

(i) Mutually unbiased bases: For all $k = 0, \dots, p$ and all $j = 0, \dots, p-1$, there holds:

$$|\langle u_{k,j}, u_{k',j'} \rangle| = \begin{cases} 1 & \text{if } k = k', j = j', \\ 0 & \text{if } k = k', j \neq j', \\ \frac{1}{\sqrt{p}} & \text{if } k \neq k'. \end{cases}$$

(ii) Time-frequency "autocorrelation":
(a) For any fixed $(f, l) \neq (0, 0)$ there exists a unique k_0 such that

$$|\langle \mathbf{M}_f \mathbf{T}_l u_{k_0,j}, u_{k_0,j} \rangle| = 1 \qquad \text{for } j = 0, \dots, p-1, \quad (7)$$

$$|\langle \mathbf{M}_f \mathbf{T}_l u_{k,j}, u_{k,j} \rangle| = 0 \qquad \text{for } k \neq k_0. \quad (8)$$

(b) For any fixed $0 \leq k \leq p-1$, there exist distinct (f_r, l_r), $r = 1, \dots, p$ such that

$$|\langle \mathbf{M}_{f_r} \mathbf{T}_{l_r} u_{k,j}, u_{k,j} \rangle| = 1 \qquad \text{for } j = 0, \dots, p-1, \quad (9)$$

(iii) Time-frequency crosscorrelation: For all $k \neq k'$ and all f and l there holds:

$$|\langle \mathbf{M}_f \mathbf{T}_l u_{k,j}, u_{k',j} \rangle| \leq \frac{1}{\sqrt{p}} \qquad \text{for } j = 0, \dots, p-1. \quad (10)$$

We emphasize though that Kerdock codes would not be very effective for a radar system with a single transmit antenna (SISO or SIMO radar). This can be easily seen as follows: Suppose we only have one antenna that transmits one waveform $\vec{s}$. Because of (9), $\vec{s}$ is equal to (up to a phase factor) $\mathbf{M}_f \mathbf{T}_l \vec{s}$ for some f, l. In practice, this prevents us from determining the distance and the speed of the object.

As a consequence of the aforementioned ambiguity we will not use *all* of the Kerdock codes as transmission signals for our MIMO radar, instead we will choose one code for each index k. The reason is that we need the waveforms to have low time-frequency crosscorrelation, while (10) only holds when k and k' are different.

*Definition 3.2 (**Kerdock waveforms**):* Let $\{\mathbf{u}_{k,j}, k = 0, \dots, p, j = 0, \dots, p-1\}$ be a Kerdock code over $\mathbb{Z}_p$. The *Kerdock waveforms* $\mathbf{k}_0, \dots, \mathbf{k}_r$, where $r < p$, are given by $\mathbf{k}_k = \mathbf{u}_{k,j}$ for some arbitrary j. In other words, for each $k = 0, \dots, r-1$ we pick an arbitrary vector from the orthonormal basis $\{\mathbf{u}_{k,j}\}_{j=0}^{p-1}$.
Note Kerdock waveforms do not include any canonical vectors, since only the first r unitary matrices $\mathbf{U}_{(0)}, \dots, \mathbf{U}_{(r-1)}$ are considered and r is strictly less than p (recall $\mathbf{U}_{(p)} = \mathbf{I}_p$).

IV. The Main Theorem

As mentioned in the introduction, a standard approach to solve (1) when $\mathbf{x}$ is sparse, is given in (2). But instead of (2), we will use the *debiased lasso*. That means first we compute an approximation $\tilde{I}$ for the support of $\mathbf{x}$ by solving (2). This is the detection step. Then, in the estimation step, we "debias" the solution by computing the amplitudes of $\mathbf{x}$ via solving the reduced-size least squares problem $\min \|\mathbf{A}_{\tilde{I}} \mathbf{x}_{\tilde{I}} - \mathbf{y}\|_2$, where $\mathbf{A}_{\tilde{I}}$ is the submatrix of $\mathbf{A}$ consisting of the columns corresponding to the index set $\tilde{I}$, and similarly for $\mathbf{x}_{\tilde{I}}$.

We assume x is drawn from a generic S-sparse target model. We are now ready to state our main result (more details of this theorem can be found in [14]).

Theorem 4.1: Consider $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$, where $\mathbf{A}$ is defined as in (5) and $\mathbf{w}_j \in \mathcal{C}N(0, \sigma^2)$. Assume that the positions of the transmit and receive antennas p_j's and q_j's are chosen i.i.d. uniformly on $[0, \frac{N_R N_T}{2}]$ at random. Suppose further that each transmit antenna sends a different Kerdock waveform, i.e. the columns of the signal matrix $\mathbf{S}$ are different Kerdock waveforms. Suppose that

$$\max\left(N_R N_T, 32 N_T^3 \log N_\tau N_f N_\beta\right) \leq N_s = N_\tau, \quad (11)$$

and also

$$\log^2 N_\tau N_f N_\beta \leq N_T \leq N_R. \quad (12)$$

If $\mathbf{x}$ is drawn from the generic S-sparse scatterer model with

$$S \leq \frac{c_0 N_\tau}{\log N_\tau N_f N_\beta} \quad (13)$$

for some constant $c_0 > 0$, and if

$$\min_{k \in I} |\mathbf{x}_k| > \frac{8\sqrt{3}\sigma}{\sqrt{N_R N_T}} \sqrt{2 \log N_\tau N_f N_\beta}, \quad (14)$$

then the solution $\tilde{\mathbf{x}}$ of the debiased lasso computed with $\lambda = 2\sigma\sqrt{2 \log N_\tau N_f N_\beta}$ satisfies with high probability

$$\mathrm{supp}(\tilde{\mathbf{x}}) = \mathrm{supp}(\mathbf{x}), \quad (15)$$

and

$$\frac{\|\tilde{\mathbf{x}} - \mathbf{x}\|_2}{\|\mathbf{x}\|_2} \leq \frac{5\sigma\sqrt{3 N_R N_s}}{\|\mathbf{y}\|_2}. \quad (16)$$

Remarks:

1) The condition $N_T \leq N_R$ in (12) is by no means necessary, but rather to make our computation a little cleaner. We could change it into $N_T \leq 2N_R$, then the theorem would be true with a slightly different probability of success.
2) It may seem that the conditions in (11) and (12) are a bit restrictive. But, in practice, our method works with a broad range of parameters.

The proof of the above theorem is rather involved and too long to be included in this brief paper. The full proof of this theorem, as well as other results presented in this paper can be found in the journal version of this paper [14]. Here, we can only sketch the key steps. To prove Theorem 4.1, we use a theorem by Candès and Plan (Theorem 1.3 in [3]) which requires to estimate the operator norm of $\mathbf{A}$ and the coherence of $\mathbf{A}$. The original theorem only treats the real-valued case, it can be extended to complex-valued case after some straightforward modifications (see Appendix B in [13]).

V. Extension of the main result

In this section, we present a modified version of Theorem 4.1 that applies to waveforms that satisfy slightly more restrictive incoherence conditions. As such, Theorem 5.1 below does not hold for Kerdock waveforms, but the advantage compared to Theorem 4.1 is that the result also applies to radar systems with only one transmit antenna.

Theorem 5.1: Consider $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$, where $\mathbf{A}$ is defined as in (5) and $\mathbf{w}_j \in \mathcal{CN}(0, \sigma^2)$. Suppose the transmission waveforms satisfy the following conditions

$$|\langle \vec{s}_j, \mathbf{M}_f \mathbf{T}_\tau \vec{s}_j \rangle| \leq \frac{\gamma}{\sqrt{p}} \quad \text{for} \quad (f,\tau) \neq (0,0), \tag{17}$$

$$|\langle \vec{s}_k, \mathbf{M}_f \mathbf{T}_\tau \vec{s}_j \rangle| \leq \frac{\gamma}{\sqrt{p}} \quad \text{for} \quad k \neq j, \tag{18}$$

where $\gamma > 0$ is a fixed constant. Assume that the positions of the transmit and receive antennas p_j's and q_j's are chosen i.i.d. uniformly on $[0, \frac{N_R N_T}{2}]$. Choose the same discretization stepsizes to be $\Delta_\beta = \frac{2}{N_R N_T}$,$\Delta_\tau = \frac{1}{2B}$, $\Delta_f = \frac{1}{T}$ and suppose that

$$\max\left(\gamma^2 N_R N_T, 16\gamma^2 N_T \log^3 N_\tau N_f N_\beta\right) \leq N_s = N_\tau$$

and also

$$\gamma^2 N_T \log^4 N_\tau N_f N_\beta \leq N_s N_R, \quad \log^2 N_\tau N_f N_\beta \leq N_T \leq N_R.$$

Then if the rest of the conditions of Theorem 4.1 hold, we have the same conclusion as in Theorem 4.1.

There are several examples of signal sets that satisfy the above conditions. Perhaps the most intriguing example is the finite harmonic oscillator system (FHOS) constructed in [7]. This signal set in $\mathbb{C}^p$ (where p is a prime number) of cardinality $\mathcal{O}(p^3)$ satisfies (17) and (18) with $\gamma = 4$. An elementary construction of the FHOS for prime number $p \geq 5$ can be found in [16].

VI. Simulations

In this section we will demonstrate the performance of our algorithms via numerical simulations. We use the Matlab Toolbox TFOCS ([1]).

We choose Kerdock codes as transmission waveforms along with the parameters: $N_T = 6, N_R = 6, N_s = 37, N_f = 37$. The number of the scatters $S = 10, 20, 40$ while the SNR is chosen to be 20dB.

The values of the estimated vector $\hat{\mathbf{x}}$ corresponding to the true scatterer locations are compared to a threshold. Detection is declared whenever a value exceeds the threshold. The probability of detection P_d is defined as the number of detections divided by S. Next the values of the estimated vector $\hat{\mathbf{x}}$ corresponding to locations not containing scatterers are compared to the same threshold. A false alarm is declared whenever one of these values exceeds the threshold. The probability of false alarm P_{fa} is defined as the number of false alarms divided by $n - S$, where n is the signal dimension. The results are averaged over the 50 repetitions of the experiment. The probabilities are computed for a range of values of the threshold to produce the so-called Receiver Operating Characteristics (ROC)- the graph of P_d vs. P_{fa}.

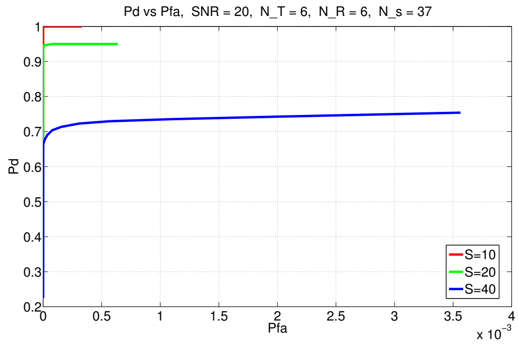

Fig. 1. MIMO, Kerdock codes, SNR=20

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