EESM AS A LINK TO SYSTEM LEVEL INTERFACE FOR MIMO OFDM SYSTEMS
BASED ON QOSFBC

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ABSTRACT

This article focuses on the physical layer abstraction of an orthogonal frequency division multiplexing (OFDM) transmission link augmented with multiple transmit and receive antennas. An existing link level modeling method, the so-called exponential effective SNR mapping (EESM), originally developed for single-antenna transmissions, is extended to handle the multi-array link enhancement. Simulative analyses investigate and validate the applicability of the proposed link level modeling scheme to multi-antenna communications. The method provides an acceptably accurate link quality estimation and can be utilized in a system level simulator as a link to system level interface.

1. INTRODUCTION

System level evaluation of multi-array techniques is of major significance, since multiple-input multiple-output (MIMO) performance gains which link level simulation promises, may not translate to equivalent gains at the system level. The simulation of a wireless cellular network involves, in general, scenarios of multiple stations and thus necessitates a simple and efficient way of physical layer modeling of each individual link. A link to system level interface is achieved, which is typically based on a prediction of the instantaneous link performance such as the link Packet Error Ratio (PER). Conventional PER prediction methods offer the average received signal to noise ratio (SNR) as a link to system level metric. Several publications considering single-antenna multi-carrier transmissions over frequency-selective channels reveal that this approach is suboptimum and correspondingly suggest advanced PER prediction schemes [1, 2].

The present work extends these investigations in the context of MIMO OFDM systems based on quasi-orthogonal space-frequency block coding (QOSFBC). Considered is one of the proposed single-antenna link modeling methods: the exponential effective SNR mapping (EESM) [4], which predicts the link PER as a Link Quality Indicator (LQI).

The rest of the paper is organized as follows. QOSFBC is introduced in section 2 and the corresponding effective channel representation is derived. Section 3 explains the basics of EESM and the methodology to apply this mapping technique as a link to system level interface in the considered MIMO OFDM system. In Section 4 simulation results are presented assuming different channel conditions. Finally, conclusions are drawn in Section 5.

2. LINK MODEL

The considered system represents a single user point-to-point MIMO OFDM link with \( n_T = 4 \) transmit and \( n_R = 4 \) receive antennas. The MIMO processing is carried out in the subcarrier domain within subsets of \( F \) subcarriers. Rate-one QOSFBC with codeword length \( n_T = F \) is employed.

The signal model is defined like [3]:

\[
Y = \sqrt{\frac{E_s}{n_T}}GH + N, \tag{1}
\]

where \( Y \) denotes the \((F \times n_T)\) receive codeword matrix, \( H \) the \((n_T \times n_R)\) channel matrix with elements \( h_{ij} \), and \( N \) the complex \((F \times n_R)\) white Gaussian noise (AWGN) matrix, respectively. The average signal energy is \( E_s \) and noise energy is \( N_0 \). \( G \) represents the \((F \times n_T)\) transmit matrix:

\[
G = \begin{bmatrix}
x_1 & x_2 & x_3 & x_4 \\
x_2^* & -x_1^* & x_4^* & -x_3^* \\
x_3 & -x_4 & -x_1 & x_2 \\
x_4^* & x_3^* & -x_2^* & -x_1^* 
\end{bmatrix}. \tag{2}
\]

The channel matrix is assumed to be constant for a subband of \( F \) subcarriers, which is a general assumption in
space-frequency block coding. In addition, a temporal block-fading channel model is considered, i.e. the channel is assumed to remain constant for the duration of a packet.

The information vector \( \mathbf{x} = [x_1, x_2, x_3, x_4]^T \) is gained from a vector \( \mathbf{s} = [s_1, s_2, s_3, s_4]^T \) containing complex constellation symbols, according to the precoding scheme proposed in [3]:

\[
\begin{bmatrix}
x_1 \\
x_3
\end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix}
s_1 \\
s_2
\end{bmatrix}, \quad \begin{bmatrix}
x_2 \\
x_4
\end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix}
s_4 \\
s_3
\end{bmatrix},
\]

where \( \mathbf{\Gamma} \) is a unitary matrix:

\[
\mathbf{\Gamma} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
-1 & i
\end{bmatrix}.
\]

Block-diagonalizing and accounting for the pre-processing at the receiver (channel matched filtering and noise pre-whitening), the system model of equation (1) reduces to the following equivalent [3]:

\[
\mathbf{y} = \sqrt{\frac{E_s}{nT}} \mathbf{H}_{eq} \begin{bmatrix}
\mu_1 & 0 & 0 & 0 \\
0 & \mu_2 & 0 & 0 \\
0 & 0 & \mu_3 & 0 \\
0 & 0 & 0 & \mu_2
\end{bmatrix} \begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4
\end{bmatrix} + \mathbf{w},
\]

where \( \mathbf{y} \) denotes the \((n_T \times 1)\) receive vector. The terms \( \mu_1 = \sqrt{\alpha_1 + \alpha_2} \) and \( \mu_2 = \sqrt{\alpha_1 - \alpha_2} \) are a function solely of the channel state, since:

\[
\alpha_1 = ||\mathbf{H}||^2_F, \\
\alpha_2 = \sum_{i=1}^{n_T} 2 \Im (h_{1i}^* h_{3i} + h_{4i}^* h_{2i}).
\]

The signals transmitted from different antennas are completely decoupled: the constellation symbols do not experience interference from each other and can be decoded independently. The matrix \( \mathbf{H}_{eq} \) in equation (5) can be seen as an equivalent channel per subband of \( F \) subcarriers, where a term \( h_{eq,k} = \mu_k \sqrt{E_s/nT} \) represents the effective scalar transfer factor of the \( k \)-th subcarrier.

### 3. EXPONENTIAL EFFECTIVE SNR MAPPING

The exponential effective SNR mapping (EESM) has been proposed as a tool to model the single-input single-output (SISO) OFDM link-level performance [4]. Many contributions show that this technique provides an accurate instantaneous PER estimation independent of the channel type.

The following mapping function, which maps the set of subcarrier SNRs \( \{\gamma_k\} \) to a scalar \( \gamma_{\text{eff}} \), is offered [4]:

\[
\gamma_{\text{eff}} = -\beta \ln \left( \frac{1}{N} \sum_{k=1}^{N} e^{-\frac{\gamma_k}{\beta}} \right),
\]

where \( \gamma_k \) is the SNR experienced on the \( k \)-th subcarrier, \( N \) is the total number of subcarriers and \( \gamma_{\text{eff}} \) is the instantaneous effective SNR. The parameter \( \beta \) depends on the modulation and coding scheme (MCS), but does not rely on the channel type. Its value is to be optimized through link level simulations to provide the best matching.

The idea behind this compression is that the computed \( \gamma_{\text{eff}} \) should be approximately equal to the SNR \( \gamma_{awgn} \) that would yield in an AWGN channel a PER equal to the actual instantaneous PER in a frequency-selective channel, i.e.:

\[
\text{PER}_{\text{AWGN}}(\gamma_{\text{eff}}) \approx \gamma_{awgn} \approx \text{PER}_{\text{inst}}(\{\gamma_k\}).
\]

\( \text{PER}_{\text{AWGN}}(\gamma_{\text{eff}}) \) is the PER vs. SNR curve for the AWGN channel and \( \text{PER}_{\text{inst}}(\{\gamma_k\}) \) is the actual PER for the instantaneous channel state \( \{\gamma_k\} \). This means that the link error prediction algorithm depends only on the single instantaneous variable \( \gamma_{\text{eff}} \) and the \( a\text{-priori} \) knowledge of the PER curve for the AWGN channel: the latter can be read at the effective SNR value \( \gamma_{\text{eff}} \) in order to predict the instantaneous PER.

Now the EESM algorithm, which is originally developed and applicable for single-antenna systems, can be applied as a link to system level interface in the considered MIMO system based on QOSFBC. Using the channel state information (CSI), the equivalent channel of each independent channel realization can be evaluated according to equation (5). Since there is no interaction between the constellation symbols, the EESM can be directly applied to this effective channel.

Although initially derived for soft-decision Viterbi decoding, the EESM formula in the form of equation (8) can also be applied for hard-decision decoders [5]. In the following, unless otherwise stated, results of hard-decision Viterbi decoding are presented.

### 4. SIMULATION RESULTS

The system under consideration is based on the IEEE 802.11a standard, which is extended with 4x4 MIMO processing based on QOSFBC; table 1 summarizes the most important system parameters. For evaluation the MIMO wireless LAN channel models proposed in [6] are considered: multipath, multicluster channels are generated. A temporal block-fading channel model is considered throughout this contribution. In subsection 4.1 simulation results are presented considering channels for which, in addition, block-fading in frequency is assumed. Subsection 4.2 deals with temporal block-fading frequency-selective channels.
### 4.1. EESM performance for channels assuming block-fading in frequency

The general premise in space-frequency block coding (SFBC) is the assumption that the channel is constant over the subcarriers on which the coding is performed, i.e. within a subband of \( F \) subcarriers. In order to meet this assumption the frequency-selective channels suggested in [6] are modified accordingly.

Figure 1 illustrates the EESM performance for QPSK modulation with code rate \( R_C = 3/4 \). Instantaneous PER vs. \( E_s/N_0 \) curves for many independent channel realizations obtained by link-level simulation are shown in red color. For each channel realization the equivalent channel \( H_{eq} \) is evaluated and at a certain \( E_s/N_0 \) the SNR on each subcarrier \( \gamma_k \) is found as \( \gamma_k = |h_{eq,k}|^2 E_s/N_0 \). With the help of equation (8), the set of subcarrier SNRs \( \{ \gamma_k \} \) is mapped into the scalar \( \gamma_{eff} \). Thus, to each \((E_s/N_0, \text{PER})\) point (red circles), an equivalent \((\gamma_{eff}, \text{PER})\) point is found (green crosses). The nearer the mapped point to the AWGN PER curve, the smaller the mapping error \( \Delta \gamma = \gamma_{eff} - \gamma_{awgn} \). As can be inferred from figure 1, the estimation accuracy of the EESM algorithm decreases with a declining \( \text{PER} \). This is due to the fact that at lower \( \text{PERs} \), the slope of the instantaneous \( \text{PER} \) curves deviates more and more from that one of the AWGN \( \text{PER} \) curve.

For the mapping already shown, the parameter \( \beta \) is optimized in the MMSE sense at \( \text{PER}_{\text{target}} = 10^{-1} \). Consequently, with the obtained parameter value \( \beta_{\text{opt}} \). EESM is carried out on an independent set of 120 channel realizations. Figure 2 depicts the performance of the mapping at \( \text{PER}_{\text{target}} \): most of the prediction errors fall within a \( \pm 0.5 \text{ dB} \) range. As a reference the histogram of the error of a conventional method is also plotted, where for PER prediction the \( \text{PER} \) vs. \( E_s/N_0 \) curve averaged over many frequency-selective channel realizations is taken into account (the black dotted curve in figure 1).

### Table 1. System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Sampling period</td>
<td>50 ns</td>
</tr>
<tr>
<td>FFT size</td>
<td>64</td>
</tr>
<tr>
<td>Number of used subcarriers</td>
<td>48</td>
</tr>
<tr>
<td>Guard interval</td>
<td>16 samples</td>
</tr>
<tr>
<td>Maximum path delay</td>
<td>0.8 ( \mu s )</td>
</tr>
<tr>
<td>Bits per subcarrier used</td>
<td>1, 2, 4, 6</td>
</tr>
<tr>
<td>Convolutional code</td>
<td>(133,171)</td>
</tr>
<tr>
<td>Coding rate</td>
<td>3/4</td>
</tr>
<tr>
<td>Packet size</td>
<td>432 bits</td>
</tr>
<tr>
<td>Antenna element spacing (BS,MS)</td>
<td>0.5 ( \lambda )</td>
</tr>
</tbody>
</table>

Fig. 1. Exponential Effective SNR Mapping in 4x4 QOSFBC for QPSK modulation and code rate 3/4: each instantaneous \( \text{PER} \) (red) is mapped into an effective one (green). Perfect mapping occurs when an effective point lies on the \( \text{PER} \) curve in a SISO AWGN channel. 120 channel realizations [6] with frequency block-fading, \( \beta_{\text{opt}} = 1.61 \).

Fig. 2. Histogram of: a) the EESM error, i.e. the difference between the estimated effective SNR \( \gamma_{eff} \) and the SNR \( \gamma_{awgn} \) necessary in an AWGN channel for \( \text{PER}_{\text{target}} \), b) the prediction error of a conventional scheme, i.e. the difference between the instantaneous frequency-averaged receive SNR \( \bar{\gamma} \) and the SNR \( \gamma_{awgn} \) averaged over many channel realizations. QPSK modulation with code rate 3/4 in 4x4 QOSFBC. \( \text{PER}_{\text{target}} = 10^{-1} \), 120 channel realizations [6] with frequency block-fading, \( \beta_{\text{opt}} = 1.61 \).

Table 2 summarizes statistics of the mean absolute EESM error for different physical layer (PHY) modes. For each constellation order, the mapping reliability at \( \text{PER}_{\text{target}} = 10^{-1} \) is examined for 240 frequency block-fading channels of different type [6]. Results for soft decision Viterbi decoding with four-level (2 bits) quantized inputs are listed in addition as a reference. In contrast to the soft decision case, in the hard decision case the prediction accuracy for higher modulation orders decreases slightly.
The sensitivity of the EESM error with respect to the parameter $\beta$ is similar for all considered PHY-Modes (figure 3). An acceptable performance of the algorithm within a relatively wide range of $\beta$ values is guaranteed for each MCS.

Figure 4 depicts the optimum $\beta$ values per MCS at different target PERs. For each constellation order $\beta_{opt}$ is almost constant with respect to $\text{PER}_{\text{target}}$. Having in mind the low sensitivity of the EESM performance with respect to slight variations of $\beta$ (see figure 3), one and the same $\beta_{opt}$ value can be considered per MCS in the mapping process within the whole target PER range of 0.05 - 0.5. This is of advantage, since the PER prediction over different target PER levels remains a one-dimensional task. The value $\beta_{opt}$ for each MCS is found by averaging the $\beta_{opt}$ values at all $\text{PER}_{\text{target}}$ in the considered range.

The performance of the EESM scheme over many realizations of frequency block-fading channels of different type [6] is depicted as a function of $\text{PER}_{\text{target}}$ in figure 5.

**Table 2.** EESM error statistics and optimized $\beta$ values for hard and 2 bits quantized soft decision Viterbi decoding. 240 channel realizations [6] with frequency block-fading. $R_C = 3/4$.

<table>
<thead>
<tr>
<th>PHY-mode</th>
<th>Hard-decision</th>
<th>Soft-decision</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean abs. error [dB]</td>
<td>$\beta_{opt}$</td>
</tr>
<tr>
<td>BPSK</td>
<td>0.265</td>
<td>0.89</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.247</td>
<td>1.64</td>
</tr>
<tr>
<td>16QAM</td>
<td>0.315</td>
<td>8.34</td>
</tr>
<tr>
<td>64QAM</td>
<td>0.345</td>
<td>29.80</td>
</tr>
</tbody>
</table>

Fig. 3. Mean Squared Error of EESM as a function of the mapping parameter $\beta$. Modulations with code rate $3/4$ in 4x4 QOSFBC. $\text{PER}_{\text{target}} = 10^{-1}$. 240 channel realizations [6] with frequency block-fading.

Fig. 4. Optimum values of the mapping parameter $\beta$ for different target PERs. Modulations with code rate $3/4$ in 4x4 QOSFBC. 240 channel realizations [6] with frequency block-fading.

Fig. 5. EESM performance at different target PER levels. Modulations with code rate $3/4$ in 4x4 QOSFBC. 240 channel realizations [6] with frequency block-fading.
wireless system. The stringent PER requirements at the physical layer are alleviated through allowing packet retransmissions at the data link layer [8]. Hence, due to the lower target PERs that have to be guaranteed, a higher reliability and accuracy of the EESM algorithm can be expected.

4.2. EESM performance for frequency-selective channels

This subsection presents results for the case when the assumption of block-fading in frequency is relaxed, i.e., now each subcarrier has a distinct channel gain.

The EESM performance is examined again at $\text{PER}_{\text{target}} = 10^{-1}$ for 240 frequency-selective channels of different type [6]. Table 3 summarizes statistics of the mean absolute EESM error for different PHY-Modes. While the mapping error for BPSK and QPSK is comparable with the one over frequency block-fading channels (see table 2), for higher modulation orders it becomes infinite.

The origin of this problem does not lie in the EESM algorithm, but in the applied MIMO processing. The assumption in SFBC of constant subcarriers within a codeword will not always be met in wireless LAN channels. It has been shown that, especially for high modulation orders, the 4x4 QOSFBC transmission over frequency-selective channels is severely affected [9].

In channels with large multipath spreading, the slope of the instantaneous PER vs. SNR curves deviate significantly from that one of the AWGN SISO PER curve. Some of the PER curves even saturate at a certain PER level. In the former case this results in a high mapping error, while in the latter case in an infinite one.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>0.250</td>
<td>0.248</td>
<td>0.75</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.370</td>
<td>0.313</td>
<td>1.10</td>
</tr>
<tr>
<td>16-, 64-QAM</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>-</td>
</tr>
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</table>

5. CONCLUSIONS

The presented work offers a new link to system level interface metric for MIMO multi-carrier communications based on QOSFBC. It should be noted that the herein presented methodology for applying the EESM technique in multi-array transmissions can readily be applied in any type of MIMO communication system by deriving the corresponding equivalent system model representation.

As the simulative investigations show, the accuracy of the proposed algorithm is acceptable and its performance is robust. Besides its application as a component for a more efficient and accurate system-level evaluation, this interface can be utilized in real system architecture to improve the performance of link adaptation techniques and thus increase system throughput and spectral efficiency.

6. REFERENCES