# SOFT DECISION SOLUTION TO ILL CONVERGENCE OF BLIND DECISION FEEDBACK EQUALIZERS


Sofiane Cherif (1)(2), Meriem Jaidane (1), Sylvie Marcos (3)

(1) Laboratoire des Systèmes de Télécommunications, ENIT, BP 37, Le Belvédère-Tunis, TUNISIA

Tel : + 216 (1) 514 700; fax : + 216 (1) 510 729; e-mail : jaidane@enit.rnrt.tn

(2) Ecole Supérieure des Postes et des Télécommunications de Tunis, 2083 Cité El Ghazala, TUNISIA

Tel : + 216 (1) 762 000; fax : + 216 (1) 762 819; e-mail : cherif@espttn.esptt.tn

(3) Laboratoire des Signaux et Systèmes, CNRS-ESE, 91192 Gif/Yvette cedex, FRANCE

Tel : + 33 (1) 69 85 17 29; fax : + 33 (1) 69 41 30 60; e-mail : marcos@lss.supelec.fr



**ABSTRACT**

Decision Feedback Equalizers (DFE) for blind equalization are subject to ill-convergence. In this paper we prove that the algorithms may be blind to the global minimum due to the error surface structure. The use of a soft decision in the decision device during a pseudo-training phase solve partially the problem of ill-convergence of DFE.


## 1 INTRODUCTION

In data transmission digital networks and mobile radio communications, standard method for adaptive equalization using a known data training sequence $\{a_n\}$ is most constraining. To remove the Inter-Symbol Interference (ISI) between the successive transmitted data, blind (or self-recovering) equalizers are prefered.

Decision Feedback Equalizers (DFE), which consist of a recursive filter in conjunction with the decision device in the feedback loop, offer improved performances in blind equalization compared to the Linear Transversal Equalizer (LTE) in case of difficult channels.

In the literature, different approachs to blind equalization were proposed, especially by Sato [1], Godard [2] and Picchi and Pratti [3]. Bellini, in [4], has discussed how the previously proposed algorithms, the Sato, the Godard and the Stop-and-Go Decision-Directed algorithms respectively, are related to the Bussgang techniques. Whereas these algorithms were initially introduced and analysed for LTE, they may however be implemented for DFE.

In a previous paper [5], we have presented the Inter-Symbol Interference Cancellation (ISIC) as a criterion for the blind adaptation of DFE. We showed that this criterion is an alternative to the Decision-Directed (DD) algorithm.

This paper will first point out, in section 3, the ill-convergence of the ISIC and the DD algorithms. Moreover, interesting conditions of convergence are given.

This paper will also prove, in section 4, that the use of a soft decision, instead of the decision device (hard decision), during an initialization period, improve the performances of adaptive DFE, avoiding the ill-convergence. Based on this property, modified ISIC and DD algorithms are proposed.

## 2 BLIND DECISION FEEDBACK EQUALIZATION

Let $h_i, i = 0, \cdots, M$, denote the channel impulse response and $\{a_n\}$ an independent identically distributed (iid) sequence taking a finite number $L$ of levels (i.e. $\{\pm 1, \pm 3\}$ for $L = 4$) in such a way that the equalizer input at time $(n)$ is in the absence of noise :

$$x_n = \sum_{i=0}^{M} h_i a_{n-i} \qquad (1)$$

The coefficients $g_i(n), i = 1, \cdots, N$, denote the DFE impulse response, at time $(n)$, so that the signal at the input of the decision device is :

$$y_n = x_n - \sum_{i=1}^{N} g_i(n)\widehat{a}_{n-i}, \;\; \widehat{a}_{n-i} = q(y_{n-i}) \qquad (2)$$

where $q(.)$ is the nonlinear function implemented in the decision device (a hard limiter), defined by :

$$q(x) = \sum_{k=1-L/2}^{L/2-1} sgn(x + 2k); \qquad (3)$$

with $sgn(x) = +1$ if $x > 0$, $sgn(x) = -1$ if $x < 0$ and $sgn(0) = 0$. The channel and the DFE equalizer are presented in figure 1.

The optimisation of the DFE is usually performed by minimizing the MSE cost function $J = E\{(a_n - y_n)^2\}$ between the reference symbol and the signal $y_n$ before decision. Since the corresponding LMS algorithm requires the knowledge of the transmitted data, we have to consider a modified criterion. For example, in the Decision-Directed algorithm, the reference symbol is replaced by the estimated symbol so that the criterion being performed is then $J_D = E\{(\widehat{a}_n - y_n)^2\}$. However, this cost function may have local minima preventing the convergence of the corresponding algorithm to the global minimum.

The proposed ISIC algorithm, which corresponds to the cancellation of the causal ISI before decision, is obtained from the ISIC criterion by minimizing $J_I = E\{y_k^2\}$. The adaptation of the equalizer filter with the ISIC and DD algorithms are respectively given by the equations (4) and (5), for $i = 1, \cdots, N$ :

$$g_i^I(n+1) = g_i^I(n) + \mu y_n \widehat{a}_{n-i} \qquad (4)$$

$$g_i^D(n+1) = g_i^D(n) + \mu(\widehat{a}_n - y_n)\widehat{a}_{n-i} \qquad (5)$$

The ISIC and the DD criterions are both subject to local minima which may lead to the ill-convergence of the algorithms.

# 3 ILL CONVERGENCE OF BLIND DFE

We proved in [5], for $M = 1$ and $N = 1$, that both the ISIC and DD criterions are subject to ill-convergence. However, the DD criterion may have more local minima than the proposed one. Then, the ISIC algorithm is less subject to getting stuck into a local minimum than the DD algorithm.

This result can be generallied, for any $M$ and $N$. The number and the nature of these minima depend on the criterion used for the adaptation, on the order of the channel and the equalizer filters, but also on the number of levels taken by $a_n$ and on the nonlinearity $q(.)$.

In the adaptative context, the convergence of the algorithm to the global minimum is related to the initial conditions and to the value of the step-size $\mu$. However, the algorithm may be blind to the global minimum.

Even for a good initialization on the global minimum, the algorithm may leap over to a local minimum for some channel and some value of $\mu$. Even for high values of $\mu$, it may be not possible to reintegrate the global minimum. The attractive character of the global minimum is related to the structure of the error surface.

Kennedy and al., in [6], have presented a partition of the DFE parameter space independant from the blind algorithm under study. Thus a set of polytopes are defined. Into each polytope, the error surface is reduced to a parabola which has a corresponding minimum. If the minimum belongs to the polytope, it is said to be attainable.

It is interesting to note that the ill-convergence of the algorithm depends on the area of the polytope corresponding to the global minimum. The algorithms can converge to the global minimum if its corresponding polytope is more wide than the areas corresponding to the local minima. Then we can choose an appropriate value of $\mu$ which allows the algorithms to escape from a local minimum.

This is illustrated by considering a second order channel transfer function $H(z) = 1 + h_1 z^{-1} + h_2 z^{-2}$ and a first order equalizer transfer function $G(z) = gz^{-1}$ (submodelling case). The ISIC and DD error surfaces have a local minimum at $g = h_2$ and a global minimum at $g = h_1$. For this case, we have investigated the partition of the DFE parameter $(g)$ space and we have established the width of the two polytopes corresponding respectively to the global and local minima. The convergence to the global minimum occurs when the width of the first polytope is larger than the width of the second. The equalization of the channel is then possible if $(h_1, h_2) \in E$, as :

$$E = \{(h_1, h_2) \mid (1 - |h_2|) > (|h_1| - 1) \text{ and } |h_2| < 1\} \qquad (6)$$

This result is verified for the ISIC and DD algorithms by simulations.

It is worth noting that $E$ doesn't correspond to the minimum phase channel case.

Figure 2 exhibits the simulated cost function $J_I$ as a function of the equalizer parameter $g$ for the minimum phase channel $H_1(z) = (1+0.8z^{-1})(1+0.8z^{-1}) = 1 + 1.6z^{-1} + 0.64z^{-2}$. The channel $H_1(z)$ is not of type $E$ : the width of the local minimum $l = 1.27$ is larger than the width of the global minimum $L = 0.72$. For the adaptation of the equalizer parameter $g$, the ISIC algorithm, as the DD algorithm, doesn't allow the convergence to the global minimum even for high values of $\mu$. Figure 3 shows how g jump between the local and global minima without reaching the convergence.

To avoid the ill-convergence, we have to modify the error surface and to improve the attraction of the global minimum. The use of a soft decision in the decision device seems to be a good idea to solve this problem.

# 4 SOFT DECISION SOLUTION

## 4.1 Effect on the Behaviour of the DFE

We showed in [5] that the use of a soft decision reduces the influence of false decisions and smoothes the local minima, so that the global convergence may occur for any step-size and any initialization. This result was proved for the ISIC Algorithm.

The main modification, considered in this paper, is to replace the hard limiter $q(.)$ in the decision device by soft decision $\widetilde{q}(.)$ implemented by $erf(.)$ function, during the initialization of both ISIC and DD algorithms. The equations (2) and (3) become :

$$y_n = x_n - \sum_{i=1}^{N} g_i(n)\widetilde{a}_{n-i}, \;\; \widetilde{a}_{n-i} = \widetilde{q}(y_{n-i}) \qquad (7)$$

$$\widetilde{q}(x) = \sum_{k=1-L/2}^{L/2-1} erf(\frac{x+2k}{s}); \qquad (8)$$

with $erf(x) = \frac{2}{\pi}\int_0^x e^{-t^2}dt$. The factor $s$ denotes the so called saturation degree of the nonlinear function $\widetilde{q}(.)$.

Let us consider a non-minimum phase channel transfer function $H_2(z) = (1+0.5z^{-1})(1+1.5z^{-1})$ and an equalizer $G(z) = gz^{-1}$ of length 1 ($N = 2$). The transmitted data take 2 levels $\pm 1$ ($L = 2$). For these considerations the ISIC and DD algorithms present a local minimum at $g = 0.75$ and a global minimum at $g = 2.0$. We note that this channel is not of type $E$. Figure 4 exhibits two curves corresponding to the equalizer parameter ($g$) evolution. The first corresponds to the ISIC algorithm. The initialization is done at $g = 0.75$ (i.e. at the local minimum). We can observe that the ISIC algorithm converges to the local minimum. For the second, we have used the soft decision during initialization, where $s = 1.2$. First the ISIC algorithm succeeds to escape from the local minimum and the algorithm is stabilized at the minimum ($g = 2.35$) relating to the soft decision. After, when we switch to the hard decision (at n=15000) it converges to the global minimum.

Two efficient algorithms to implement the proposed soft decision during initialization are next introduced.

## 4.2 The Modified Blind DFE Algorithms

The purpose of this paragraph is to introduce a modified ISIC algorithm that switches automatically between the pseudo-training phase, during witch a smooth function (the soft decision) is used instead of the hard decision device, and the tracking phase.

The idea is that the soft decision allows to smooth the local minima as $s$ increases, until they disappear, while the global minimum is only slightly translated. Then, if we initialize $s$ with a high value and we choose a decreasing evolution of $s = f(n)$, witch tends to 0 at the convergence of the algorithm (at the end of the pseudo-training phase), so $\widetilde{q}(.)$ becomes the hard decision $q(.)$ and we avoid the possible local minima.
Let us consider the following decreasing evolution of $s$ :

$$s(n) = s_0 \exp\left(-\frac{n}{n_T}\right) \qquad (9)$$

where $s_0$ corresponds to the initial value of $s$ and $n_T$ corresponds to the duration of the pseudo-training phase.

The *Modified ISIC* algorithm is obtained by the substitution of $s$ by $s(n)$ in accordance with (10). The extension to the DD algorithm leads to the *Modified Decision-Directed* algorithm.

To illustrate this paragraph, we consider the non-minimum phase channel $H_2(z)$ used above. Figure 5 exhibits respectively the equalizer coefficient $g$ (5a) and the probability error (5b) evolution, corresponding to the DD and to the *Modified DD* algorithms. We can observe the convergence of the proposed *Modified DD* algorithm to the global minimum while the DD algorithm is subject to ill-convergence to a local minimum.

# 5 CONCLUSION

Decision Feedback Equalizers for blind equalization are subject to ill-convergence. In particular the algorithm may be blind to the global minimum, due to the error surface structure. The global minimum may be not attractive even for minimum phase channel. The use of a soft decision in the decision device smooths the local minima during a pseudo-training phase. The proposed *Modified DD* and *ISIC* algorithms seems to solve the problem of ill-convergence of DFE.

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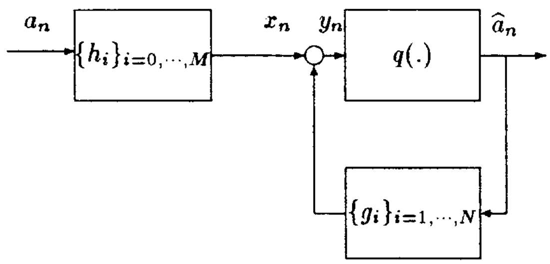


Figure 1: Channel and DFE equalizer modelling

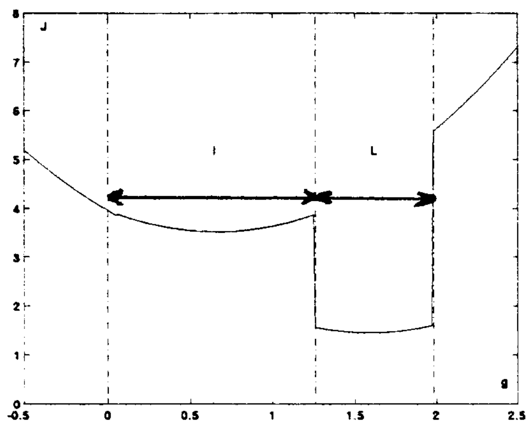


Figure 2: Error surface corresponding to ill-convergence condition of the ISIC algorithm for a minimum phase channel

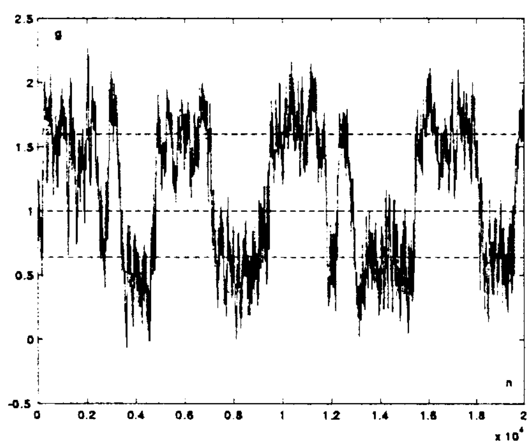


Figure 3: Ill-convergence of the ISIC algorithm for a minimum phase channel ($g(0) = 0.64$, $\mu = 0.05$)

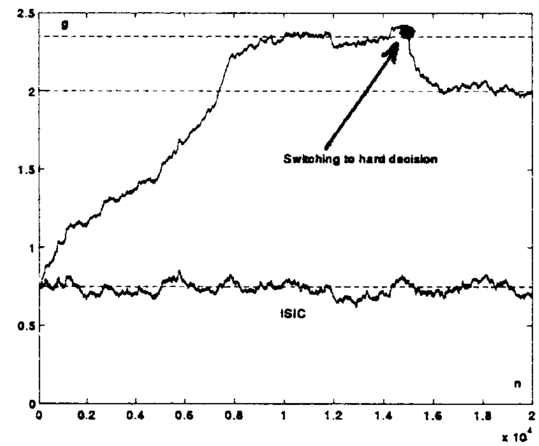


Figure 4: Convergence of the ISIC algorithm with soft decision for a non-minimum phase channel ($g(0) = 0.75$, $\mu = 0.001$, $s = 1.2$)

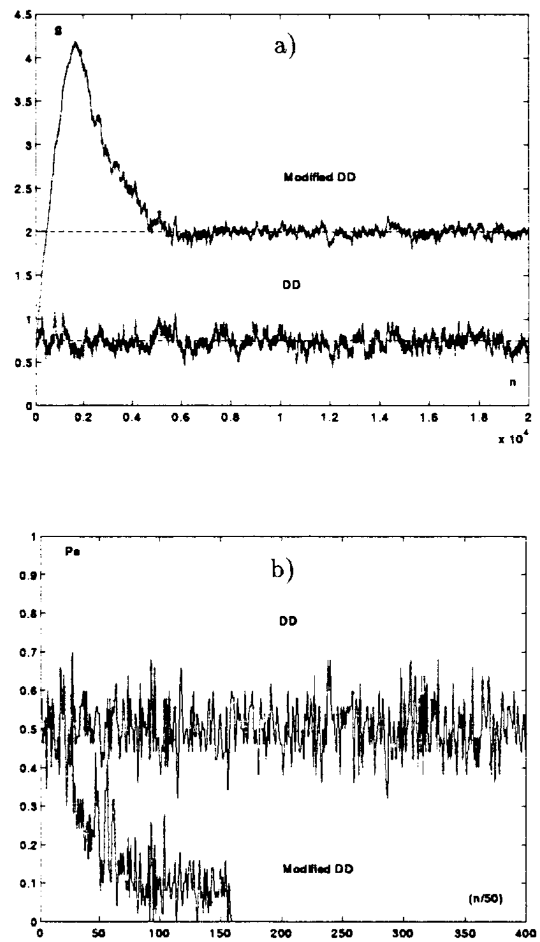


Figure 5: Convergence of the *Modified DD* algorithm for a non-minimum phase channel compared to DD; a) : DFE parameter evolution; b) : error rate evolution ($g(0) = 0.75$, $\mu = 0.01$)