Source Independent Blind Equalization with Fractionally-Spaced Sampling

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ABSTRACT

A generalization of the super-exponential blind equalization algorithm for fractionally-spaced sampling is presented. Taking advantage of the increased degrees of freedom in selecting higher order statistics of cyclostationary signals, two different cost functions are proposed for blind equalization. One of them allows the inverse of a bandlimited continuous channel to be identified without aliasing, and the other leads to a blind counterpart of a decision-directed fractionally-spaced equalizer (FSE). Simulation results document the performance of these algorithms.

1 INTRODUCTION

This paper addresses the problem of equalization and channel identification when no reference signal is available. Recently, a considerable research effort has been devoted to this problem, due to the various applications that can benefit from it. In digital communications, for example, the communication channel may experience severe fading during some transmissions, and it is typically necessary to restart the receiver algorithms after such occurrences. This restarting process cannot rely on the knowledge of the transmitted message, and blind equalization techniques are well suited for this purpose.

Most blind equalization/identification methods can be divided into two major categories. Equalization methods based on higher order statistics usually use a purely discrete time model, and consider all the filtering and distortion to be included in the discrete channel. In another class of blind algorithms, advantage is taken of the cyclostationarity of the continuous received signal, which is sampled faster than the symbol rate to identify the channel from second order statistics [1]. Although the results are promising, these algorithms require rather complex computations, which may be difficult to perform in real time.

In this paper, we combine the features of both classes of algorithms to improve the performance, while retaining a relatively simple implementation. This work is an extension of the *Shalvi-Weinstein batch algorithm* [2, 3], which operates with symbol rate sampling and higher order statistics. Unlike most other gradient based algorithms, this one converges rapidly, and avoids the selection of an appropriate learning rate parameter. Moreoever, the method is universal in the sense that it does not require a specific source probability distribution to guarantee convergence. We have reformulated the Shalvi-Weinstein method to use fractionally-spaced sampling (i.e., multiple of the symbol rate), and the result is a new algorithm with some degrees of freedom in the choice of an optimization criterion. Depending on this choice, it is possible to obtain an algorithm that can identify a bandlimited continuous channel without the aliasing inherent in symbol rate sampling. With a different criterion, it is also possible to obtain the blind counterpart of a decision-directed fractionally-spaced equalizer [4].

This work was partially funded by FEDER and PRAXIS XXI, under contract PRAXIS/3/3.1/TPR/23/94.

2 FRACTIONALLY-SPACED SAMPLING

Equalization is based on the following continuous PAM signal

$$y_c(t) = \sum_{k=-\infty}^{+\infty} a(k) h_c(t - kT_b)\,, \qquad (1)$$

where $a(k)$ is the sequence of complex symbols, $h_c(t)$ is the complex bandlimited baseband pulse shape, and T_b is the signaling interval. The transmitted symbols are uncorrelated, with zero mean and variance σ_a^2. This signal is sampled at L samples per symbol, and generates the discrete sequence

$$y(n) = \sum_{k=-\infty}^{+\infty} a(k) h(n - kL)\,, \qquad (2)$$

where $h(n) = h_c(nT_b/L)$. For equalization purposes, $y_c(t)$ should be obtained at the output of a matched filter when symbol-spaced sampling is used ($L = 1$). Naturally, the pulse shape at the receiver is imperfectly known, and a suboptimal filter causes some performance degradation due to information loss. When fractional sampling is used, the oversampling factor L is assumed to be high enough, so that the pulse shape suffers no aliasing and a continuous matched filter is not needed [4]. The discrete signal $y(n)$ is then processed by an equalizer with impulse response $c(n)$, whose output is the convolution $z(n) = c(n) * y(n)$. The global channel/equalizer impulse response will be denoted by $s(n) = c(n) * h(n)$. For $L > 1$, $y(n)$ is a cyclostationary process that can be separated in L stationary signals $y^{(0)}(n), \ldots, y^{(L-1)}(n)$, with $y^{(i)}(n) = y(nL + i)$. The subsequences $z^{(i)}(n)$, $s^{(i)}(n)$, $h^{(i)}(n)$ and $c^{(i)}(n)$ are defined similarly, and

$$s^{(i)}(n) = \sum_{k=0}^{L-1} c^{(k)}(n) * h^{(i-k)}(n) \tag{3}$$

holds. In (3), a negative subsequence index can be rewritten in the range $0 \ldots L-1$ with a suitable delay, according to $h^{(i)}(n) = h(nL+i) = h((n-1)L+i+L) = h^{(i+L)}(n-1)$.

3 EQUALIZATION CRITERIA

The Shalvi-Weinstein equalization criterion maximizes

$$\left| \sum_{k=-\infty}^{+\infty} s(k)^p s^*(k)^q \right| , \text{ subject to } \sum_{k=-\infty}^{+\infty} |s(k)|^2 = 1 . \tag{4}$$

For $p + q > 2$, the maxima of (4) are of the form $s(n) = e^{j\phi}\,\delta(n-d)$, with arbitrary delay d and phase shift ϕ. Under such conditions, the blind equalization problem is considered to be solved. A practical formulation of (4) makes use of a family of complex cumulants

$$\begin{aligned} C^z_{p,q} &= \operatorname{cum}(\underbrace{z(n); \ldots; z(n)}_{p \text{ times}}; \underbrace{z^*(n); \ldots; z^*(n)}_{q \text{ times}}) \\ &\triangleq \operatorname{cum}(z(n):p; z^*(n):q) , \end{aligned} \tag{5}$$

and maximizes $|C^z_{p,q}|$ subject to the variance constraint $C^z_{1,1} = C^a_{1,1}$ [3]. With fractional sampling, cumulants can be defined between pairs of subsequences, forming an $L \times L$ matrix $\mathbf{C}^z_{p,q}$, where

$$\begin{aligned} \left[\mathbf{C}^z_{p,q}\right]_{ij} &\triangleq C^{z^{(i)},z^{(j)}}_{p,q} = \operatorname{cum}(z^{(i)}(n):p; z^{(j)*}(n):q) \\ &= C^a_{p,q} \sum_{k=-\infty}^{+\infty} s^{(i)}(k)^p s^{(j)*}(k)^q . \end{aligned} \tag{6}$$

Identification. The criterion used for channel identification maximizes the trace

$$\begin{aligned} J &= \left|\operatorname{tr}\left\{\mathbf{C}^z_{p,q}\right\}\right| = |C^a_{p,q}| \left| \sum_{i=0}^{L-1} \sum_{k=-\infty}^{+\infty} s^{(i)}(k)^p s^{(i)*}(k)^q \right| \\ &= |C^a_{p,q}| \left| \sum_{k=-\infty}^{+\infty} s(k)^p s^*(k)^q \right| \end{aligned} \tag{7}$$

subject to $\operatorname{tr}\{\mathbf{C}^z_{1,1}\} = C^a_{1,1}$, or equivalently

$$\sum_{i=0}^{L-1} \sum_{k=-\infty}^{+\infty} |s^{(i)}(k)|^2 = \sum_{k=-\infty}^{+\infty} |s(k)|^2 = 1 . \tag{8}$$

This cost function is formally identical to (4), and is maximized when $s(n) = e^{j\phi}\,\delta(n-d)$. In practice, this solution is not possible because $h(n)$ cannot be inverted due to the null portion of its spectrum. However, (7) still leads to a useful algorithm, which is described in section 4.

Equalization. For blind equalization, we select

$$J = \left| \left[\mathbf{C}^z_{p,q}\right]_{0,0} \right| = |C^a_{p,q}| \left| \sum_{k=-\infty}^{+\infty} s^{(0)}(k)^p s^{(0)*}(k)^q \right| , \tag{9}$$

subject to $\sum_{k=-\infty}^{+\infty} |s^{(0)}(k)|^2 = 1$. This is again mathematically equivalent to (4), and is therefore maximized when perfect equalization is achieved on $z^{(0)}(n)$, but no restriction is imposed on the behavior of the other subsequences. This criterion is conceptually similar to the one proposed for the FSE in [4].

4 SUPER-EXPONENTIAL ALGORITHMS

In this section, adaptive algorithms that converge to maxima of the previous cost functions are presented.

Identification. As in [2, 3], the following *super-exponential ideal iteration* is defined for criterion (7)

$$\mathbf{s}' = \mathbf{g} , \qquad \mathbf{s}'' = \frac{\mathbf{s}'}{||\mathbf{s}'||} , \tag{10}$$

where $\mathbf{s} = [\ldots s(-1)s(0)s(1)\ldots]^T$ is the infinite dimensional vector of combined coefficients and $\mathbf{g}$ is a vector formed by the sequence $g(n) = s(n)^p s^*(n)^q$. When $p+q \geq 2$, (10) converges to a vector $\boldsymbol{\delta}_d$ whose associated sequence maximizes (7).

In vector form, the convolution operation that relates $s(n)$ and $c(n)$ is written as $\mathbf{s} = \mathbf{H}\mathbf{c}$, where $\mathbf{H}_{ij} = h(i-j)$ is the channel convolution matrix, and $\mathbf{c}$ is the vector of equalizer coefficients $\mathbf{c} = [c(N_1)\ldots c(N_2)]^T$. This allows an approximate iteration to be written for $\mathbf{c}$, by projecting (10) in the column space of $\mathbf{H}$

$$\mathbf{c}' = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{g} , \quad \mathbf{c}'' = \frac{\mathbf{c}'}{\sqrt{\mathbf{c}'^H\mathbf{H}^H\mathbf{H}\mathbf{c}'}} , \tag{11}$$

where $(\cdot)^H$ denotes conjugate transpose. The iteration in $\mathbf{s}$ space is

$$\mathbf{s}' = \mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{g} , \qquad \mathbf{s}'' = \frac{\mathbf{s}'}{||\mathbf{s}'||} . \tag{12}$$

Being a Toeplitz matrix, the asymptotic eigenvalues of $\mathbf{H}^H\mathbf{H}$ for an unconstrained equalizer ($N_1 \to -\infty$, $N_2 \to +\infty$) are obtained from samples of the channel spectrum $|H(\omega)|^2$. When $L = 1$ and the channel is invertible, the projection matrix in (12) tends to an identity as the equalizer order increases, and (10) is approximately recovered. However, the high frequency portion of $H(\omega)$ is null for $L > 1$, hence $\mathbf{H}$ is asymptotically rank deficient, and the inverse matrix in (11)–(12) is at least numerically ill-conditioned for a large equalizer. When $(\mathbf{H}^H\mathbf{H})^{-1}$ is replaced by the Moore-Penrose pseudoinverse $(\mathbf{H}^H\mathbf{H})^{+}$, (12) does not necessarily converge to (10), but meaningful results are still obtained, as explained next. In (11), $\mathbf{c}'$ minimizes the distance between $\mathbf{g}$ and its projection in $\mathbf{H}$, given by

$$\begin{aligned}\|\mathbf{H}\mathbf{c}' - \mathbf{g}\|^2 &= \sum_{n=-\infty}^{+\infty} |h(n) * c'(n) - g(n)|^2 \\ &= \frac{1}{2\pi}\int_{-\pi}^{+\pi} |H(\omega)C'(\omega) - G(\omega)|^2 \, d\omega \,. \quad (13)\end{aligned}$$

Let W_H denote the frequency band where $H(\omega)$ does not vanish[1]. Then, $H(\omega)C'(\omega) = 0$ outside W_H, and (13) should only be minimized inside W_H. Note that $G(\omega)$ is essentially the convolution of $p+q$ replicas of $H(\omega)C(\omega)$, and it takes an almost constant value G_0 in W_H for sufficiently high $p+q$ and L. The minimization of (13) *inside* W_H then yields $C'(\omega) = G_0/H(\omega)$, thereby inverting $H(\omega)$ up to a constant that can be easily compensated after the normalization step in (11). Given $C'(\omega)$ in W_H, an infinite equalizer can synthesize an arbitray transfer function at the remaining frequencies, which explains the ill-conditioned nature of $\mathbf{c}'$. When the minimum norm solution $\mathbf{c}' = \mathbf{H}^{+}\mathbf{g} = (\mathbf{H}^H\mathbf{H})^{+}\mathbf{H}^H\mathbf{g}$ is selected, the ambiguity is removed because then $C'(\omega)$ (approximately) vanishes outside W_H.

To obtain a practical algorithm based on (11), the unobservable quantities $\mathbf{H}^H\mathbf{H}$ and $\mathbf{H}^H\mathbf{g}$ should be expressed with cumulants of $y(n)$ and $z(n)$. Following an argument similar to that in [2, 3], it may be shown that

$$\begin{aligned}\mathbf{R}_{ij} &\triangleq \frac{\sum_{k=0}^{L-1} \mathrm{cum}(y^{(k-j)}(n);\, y^{(k-i)*}(n))}{C^a_{1,1}} \\ &= \sum_{n=-\infty}^{+\infty} h(n-j)h^*(n-i) = [\mathbf{H}^H\mathbf{H}]_{ij} \quad (14)\\ \mathbf{d}_i &\triangleq \frac{\sum_{k=0}^{L-1} \mathrm{cum}(z^{(k)}(n):p;\, z^{(k)*}(n):q;\, y^{(k-i)*}(n))}{C^a_{p,q+1}} \\ &= [\mathbf{H}^H\mathbf{g}]_i \,. \quad (15)\end{aligned}$$

With $\mathbf{R}$ and $\mathbf{d}$, (11) can be rewritten as

$$\mathbf{c}' = \mathbf{R}^{+}\mathbf{d}\,, \qquad \mathbf{c}'' = \frac{\mathbf{c}'}{\sqrt{\mathbf{c}'^H\mathbf{R}\mathbf{c}'}}\,. \quad (16)$$

The cumulants in (14) and (15) are estimated using time averages. While this operation conceptually requires the separation of $y(n)$ and $z(n)$ in L subsequences, it may be avoided for some simple averages, and the expressions originally proposed in [2, 3] can be computed (up to a factor of L) *as if* $y(n)$ *and* $z(n)$ *were stationary*. This can be seen for $\mathbf{d}$ when it is estimated as

$$\begin{aligned}\hat{\mathbf{d}}_i &= \frac{1}{C^a_{p,q+1}}\sum_{k=0}^{L-1}\frac{1}{N}\sum_{n\in\langle N\rangle} f\left(z^{(k)}(n);\, y^{(k-i)*}(n)\right) \\ &= \frac{1}{NC^a_{p,q+1}}\sum_{k=0}^{L-1}\sum_{n\in\langle N\rangle} f\left(z(nL+k);\, y^*(nL+k-i)\right) \\ &= \frac{L}{C^a_{p,q+1}}\frac{1}{NL}\sum_{n\in\langle NL\rangle} f\left(z(n);\, y^*(n-i)\right)\,, \quad (17)\end{aligned}$$

where $\langle N\rangle$ and $\langle NL\rangle$ denote data records of length N and NL, respectively, and f is a function that depends on the order of the cumulant being estimated. The empirical covariance matrix $\hat{\mathbf{R}}$ is obtained in a similar way.

Equalization. As for the equalization criterion (9), an ideal super-exponential iteration that converges to its maxima may also be defined. This was developed in [5], and leads to the following iteration for the coefficient vector[2]

$$\bar{\mathbf{c}}' = \bar{\mathbf{R}}^{-1}\bar{\mathbf{d}}\,, \qquad \bar{\mathbf{c}}'' = \frac{\bar{\mathbf{c}}'}{\sqrt{\bar{\mathbf{c}}'^H\bar{\mathbf{R}}\bar{\mathbf{c}}'}}\,, \quad (18)$$

with

$$\bar{\mathbf{R}} = \begin{bmatrix} \mathbf{R}^{(0,0)} & \dots & \mathbf{R}^{(0,-L+1)} \\ \vdots & & \vdots \\ \mathbf{R}^{(-L+1,0)} & \dots & \mathbf{R}^{(-L+1,-L+1)} \end{bmatrix},$$

and $\bar{\mathbf{d}} = [\mathbf{d}^{(0)T} \dots \mathbf{d}^{(-L+1)T}]^T$. The blocks in $\bar{\mathbf{R}}$ and $\bar{\mathbf{d}}$ are given by

$$\begin{aligned}\mathbf{R}^{(k,l)}_{ij} &= \frac{\mathrm{cum}(y^{(l)}(n-j);\, y^{(k)*}(n-i))}{C^a_{1,1}} \quad (19)\\ \mathbf{d}^{(k)}_i &= \frac{\mathrm{cum}(z^{(0)}(n):p;\, z^{(0)*}(n):q;\, y^{(k)*}(n-i))}{C^a_{p,q+1}}\,. \quad (20)\end{aligned}$$

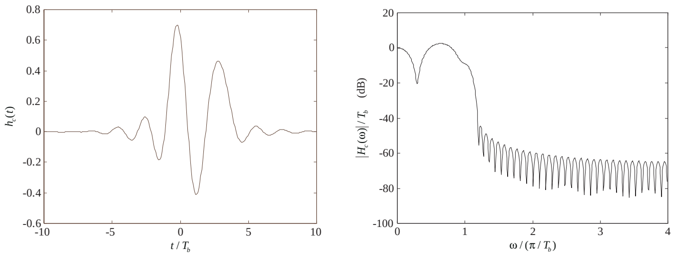
It may be shown that this algorithm converges to the FSE solution of [4], and so (18) describes a blind adaptive FSE. As expected, it shares the most important properties of the FSE, such as an almost perfect insensitivity to sampling time for bandlimited inputs, and the ability to adaptively synthesize in discrete time the equivalent of a continuous matched filter and symbol-spaced equalizer.

5 EXPERIMENTAL RESULTS

A three ray multipath channel is used for the simulations in this section. The total impulse response is $h_c(t) = r(t) - 0.5r(t - T_b/2) + 0.5r(t - 3T_b)$, where $r(t)$ stands for a raised cosine transmitted pulse with a rolloff factor of $\beta = 0.2$. Figure 1 depicts $h_c(t)$ in *continuous* time and frequency. The received $y_c(t)$ is then sampled

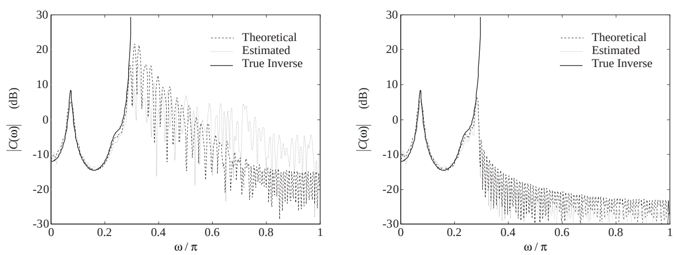
[1] Positive and negative frequencies are considered.

[2] In (18), $\bar{\mathbf{c}}$ is obtained by rearranging the elements of $\mathbf{c}$ in L subsequences $[\mathbf{c}^{(0)} \dots \mathbf{c}^{(L-1)}]^T$.



(a) (b)

Figure 1: Received pulse shape (a) Time domain (b) Continuous frequency domain



(a) (b)

Figure 2: Identified inverse channel in discrete frequency domain (a) $\mathbf{c}' = \mathbf{R}^{-1}\mathbf{d}$ (b) $\mathbf{c}' = \mathbf{R}^{+}\mathbf{d}$

at 16 samples per symbol, and Gaussian white noise with an SNR of 20 dB is added. This signal is decimated as needed to $L = 1$, 2 or 4, and processed by a 200 tap equalizer, initialized with a single nonzero central coefficient. Fourth-order algorithms ($p = 2$ and $q = 1$) are used, which correspond to the kurtosis maximization condition that is often used in blind deconvolution. Cumulant estimates are based on time averages over 2000 symbols.

Figure 2a shows the identified discrete inverse channel for $L = 4$ using both the theoretical algorithm (11) and the practical iteration (16) (with standard matrix inversions). It can be readily seen that $H^{-1}(\omega)$ is well approximated inside $W_H/2 = \pi(1+\beta)/L$. However, the two solutions differ considerably in the null portion of the input spectrum due to numerical ill-conditioning. Figure 2b shows the equalizer frequency response when pseudoinverses are computed, discarding the smallest, nearly constant, singular values of $\mathbf{R}$. The fraction of significant singular values is approximately given by $(1+\beta)/L$, and can be estimated with a number of model-order selection techniques, such as MDL or AIC [6]. As expected, $C(\omega)$ takes small values outside W_H, and (11), (16) yield similar results. Note that figure 2 reflects steady-state values, that were reached after five iterations. Thus, (11) and (16) retain the high convergence rate of the super-exponential algorithm, even with nearly singular matrices.

Figure 3 illustrates the advantage of fractionally-spaced equalization using (18) over synchronous equalization. When $L = 1$, $y_c(t)$ should be sampled at $t = nL$, and some degradation in output mean-square error (MSE) occurs for other choices of t. On the contrary, there is little sensitivity to the sampling instant when $L \geq 2$, and the residual MSE is lower. The ideal solution obtained with $L = 1$ and a matched filter (MF) is also shown. Although it is only outperformed by the fractional equalizer for large sampling errors, the exact MF approach is unrealizable in practice, and must be regarded as a benchmark.

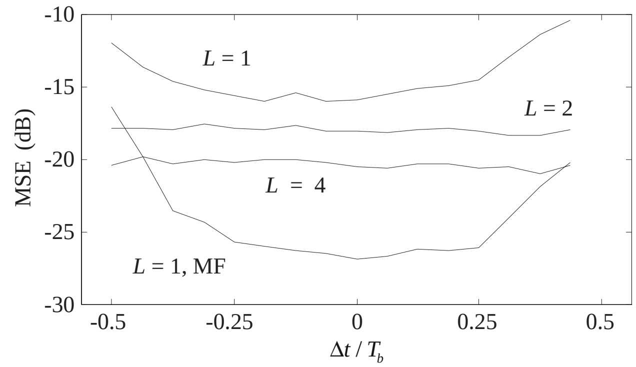


Figure 3: Sensitivity to sampling instant

6 CONCLUSION

Two different strategies for blind identification and equalization of bandlimited cyclostationary signals were presented. These block algorithms extend the symbol-spaced Shalvi-Weinstein algorithm, and converge very rapidly. As in a decision-directed FSE, it was shown that oversampling is highly beneficial for equalizer performance in terms of residual MSE and timing sensitivity. Our simulation results show that the algorithms perform well for channels having impulse responses that may frequently be encountered in practice.

References

[1] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach", *IEEE Trans. on Inf. Theory*, vol. 40, pp. 340–349, March 1994.

[2] O. Shalvi and E. Weinstein, "Super-Exponential methods for blind deconvolution", *IEEE Trans. on Inf. Theory*, vol. 39, pp. 504–519, March 1993.

[3] O. Shalvi and E. Weinstein, "Universal methods for blind deconvolution", in S. Haykin, ed., *Blind Deconvolution*, pp. 121–180. PTR Prentice-Hall, 1994.

[4] R. D. Gitlin and S. B. Weinstein, "Fractionally-Spaced equalization: An improved digital transversal equalizer", *Bell Syst. Tech. J.*, vol. 60, pp. 275–296, February 1981.

[5] J. Gomes and V. Barroso, "A super-exponential algorithm for blind fractionally-spaced equalization", *Submitted for publication.*

[6] S. Kay, *Modern Spectral Estimation: Theory and Application*, Prentice-Hall, 1988.