ON THE PROBLEM OF BLIND EQUALIZATION CONSIDERING ABRUPT CHANGES IN THE CHANNEL CHARACTERISTICS

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ABSTRACT

The problem of blind equalization in a digital communication system is considered. Unfortunately, the circuit might suffer from abrupt changes. Thus, it is critical not to ignore this phenomenon when the problem of blind equalization is analyzed. The proposed method, which is based on an Itô stochastic differential calculus approach, describes the dynamics of the output signal with an infinite impulse response (IIR) model where the involved taps are modeled as time-varying cadlag (continu à droite limites à gauche) processes. Therefore, nonlinear and time-variant changes in the channel characteristics are included.

1 INTRODUCTION

The purpose of this paper is to study the problem of blind equalization when abrupt changes in the channel characteristics might have occurred. With use of a stochastic differential calculus approach, suggested in [2], the posed problem is analyzed.

The problem of blind equalization has been extensively studied (see *e.g.* [2], [3], [4], [5], [6]). Though, the possibility of abrupt changes has been ignored.

Equalization is of major importance in data communication. A sequence of output signals is received and the transmitted unknown input signals are to be estimated. If the channel which causes the distortion of the input signals were known, the problem would be simple to solve. Assuming that the channel is modeled as a known finite impulse response filter and that the input signals belong to a finite alphabet, the Viterbi algorithm, suggested in [7], determines the optimal estimates of the input signals. When the problem of blind equalization is considered, the channel as well as the input signals are assumed to be unknown.

In a digital communication system abrupt changes in the channel characteristics are plausible to occur. Thus, a method developed for blind equalization must not ignore the phenomenon abrupt changes.

In speech echo cancellation, the sources of nonlinearity are not of serious concern. When data transmission is concerned though, the nonlinear component, which is introduced by the circuit components of the digital transceiver [8], must be considered because of the need for greater suppression of abrupt changes in the channel dynamics [9].

In this paper new and more advanced methods based on an Itô stochastic differential calculus approach will be applied on the posed problem. The primary contribution of this paper is that an algorithm is suggested which includes the possibility of abrupt changes in the channel characteristics.

This paper is organized as follows. In Section 2 the problem is defined. In Section 3 the dynamics of the channel parameters are derived and a solution algorithm for the posed problem is suggested. Finally, Section 4 gives some concluding remarks.

2 PRELIMINARIES

2.1 Notation, Models and Assumptions

Consider the digital communication system illustrated in Fig. 2.

An unknown sequence $\{u_k\}$ is sent over an unknown channel. The observed output signal at time t is denoted y_t.

In order to model the channel accurately, the possibility of abrupt changes in the dynamics of the taps must not be ignored.

Assumption 1: The input signal u_k is transmitted at the time instant kT, where T is a known fixed time interval and $k \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ denotes the set of positive integers.

Consider the system illustrated in Fig. 2 during the time interval $[kT, (k+1)T)$, where $k \in \mathbb{Z}^+$.

Assume during this time interval that $u_1, u_2, \ldots, u_{k-2}$ have been estimated, *i.e.* $\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_{k-2}$ have been determined. u_{k-1} and u_k have not yet been estimated.

Introduce the notation $\xi_k(t)$. Using relations derived in Section 3, $\xi_k(t)$ will be estimated. u_k will be estimated according to

$$\hat{u}_k = \begin{cases} \xi_k(kT) & \text{if } k \text{ is odd} \\ \xi_k((k+1)T) & \text{if } k \text{ is even} \end{cases} \tag{1}$$

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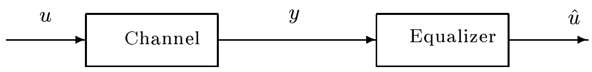

Figure 1: *The model used to describe a digital communication system.*

Model 1:

$$y(t) = \sum_{i=1}^{k} b^i(t) u_i, \text{ where } t \in [kT, (k+1)T),\ k \in \mathbb{Z}^+ . \quad (2)$$

Consider the first tap, *i. e.*, the one connected with the first input signal. The dynamics of this tap is in this paper suggested to be described by the following model

Model 2:

$$\begin{aligned} db^1(t) &= \hat{\alpha}_1 b^1(t) dt + \hat{\sigma}_1 b^1(t^-) dW(t) \\ &+ \hat{\beta}_1 b^1(t^-) dN(t), \end{aligned} \quad (3)$$

where W is a wiener process and N is a jump process.

Remark 1: The last term models the abrupt changes.

Remark 2: The solution to equation (3) is according to [1]

$$b^1(t) = b_0^1 \exp\left\{ \left(\hat{\alpha}_1 - \frac{1}{2}\hat{\sigma}_1^2 \right) t + \hat{\sigma}_1 W(t) \right. \quad (4)$$
$$\left. + \ln\{1 + \hat{\beta}_1\} N(t) \right\},$$

i. e. by choosing $\hat{\alpha}_1, \hat{\beta}_1$ and $\hat{\sigma}_1$ appropriately an exponentially decreasing behaviour of the tap dynamics is achieved, which is physically plausible.

Assumption 2:

$$b^i(t) = F^i(t, b^1) \quad \forall i \geq 2, \quad (5)$$

i. e. tap *no.* i is assumed to be a function of the first tap.

Assumption 3: $\hat{\alpha}_1, \hat{\sigma}_1, \hat{\beta}_1, u_1$ are known constants. y is known. The growth, ρ_{2k-1}, during the time interval $[(2k-1)T, 2kT)$, $k \geq 2$, of the output signal, y, is known.

2.2 Statement of the Problem

The posed problem can now be stated as follows.

Problem: Given the sequence $(y_1, y_2, \ldots, y_t)$ and the model structures (2), (3) and (5), determine $\hat{u}_i\ \forall i$.

3 ANALYSIS

In this section the dynamics of the taps and the output signal are derived. Furthermore, the blind equalization algorithm is described. Finally, an alternative to the model (2) is suggested.

3.1 Determination of the Involved Parameters

The purpose of this section is to derive the dynamics of the output signal, y, using an Itô stochastic differential calculus approach, suggested in [2].

Result: *Consider the model (2). Using Itô–notation, the following holds*

$$dy(t) = u'(t) db(t), \quad (6)$$

where $u(t) = \left[u_1, u_2, \ldots, u_{int(\frac{t}{T})}, 0, \ldots\right]'$ *and* $db(t) = \left[db^1(t), db^2(t), \ldots, db^{int(\frac{t}{T})}(t), 0, \ldots\right]'$, *where* $'$ *denotes transposition and int(·) denotes the integer part.*

Proof: The vector $u(t) = \left[u_1, u_2, \ldots u_{int(\frac{t}{T})}, 0, \ldots\right]'$ is assumed to be constant during the time interval $[t, t+h)$.

At the *entrance* of the time interval $[t, t+h)$, it holds that

$$y(t) = u'(t-h) b(t). \quad (7)$$

But at time t,

$$y(t) = u'(t) b(t). \quad (8)$$

Subtracting Equation (7) from Equation (8) gives

$$0 = [u(t) - u(t-h)]' b(t). \quad (9)$$

Remark: There is a "backward difference" in Equation (9). To achieve Itô-differentials when $h \to 0$ proceed as follows: Add and subtract $[u(t+h) - u(t)]' b(t)$ in Equation (9). The result is

$$\begin{aligned} & [u(t+h) - u(t)]' b(t) \\ + & [u(t+h) - u(t)]' [b(t+h) - b(t)] = 0. \end{aligned} \quad (10)$$

Taking the limits as $h \to 0$ in (10), results in a continuous version of (10).

$$du'(t) b(t) + du'(t) db(t) = 0. \quad (11)$$

Letting $h \to 0$ in Equation (7) gives,

$$y(t) = u'(t) b(t). \quad (12)$$

Applying Itô's formula on Equation(12) results in

$$\begin{aligned} dy(t) = u'(t) db(t) + du'(t) b(t) \\ + du'(t) db(t). \end{aligned} \quad (13)$$

The Equations (11) and (13) give

$$dy(t) = u'(t) db(t) \quad (14)$$

□

Remark: Equation (6) must be interpreted in the meaning of Itô!

Introduce:

$$r_i(t) = \begin{cases} \frac{\xi_i b^i(t)}{y(t)} & \text{if } i = k-1, k \\ \frac{u_i b^i(t)}{y(t)} & \text{if } i < k-1 \end{cases}, \; t \in [kT, (k+1)T) \tag{15}$$

Equation (6) can therefore be reformulated

$$dy(t) = y(t) \left\{ \sum_{i=1}^{k} r_i(t) \frac{db^i(t)}{b^i(t)} \right\}, \quad t \in [kT, (k+1)T). \tag{16}$$

According to definition (15) and equation (2) the following holds

$$\sum_{i=1}^{k} r_i(t) = 1, \quad t \in [kT, (k+1)T). \tag{17}$$

Applying Itô's formula on Assumption 1 gives

$$db^i = \alpha_i b^i dt + \sigma_i b^i dW + \beta_i b^i dN, \tag{18}$$

where

$$\alpha_i = \frac{1}{F^i} \left(F_t^i + \tilde{\alpha}_1 b^1 F_{b^1}^i + \frac{\hat{\sigma}_1^2}{2} (b^1)^2 F_{b^1 b^1}^i \right), \tag{19}$$

$$\sigma_i = \frac{1}{F^i} \hat{\sigma}_1 b^1 F_{b^1}^i, \tag{20}$$

and

$$\beta_i = \frac{1}{F^i} \left(F^i(t, b^1(t) + \hat{\beta}_1 b^1(t)) - F^i(t, b^1(t)) \right), \; i \geq 2. \tag{21}$$

The subindices $t, b^i, b^i b^i$ indicate differentiation with respect to $t, b^i, b^i b^i$.

In order to reduce the computational complexity, the following approximations are advisable

$$\begin{aligned} \hat{\alpha}_i &= \alpha_i((2k-1)T), \\ \hat{\beta}_i &= \beta_i((2k-1)T), \\ \hat{\sigma}_i &= \sigma_i((2k-1)T), \\ i &= 2k-2, 2k-1, \; \forall k \geq 2. \end{aligned} \tag{22}$$

3.2 Solution Strategy

Consider the system at the time instant
$t \in [(2k-1)T, 2kT)$.

Knowns: $\hat{\alpha}_i, \hat{\sigma}_i, u_i, \rho_{2k}, r_i = \frac{u_i b^i(t)}{y(t)}, \; i = 1, 2, \ldots, 2k-3$

Unknowns: $\xi_{2k-2}, \xi_{2k-1}, \alpha_{2k-2}, \alpha_{2k-1}, \beta_{2k-2}, \beta_{2k-1}, \sigma_{2k-2}, \sigma_{2k-1}, r_{2k-2}, r_{2k-1}, F^{2k-2}, F^{2k-1}$

Available equations: (21) for $i = 2k-2, 2k-1$, (19) for $i = 2k-2, 2k-1$, (20) for $i = 2k-2, 2k-1$, (15) for $i = 2k-2, 2k-1$, (25), (17), (26), (24).

I.e., there are 12 unknowns and 12 available equations!

3.3 Derivation of a Partial Differential Equation Determining the Dynamics of the Taps

During the time interval $[(2k-1)T, 2kT)$, $k = 2, 3, \ldots$ it holds according to (16), (22) and (18)

$$\begin{aligned} dy(t) = y(t) \Bigg\{ & \left[\sum_{i=1}^{2k-3} r_i \hat{\alpha}_i + r_{2k-2}\alpha_{2k-2} + r_{2k-1}\alpha_{2k-1} \right] dt \\ & + \left[\sum_{i=1}^{2k-3} r_i \hat{\sigma}_i + r_{2k-2}\sigma_{2k-2} + r_{2k-1}\sigma_{2k-1} \right] dW(t) \\ & + \left[\sum_{i=1}^{2k-3} r_1 \hat{\beta}_i + r_{2k-2}\beta_{2k-2} + r_{2k-1}\beta_{2k-1} \right] dN(t) \Bigg\} \end{aligned} \tag{23}$$

If

$$\sum_{i=1}^{2k-3} r_i \hat{\sigma}_i + r_{2k-2}\sigma_{2k-2} + r_{2k-1}\sigma_{2k-1} = 0 \tag{24}$$

and

$$\sum_{i=1}^{2k-3} r_i \hat{\beta}_i + r_{2k-2}\beta_{2k-2} + r_{2k-1}\beta_{2k-1} = 0 \tag{25}$$

the dN- and dW-terms in equation (23) will be completely eliminated. The growth, ρ_{2k-1}, of the output signal, y, is known according to Assumption 3, *i.e.*

$$\sum_{i=1}^{2k-3} r_i \hat{\alpha}_i + r_{2k-2}\alpha_{2k-2} + r_{2k-1}\alpha_{2k-1} = \rho_{2k-1} \tag{26}$$

Equations (17) and (24) result in

$$\sum_{i=1}^{2k-3} r_i(\hat{\sigma}_i - \sigma_{2k-1}) + r_{2k-2}(\sigma_{2k-2} - \sigma_{2k-1}) + \sigma_{2k-1} = 0. \tag{27}$$

Equations (27), (20) and (15) give

$$\begin{aligned} \frac{\xi_{2k-2}F^{2k-2}}{y} = & \frac{\sum_{i=1}^{2k-3} r_i \left(\hat{\sigma}_i - \frac{1}{F^{2k-1}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-1} \right)}{\frac{1}{F^{2k-1}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-1} - \frac{1}{F^{2k-2}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-2}} \\ & + \frac{\frac{1}{F^{2k-1}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-1}}{\frac{1}{F^{2k-1}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-1} - \frac{1}{F^{2k-2}} \hat{\sigma}_1 b^1 F_{b^1}^{2k-2}} \end{aligned} \tag{28}$$

Equations (15), (17) and (25) give

$$\frac{\xi_{2k-2}F^{2k-2}}{y} = \frac{\sum_{i=1}^{2k-3} r_i \left(\hat{\beta}_i - \beta_{2k-1} \right) + \beta_{2k-1}}{\beta_{2k-1} - \beta_{2k-2}} \tag{29}$$

Using (21), Equation (29) can be expressed as a function of F^{2k-2} and F^{2k-1}. Comparing this expression with (28) makes it possible to express F^{2k-2} as a function of F^{2k-1}. Equations (17) and (26) give

$$\begin{aligned} \sum_{i=1}^{2k-3} r_i (\hat{\alpha}_i - \alpha_{2k-1}) + r_{2k-2} (\alpha_{2k-2} - \alpha_{2k-1}) & \\ + \alpha_{2k-1} = \rho_{2k-1} & \end{aligned} \tag{30}$$

Using (19), (30) and the derived expression of F^{2k-2} in terms of F^{2k-1} gives ξ_{2k-2} as a function of F^{2k-1} and its derivatives. Substituting ξ_{2k-2} in (28) with this expression results in a partial differential equation for F^{2k-1} with no other unknowns than F^{2k-1}.

Considering (16) at the time instant $(2k-1)T$, gives the following boundary condition.

Boundary condition:

$$\begin{aligned} &F^{2k-1}((2k-1)T, b^1((2k-1)T)) \\ &= \frac{y((2k-1)T) - b^1((2k-1)T)u_1}{\xi_{2k-1}} - \\ &- \frac{\sum_{i=3}^{2k-3} F^i((2k-1)T, b^1((2k-1)T))u_i}{\xi_{2k-1}} - \\ &- \frac{F^{2k-2}((2k-1)T, b^1((2k-1)T))\xi_{2k-2}}{\xi_{2k-1}} \end{aligned} \tag{31}$$

3.4 Algorithm for Blind Equalization

In this section the algorithm for the proposed method for blind equalization when abrupt changes might have occurred is presented.

Algorithm 1:

0. Let $k := 2$.
1. Wait until the time $t \in [(2k-1)T, 2kT)$.
2. Solve the partial differential equation derived above in Section 3.3 $\Rightarrow F^{2k-2}, F^{2k-1}$.
3. Determine α_i, σ_i and β_i using (21), (19), (20) for $i = 2k-2, 2k-1$.
4. Estimate α_i, σ_i and β_i using (22).
5. Determine ξ_{2k-2}, ξ_{2k-1} using results derived in Section 3.3.
6. Determine $\hat{u}_{2k-2}, \hat{u}_{2k-1}$ using (1).
7. Let $k := k+1$ and iterate the steps 1-7.

3.5 A Sliding Window Approach

In this section the Model (2) is modified so that the output signal only depends on the Q last input signals, *i. e.* a finite impulse response (FIR) model is used.

Model:

$$y(t) = \sum_{i=k-Q}^{k} b^i(t)u_i, \text{where } t \in [kT, (k+1)T) \tag{32}$$

Assumption: $\text{int}(\frac{t}{T}) > Q$, where $\text{int}(\cdot)$ denotes the integer part.

The analysis of this model will be very similar to that of Model (2), so, due to space limitations, these are not presented here.

4 DISCUSSION AND CONCLUSIONS

The primary contribution of this paper is twofold.

Firstly, an algorithm solving the problem of blind equalization not ignoring the possibility of abrupt changes in the channel characteristics is suggested.

Secondly, nonlinear and time-variable abrupt changes are included.

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