OPTIMAL TIME INVARIANT AND WIDELY LINEAR SPATIAL FILTERING FOR RADIOCOMMUNICATIONS

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ABSTRACT

The classical optimal array filtering problem assumes stationary signals and consists to implement a complex linear and Time Invariant (TI) filter, optimizing a second order criterion at the output under some possible constraints. Optimal for stationary signals this approach is sub-optimal for non stationary signals for which the optimal complex filters are Time Variant (TV) and, under some conditions of non circularity, Widely Linear (WL). The purpose of this paper is to present the interest of WL structures of spatial filtering with respect to linear ones in non stationary radiocommunications environments.

1. INTRODUCTION

The classical optimal array filtering problem assumes stationary signals and consists to implement a complex *linear* and *TI* filter, $\boldsymbol{h}$, which output $y(t) = \boldsymbol{h}^{\dagger}\boldsymbol{x}(t)$, where $\boldsymbol{x}(t)$ is the vector of the complex envelopes of the spatio-temporal observations at the output of the sensors, optimizes a second order criterion under some possible constraints [1]. Limiting the analysis to the exploitation of the second order statistics of the data, it has been shown recently [2] that although optimal for stationary signals this classical approach becomes sub-optimal for non stationary signals, which statistics are time-dependent (TD) and which complex envelope may be second order non circular [3]. More precisely, for non stationary signals, it has been shown in [2] that the optimal complex filters become TV and, under some non circularity conditions, WL [4], i.e. of the form $y(t) = \boldsymbol{h}_1(t)^{\dagger}\boldsymbol{x}(t) + \boldsymbol{h}_2(t)^{\dagger}\boldsymbol{x}(t)^*$, where * means complex conjugate and where $\boldsymbol{h}_1(t)$ and $\boldsymbol{h}_2(t)$ are TV complex filters.

Removing only the linearity constraint of classical complex filters, the purpose of this paper is to present the interest of TI and WL structures of spatial filtering with respect to linear ones in non stationary radiocommunications environments.

2. PROBLEM FORMULATION

Let us consider an array of N narrow-band (NB) sensors and let us call $\boldsymbol{x}(t)$ the vector of the complex amplitudes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of a useful signal corrupted by undesired signals such as noise or interference. Under these assumptions, the observation vector can be written as follows

$$\boldsymbol{x}(t) = s(t)\, e^{j\phi_0}\, \boldsymbol{s} + \boldsymbol{b}(t) \quad (2.1)$$

where $\boldsymbol{b}(t)$ is the undesired signals vector, called in the following noise vector, uncorrelated with the useful signal and $s(t)$, ϕ_0 and $\boldsymbol{s}$ are the complex envelope, the phase and the steering vector of the received useful signal respectively. Under these assumptions, the problem is then to find the TI complex filters $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ which minimize the temporal mean, $<\mathcal{E}(t)>$, over the observation window [0, T], of the instantaneous mean square error (MSE) $\mathcal{E}(t)$, defined by

$$\mathcal{E}(t) \triangleq \mathrm{E}[|d(t) - \boldsymbol{h}_1^{\dagger}\boldsymbol{x}(t) - \boldsymbol{h}_2^{\dagger}\boldsymbol{x}(t)^*|^2] \quad (2.2)$$

where $d(t)$ is a training sequence correlated with $s(t)$ and not correlated with $\boldsymbol{b}(t)$ and where $<\mathcal{E}(t)>$ is defined by

$$<\mathcal{E}(t)> \triangleq (1/\mathrm{T}) \int_0^T \mathcal{E}(t)\, dt \quad (2.3)$$

Note that in non stationary contexts, the concept of optimal TI filters has a meaning only with respect to criterions which are temporal averages of instantaneous criterions [2].

3. OPTIMAL WL SPATIAL FILTER

Noting $\boldsymbol{H} \triangleq (\boldsymbol{h}_1^T, \boldsymbol{h}_2^T)^T$ and $\boldsymbol{X}(t) \triangleq (\boldsymbol{x}(t)^T, \boldsymbol{x}(t)^{\dagger})^T$, the optimal WL spatial filter, $\boldsymbol{H}_o$, is given by

$$\boldsymbol{H}_o = \langle R_X(t)\rangle^{-1} \langle \boldsymbol{r}_{Xd}(t)\rangle \tag{3.1}$$

where $\langle R_X(t)\rangle$ and $\langle \boldsymbol{r}_{Xd}(t)\rangle$ are the temporal mean of the matrix $R_X(t) \triangleq \mathrm{E}[\boldsymbol{X}(t)\boldsymbol{X}(t)^{\dagger}]$ and the vector $\boldsymbol{r}_{Xd}(t) \triangleq \mathrm{E}[\boldsymbol{X}(t)d(t)^*]$ respectively.

In the linear case ($\boldsymbol{h}_2 = \boldsymbol{0}$), the expression (3.1) gives the classical Wiener solution, where the vector $\boldsymbol{X}(t)$ is replaced by the observation vector $\boldsymbol{x}(t)$, which finally gives, to within a constant, the well known spatial matched filter (SMF) [1] defined by $\boldsymbol{h}_{1o} = \langle R(t)\rangle^{-1}\boldsymbol{s}$, where $R(t) \triangleq \mathrm{E}[\boldsymbol{b}(t)\boldsymbol{b}(t)^{\dagger}]$.

However, in the WL case, the optimal TI spatial filter $\boldsymbol{H}_o$ is a function of the temporal mean of the first and second correlation matrices of the data, defined by $R_x(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^{\dagger}]$ and $C_x(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^{T}]$ respectively, and the first and second correlation vectors $\boldsymbol{r}_{xd}(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)d(t)^*]$ and $\boldsymbol{c}_{xd}(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)d(t)]$ respectively. Assuming $d(t) = s(t)$, the previous matrices and vectors are given, under the previous assumptions, by

$$R_x(t) = \pi_s(t)\, \boldsymbol{s}\boldsymbol{s}^{\dagger} + R(t) \tag{3.2}$$

$$C_x(t) = c_s(t)\, e^{j2\phi_0}\, \boldsymbol{s}\boldsymbol{s}^{T} + C(t) \tag{3.3}$$

$$\boldsymbol{r}_{xd}(t) = \pi_s(t)\, e^{j\phi_0}\, \boldsymbol{s} \tag{3.4}$$

$$\boldsymbol{c}_{xd}(t) = c_s(t)\, e^{j\phi_0}\, \boldsymbol{s} \tag{3.5}$$

where $\pi_s(t) \triangleq \mathrm{E}[|s(t)|^2]$, $c_s(t) \triangleq \mathrm{E}[s(t)^2]$ and $C(t) \triangleq \mathrm{E}[\boldsymbol{b}(t)\boldsymbol{b}(t)^T]$. Thus, from the expressions (3.1) to (3.5), we deduce that the optimal WL spatial filter differs from the classical SMF and then give better performance than the latter whenever $\langle c_s(t)\rangle$ or $\langle C(t)\rangle$ are not equal to zero, which may occur for second order non circular useful signal or noise respectively [2].

4. EXAMPLES OF 2-TH ORDER NON CIRCULAR SIGNALS

The previous results present some interest in radiocommunications contexts only if 2-th order non circular signals exist. To show some examples of such signals, let us consider digitally modulated signals with a linear modulation. The general expression of the complex envelope of such signals correspond to that of QAM signals [5] given by

$$s(t) = \sum_{n=1}^{P} \alpha_n\, e^{j\beta_n}\, v(t - nT_s) \tag{4.1}$$

where P is the number of symbols in the message, T_s is the symbol duration, $a_n \triangleq \alpha_n e^{j\beta_n}$ is the symbol n of the message, taking its values in an alphabet of $M = 2^q$ distinct values, and $v(t)$ is a real-valued pulse function. Assuming that the symbols are independent random variables, the second order statistics $\pi_s(t)$ and $c_s(t)$ of $s(t)$ can be written as

$$\pi_s(t) = \sum_{n=1}^{P} \mathrm{E}[\alpha_n^2]\; v(t - nT_s)^2 \tag{4.2}$$

$$c_s(t) = \sum_{n=1}^{P} \mathrm{E}[\alpha_n^2\, e^{j2\beta_n}]\; v(t - nT_s)^2 \tag{4.3}$$

The 2-th order non circularity of $s(t)$ can be characterized in particular by the so-called non circularity coefficient of $s(t)$, γ_s ($0 \leq \gamma_s \leq 1$), defined by

$$\gamma_s \triangleq |\langle c_s(t)\rangle| \;/\; \langle \pi_s(t)\rangle \tag{4.4}$$

In particular, for amplitude modulated signals (ASK signals), β_n has a constant value and the coefficient γ_s is maximum and equal to 1, which characterizes signals which complex envelope are real-valued to within a complex constant. Such signals are *the most* non circular ones and are qualified of *linear signals* in [6]. For classical phase modulated signals (PSK signals), α_n has a constant value, the M possible values of β_n are M symetric values and the coefficient γ_s is equal to 1 for $M = 2$ (BPSK signals) and to 0 for $M > 2$ (QPSK, 8PSK... signals). Finally, in practical situations, a carrier residue $\Delta\omega$ may affect the processed complex signals. This occurs in particular for jammers signals or when the carrier of the signals is not known or estimated exactly (passive listening applications). For such signals, the apparent non circularity coefficient becomes

$$\gamma_s \triangleq |\langle c_s(t)\, e^{j2\Delta\omega t}\rangle| \;/\; \langle \pi_s(t)\rangle \tag{4.5}$$

and, depending on the value of $\Delta\omega T$, the carrier residue tends to circularize the associated signal, i.e.

to transform linear signals ($\gamma_s = 1$) into elliptical [6] ones ($\gamma_s < 1$) or circular ones ($\gamma_s = 0$).

5. PERFORMANCE OF THE OPTIMAL WL SPATIAL FILTER

5.1 MSE and SINR

The performance of the optimal WL spatial filter $\boldsymbol{H}_o$ are characterized by the value of the associated averaged MSE $<\varepsilon(t)>$, defined by (2.2) and (2.3), and noted $<\varepsilon(t)>_o$. Considering either circular useful signals ($\gamma_s = 0$) or linear ones ($\gamma_s = 1$), it can be shown, after some algebraic manipulations, that

$$<\varepsilon(t)>_o \ = \ <\pi_s(t)> \ / \ (1 + \mathrm{SINR}_o) \qquad (5.1)$$

where SINR_o, corresponding to the Signal to Interference plus Noise Ratio at the output of $\boldsymbol{H}_o$, is defined by the ratio between the useful signal power temporal mean and the noise plus interference signal power temporal mean at the output of $\boldsymbol{H}_o$. Note that for linear useful signals $s(t)$, the conjugate signal $s(t)^*$ also corresponds to the useful signal whereas for circular useful signals, $s(t)^*$ corresponds to an interference signal. Thus, we deduce from (5.1) that the performance of $\boldsymbol{H}_o$, in terms of averaged MSE, can also be analysed in terms of SINR at the output, result which still holds in the linear case.

5.2 SINR computation

After some algebraic computations, it can be shown that the SINR at the output of $\boldsymbol{H}_o$ is given, for linear useful signals, by

$$\mathrm{SINR}_o = 2<\pi_s(t)>[\boldsymbol{s}^\dagger A\boldsymbol{s} + \mathrm{Re}(e^{-j2\phi_0}\boldsymbol{s}^\dagger B\boldsymbol{s}^*)] \qquad (5.2)$$

and for circular useful signals by

$$\mathrm{SINR}_o = <\pi_s(t)>[\boldsymbol{s}^\dagger A\boldsymbol{s} - \frac{<\pi_s(t)>}{1+<\pi_s(t)>\boldsymbol{s}^\dagger A\boldsymbol{s}} |\boldsymbol{s}^\dagger B\boldsymbol{s}^*|^2] \qquad (5.3)$$

where the matrices A and B are defined by

$$A \triangleq (<R(t)> - <C(t)><R(t)^*>^{-1}<C(t)^*>)^{-1} \qquad (5.4)$$

$$B \triangleq - <R(t)>^{-1}<C(t)> A^* \qquad (5.5)$$

Note that the SINR_o depends on the useful signal phase ϕ_0 for linear useful signals whereas it does not depend on the latter for circular useful signals.

5.3 Circular noise case

In the circular noise case, $<C(t)> = 0$ and the SINR_o can be written, for linear useful signals, as

$$\mathrm{SINR}_o \ = \ 2<\pi_s(t)> \boldsymbol{s}^\dagger<R(t)>^{-1}\boldsymbol{s} \qquad (5.6)$$

whereas for circular useful signals, it is given by

$$\mathrm{SINR}_o \ = \ <\pi_s(t)> \boldsymbol{s}^\dagger<R(t)>^{-1}\boldsymbol{s} \qquad (5.7)$$

which corresponds to the SINR at the output of the SMF. Thus, for linear useful signals and circular noise, the SINR at the output of the optimal WL spatial filter is 3 dB greater than the SINR obtained at the output of the linear SMF.

5.4 Presence of a non circular interference

In the case where the noise $\boldsymbol{b}(t)$ is composed of a NB interference and background noise, we obtain

$$\boldsymbol{b}(t) \ = \ j(t)\, e^{j(\Delta\omega t + \phi_1)} \boldsymbol{J} \ + \ \boldsymbol{b}_b(t) \qquad (5.8)$$

where $\boldsymbol{b}_b(t)$ is the background noise vector, assumed stationary (and then circular) and spatially white, $j(t)$, ϕ_1 and $\boldsymbol{J}$ are the complex envelope, the phase and the steering vector of the received interference respectively and where $\Delta\omega$ is the carrier residue which affects the interference. Under this assumption, limiting the analysis to linear useful signals, the SINR_o can be written as

$$\mathrm{SINR}_o = 2\, \boldsymbol{s}^\dagger\boldsymbol{s}\, \frac{<\pi_s(t)>}{<\eta_2(t)>} \ [1 - \varepsilon\, |\alpha_{js}|^2 \frac{X_1}{Y_1}\ ] \qquad (5.9)$$

with

$$X_1 \ = \varepsilon\,(1 - \gamma^2) + 1 + \gamma\cos\psi \qquad (5.10)$$

$$Y_1 \ = 1 + 2\varepsilon + \varepsilon^2(1 - \gamma^2) \qquad (5.11)$$

while the SINR at the output of the classical SMF is given by [1]

$$\mathrm{SINR}_{smf} = \boldsymbol{s}^\dagger\boldsymbol{s}\, \frac{<\pi_s(t)>}{<\eta_2(t)>} \ [1 - \frac{\varepsilon}{1 + \varepsilon} \ |\alpha_{js}|^2] \qquad (5.12)$$

where $<\eta_2(t)>$ is the background noise power per sensor, α_{js} is the spatial correlation coefficient between the interference and the useful signal, defined by the normalized inner product of $\boldsymbol{J}$ and $\boldsymbol{s}$, $\varepsilon \triangleq \boldsymbol{J}^\dagger\boldsymbol{J}\ <\pi_j(t)>/<\eta_2(t)>$ where $\pi_j(t) \triangleq \mathrm{E}[|j(t)|^2]$, γ $(0 \le \gamma \le 1)$ is the non circularity coefficient of the jammer, defined by $\gamma \triangleq |<c_j(t)\ e^{j2\Delta\omega t}>| / <\pi_j(t)>

where $c_j(t) \triangleq E[j(t)^2]$ and ψ is defined by $\psi \triangleq 2(\phi_0 - \phi_1 + \phi_{js})$ where ϕ_{js} is the coefficient α_{js} argument.

The expressions (5.9) to (5.12) show that contrary to the SINR at the output of the SMF, the SINR at the output of $\boldsymbol{H}_0$ is a function of the non circularity coefficient γ of the jammer, depends on the phase difference between the signal and the jammer (ψ /2) and is a decreasing function of $\cos\psi$. In particular, for a linear jammer ($\gamma = 1$) which is in quadrature with respect to the useful signal ($\cos\psi = -1$), the $SINR_o$ is the one obtained in the absence of jammer, which means that the latter is completely nulled by $\boldsymbol{H}_0$ whatever the value of $|\alpha_{js}|$, while the jammer's rejection by the SMF decreases as $|\alpha_{js}|$ increases [1]. These results are illustrated at figure 1 which shows the variation of the $SINR_o$ and $SINR_{smf}$ as a function of $\cos\psi$ for several values of γ.

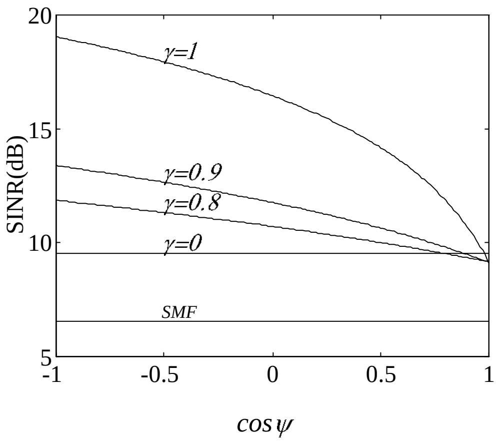

Fig.1 - *$SINR_o$ and $SINR_{smf}$ as a function of cos ψ*
N = 4, *ULA*, $\theta_s = 0°$, $\theta_j = 5°$, INR=10dB, SNR = 15dB

Finally, using (5.9) to (5.12) the gain in SINR, *G*, obtained in using $\boldsymbol{H}_0$ instead of the SMF can be computed and is given by

$$G = 2 + 2\gamma\varepsilon|\alpha_{js}|^2 \frac{X_2}{Y_2} \tag{5.13}$$

with

$$X_2 = \varepsilon\gamma - (1+\varepsilon)\cos\psi \tag{5.14}$$

$$Y_2 = [1 + \varepsilon(1 - |\alpha_{js}|^2)][(1+\varepsilon)^2 - \varepsilon^2\gamma^2] \tag{5.15}$$

The gain *G* is equal to 3 dB in the absence of jammer, when the latter is orthogonal to the useful signal or for 2-th order circular jammers. In the other cases, *G* is a decreasing function of $\cos\psi$ which may become important as $\cos\psi$ increases, as it is illustrated at figure 2.

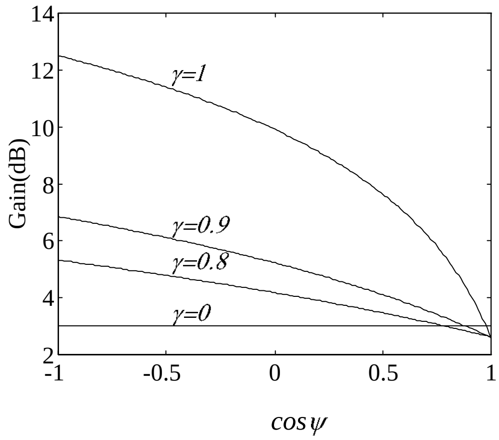

Fig.2 - *Gain G as a function of cos ψ*
N = 4, *ULA*, $\theta_s = 0°$, $\theta_j = 5°$, $\langle \pi_j(t) \rangle / \langle \eta_2(t) \rangle$ = 10 dB

6. CONCLUSION

Limiting the analysis to the exploitation of the second order statistics of the data and removing the linearity constraint of classical complex filters, the interest of TI and WL structures of spatial filtering with respect to linear ones in non stationary radiocommunications environments has been analysed. It has been shown in particular that optimal WL structures can be much more powerful than the SMF in non circular contexts, which opens new perspectives for the processing of non circular signals.

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