Wideband Blind Identification and Separation of Independent Sources

Wang Jun
DSP Division, Department of Radio Engineering
Southeast University
Nanjing 210096, P.R.China
e-mail: cwwu@seu.edu.cn

1. Introduction

During recent years, there have been much interests focused on blind identification and blind source separation. Many approaches have been proposed mainly for the special and simple case in which the MIMO system is linear memoryless[1][2][4][5]. However, in a variety of applications in which wideband sources are involved, e.g., speech dereverberation, speech enchancement in the presence of background noise and competing speakers separation, these approaches can not be applied. Several solutions have been presented for the two-source two-sensor wideband problem with two channels assumed identity systems[6-8].

In this paper, the general wideband blind identification and source separation problem is considered. All of the restrictive assumptions made by the papers above are removed in our proposal except the independence between sources and the FIR property of the channel frequency response. We have proved 1) a bispectra based criterion, which states sufficient condition for the identification and separation of asymmetriclly distributed sources, and 2) a trispectra based criterion, which states sufficient condition for the identification and separation of non-Gaussian sources. Algorithms are also developed to implement these criteria. The validity of the criteria and the efficiency of the algorithms are verified by simulations.

2. Problem Formulation

Let n denote the number of sources and sensors. The wideband formulation of the problem is written as:

$$x_i(t) = \sum_{j=1}^{n} \sum_{k=0}^{K_{ij}} a_{ij}^{(k)} s_j(t-k) + e_i(t)$$

where $x_i(t)$ is the observation of ith sensor, $s_j(t)$ is the jth source, $a_{ij}^{(k)}$ and K_{ij} are the kth coefficient and the order of the FIR filter coupling the jth source and ith sensor respectively, $e_i(t)$ is the measurement noise at the ith sensor.

Assumptions:

A.1 $s_i(t)$ is a zero-mean wide-sense stationary process. For each t, $\{s_i(t)$, i=1,...,n$\}$are mutually independent.

A.2 $e_i(t)$ is a zero-mean stationary Gaussian process. For each t, $\{e_i(t)$, i=1,...,n$\}$ are independent of $\{s_i(t)$, i=1,...,n$\}$.

A.3 Bispectra $P_{s_i s_i s_i^*}(\omega_1, \omega_2)$, $\forall \omega_1$ and ω_2, where * denotes complex conjugate, exist for all $s_i(t)$'s.

A.4 Trispectra $P_{s_i s_i s_i s_i^*}(\omega_1, \omega_2, \omega_3)$, $\forall \omega_1$, ω_2 and ω_3, exist for all $s_i(t)$'s.

Denote $\mathbf{x(t)} = [x_1(t), \ldots, x_2(t)]^t$, $\mathbf{s(t)} = [s_1(t), \ldots, s_n(t)]^t$, and $\mathbf{e(t)} = [e_1(t), \ldots, e_n(t)]^t$. The objective of blind identification and source speparation is to estimate the channel coefficients $a_{ij}^{(k)}$ and the individual signals, $s_j(t)$.We can take advantage of indeterminacy of blind identification[5] to assume: i) $a_{1j}^{(0)} = 1$, j=1,...,n; and ii) $K_{i1} = K_{i2} = \cdots = K_{in}, \forall i = 1, \ldots, n$.

3. Criterion and algorithm based on bispectra

Taking the Fourier transform of the observations

$$X(\omega) = A(\omega)S(\omega) + E(\omega)$$

in which $X(\omega)$, $S(\omega), E(\omega)$ are the Fourier transform of **x(t)**, **s(t)**, and **e(t)**, and

$$A(\omega) = \begin{bmatrix} \sum_{k=0}^{K_{11}} a_{11}^{(k)} e^{-jk\omega} & \cdots & \sum_{k=0}^{K_{1n}} a_{1n}^{(k)} e^{-jk\omega} \\ \cdots & & \cdots \\ \sum_{k=0}^{K_{n1}} a_{n1}^{(k)} e^{-jk\omega} & \cdots & \sum_{k=0}^{K_{nn}} a_{nn}^{(k)} e^{-jk\omega} \end{bmatrix}$$

We want to eliminate the contamination effects of A by using an $n \times n$ reconstruction system H

$$Y(\omega) = H(\omega)X(\omega) = H(\omega)A(\omega)S(\omega) + H(\omega)E(\omega)$$
$$= T(\omega)S(\omega) + H(\omega)E(\omega) \quad (1)$$

where $T(\omega) = H(\omega)A(\omega)$, $Y(\omega)$ denotes Fourier transform of recovered signal vector $\mathbf{y(t)} = [y_1(t), \ldots, y_n(t)]^t$ and $H(\omega), T(\omega) \in C^{n\times n}$.

Criterion: Suppose there exists a set of frequencies $\{\omega^{(l)}, l = 1, \ldots, L\}$ such that

$$P_{s_q s_q s_q^*}(\omega^{(l)}, -\omega^{(l)}) \neq 0 \quad l = 1, \ldots, L. \; q = 1, \ldots, n \quad (2)$$

$$P_{s_q s_q s_q^*}(\omega^{(l-1)}, -\omega^{(l)}) \neq 0 \quad l = 2, \ldots, L. \quad q = 1, \ldots n \quad (3)$$

Denote $\Delta\omega^{(l)} = \omega^{(l+1)} - \omega^{(l)}$. Suppose

$$\det T(0) \neq 0 \quad (4)$$

$$\det T(\omega^{(l)}) \neq 0 \qquad l = 1, \ldots, L \quad (5)$$

$$\det T(\Delta\omega^{(l)}) \neq 0 \qquad l = 1, \ldots, L-1 \quad (6)$$

Then under the assumptions A.1-A.3, $\{T(\omega^{(l)}), l = 1, \ldots, L\}$ are generalized permutation matrices

$$T(\omega^{(l)}) = P\Lambda(\omega^{(l)}) \quad (7)$$

with some permutation matrix P and some nonsingular diagonal matrix $\Lambda(\omega^{(l)})$ if

$$P_{y_i y_j y_k^*}(\omega^{(l)}, -\omega^{(l)}) = 0 \quad l = 1, \ldots, L. \quad (8)$$

$$P_{y_i y_j y_k^*}(\omega^{(l-1)}, -\omega^{(l)}) = 0 \quad l = 2, \ldots, L. \quad (9)$$

for all i, j, k=1,...,n, j<k.

The conditions in (2) and (3) are satisfied if the sources have asymmetric probability structures.By (1), equations (8) and (9) are satisfied if $\forall p \in \{1, \ldots, n\}$

$$\begin{cases} \sum_{q\neq p} H_{pq}(\omega^{(l)}) P_{y_i x_q y_k^*}(\omega^{(l)}, -\omega^{(l)}) \\ = -H_{pp}(\omega^{(l)}) P_{y_i x_p y_k^*}(\omega^{(l)}, -\omega^{(l)}). \; k > p \quad (10) \\ \sum_{q\neq p} H^*_{pq}(\omega^{(l)}) P_{y_i y_j x_q^*}(\omega^{(l)}, -\omega^{(l)}) \\ = -H^*_{pp}(\omega^{(l)}) P_{y_i y_j x_p^*}(\omega^{(l)}, -\omega^{(l)}). \; j < p \quad (11) \\ \sum_{q\neq p} H^*_{pq}(\omega^{(l)}) P_{y_i y_j x_q^*}(\omega^{(l-1)}, -\omega^{(l)}) \\ = -H^*_{pp}(\omega^{(l)}) P_{y_i y_j x_p^*}(\omega^{(l-1)}, -\omega^{(l)}). \; j < p \quad (12) \end{cases}$$

Remark 1: Since the objective is to obtain $T(\omega^{(l)}) = H(\omega^{(l)})A(\omega^{(l)}) = P\Lambda(\omega^{(l)})$, for $l = 1, \ldots, L$; where $\Lambda(\omega^{(l)})$ is an arbitary non-singular diagonal matrix, it is convenient to suppose $H_{pp}(\omega^{(l)}) = 1$, p=1,...,n. Then (10), (11) and (12) are linear in $H_{pq}(\omega^{(l)}), p \in \{1, \ldots, n\}, q = 1, \ldots, n$ and may be solved by least square procedure.

Remark 2: The solution $H_{pq}(\omega^{(l)})$, $p \in \{1, \ldots, n\}$, q=1,...,n only depends on $H_{jq}(\omega^{(l)})$, $j < p$, $H_{kq}(\omega^{(l)})$, $k > p$ and $H_{jq}(\omega^{(l-1)})$, $j < p$; $q = 1, \ldots, n$ (the reason is given in the proof of criterion), so for any given $H_{jq}(\omega^{(l)})$, $j < p$, $H_{kq}(\omega^{(l)})$, $k > p$ and $H_{jq}(\omega^{(l-1)})$, $j < p, q = 1, \ldots, n$; we shall use (10)-(12) to solve $H_{pq}(\omega^{(l)})$, $p \in \{1, \ldots, n\}$, $q = 1, \ldots, n$ and by alternating between various $p = 1, \ldots, n$, we obtain an iterative procedure to adjust $H(\omega^{(l)})$.

By (5) and (7) we have

$$H^{-1}(\omega^{(l)}) = A(\omega^{(l)})\Lambda^{-1}(\omega^{(l)})P^t$$
$$= \begin{bmatrix} A_{1j_1}(\omega^{(l)})\lambda_{j_1}(\omega^{(l)}) & \cdots & A_{1j_n}(\omega^{(l)})\lambda_{j_n}(\omega^{(l)}) \\ \cdots & & \cdots \\ A_{nj_1}(\omega^{(l)})\lambda_{j_1}(\omega^{(l)}) & \cdots & A_{nj_n}(\omega^{(l)})\lambda_{j_n}(\omega^{(l)}) \end{bmatrix} \quad (14)$$

in which $\Lambda^{-1}(\omega^{(l)}) = diag(\lambda_1(\omega^{(l)}),\ldots,\lambda_n(\omega^{(l)}))$ and $\{j_1,\ldots,j_n\}=\{1,\ldots,n\}$. To eliminate the unknown scaling of the column vectors in (14), we divide the column vectors by their first elements and get the matrix

$$\tilde{A}(\omega^{(l)}) = \begin{bmatrix} 1 & \cdots & 1 \\ \dfrac{\sum a_{2j_1}^{(k)} e^{-jk\omega^{(l)}}}{\sum a_{1j_1}^{(k)} e^{-jk\omega^{(l)}}} & \cdots & \dfrac{\sum a_{2j_n}^{(k)} e^{-jk\omega^{(l)}}}{\sum a_{1j_n}^{(k)} e^{-jk\omega^{(l)}}} \\ \cdots & \cdots & \cdots \\ \dfrac{\sum a_{nj_1}^{(k)} e^{-jk\omega^{(l)}}}{\sum a_{1j_1}^{(k)} e^{-jk\omega^{(l)}}} & \cdots & \dfrac{\sum a_{nj_n}^{(k)} e^{-jk\omega^{(l)}}}{\sum a_{1j_n}^{(k)} e^{-jk\omega^{(l)}}} \end{bmatrix}$$

Algorithm:

1. Estimate the bispectra of the observations, say $P_{x_{q1}x_{q2}x^*_{q3}}(\omega^{(l)},-\omega^{(l)})$, $P_{x_{q1}x_{q2}x^*_{q3}}(\omega^{(l-1)},-\omega^{(l)})$ where $\{\omega^{(l)},\ l=1,\ldots,L\}$ are preselected frequencies for $l=1,\ldots,L$ and $q_1,q_2,q_3=1,\ldots n;$ via the *complex demodulates* approach[3].
2. Determine $H(\omega^{(1)})$ by solving (10)-(11) and then $H(\omega^{(l)})$, $l=2,\ldots,L$ by solving (10)-(12) *in the order of precedence.*
3. Compute $\tilde{A}(\omega^{(l)})$ for $l=1,\ldots,L$ and estimate $a_{ij}^{(k)}$ by solving sets of linear equations.
4. Apply FFT to the estimates of $a_{ij}^{(k)}$'s and get $\hat{A}(\omega)=A(\omega)P^t$.
5. Compute $\hat{S}(\omega)=\hat{A}^{-1}(\omega)X(\omega)$ and inverse FFT to recover sources.

It is noted, as shown in step 4, that the ordering indeterminacy of the column vectors of A and sources is not eliminated unless some additional information or constraints are introduced[5]. Criterion and algorithm based on trispectra for the separation of non-Gaussian sources are not given for reason of space.

4. Simulations

The sources are three statistically identical linear processes, all generated as the output of the second order filter:

$$s_j(t) = s_j(t-1) - 0.5 s_j(t-2) + z_j(t) \qquad j = 1,2,3$$

Appendix

where $z_j(t), j=1,2,3$ are mutually independent computer generated i.i.d. sequences of exponentially distributed random variables. The FIR filters coefficients were chosen randomly between -1 and 1, as shown in Table I. Here we considered the noiseless case for simplicity and have 21 coefficients to determine. We computed $\tilde{A}(\omega^{(l)})$ at eight frequencies, say, $\omega^{(l)} = \frac{l+1}{10}\cdot\pi, l=1,\ldots,8;$ to estimate the filter coefficients. We have performed 50 Monte-Carlo trials using 4096 samples each source. The empirical mean and standard deviation of the estimated filter coefficients are given in Table I, indicating convergence to the desired solutions. We also note that the algorithm converges very fast, usually in 3-5 iterations to the desired $H(\omega^{(l)})$ for each $l \in \{1,\ldots,L\}$.

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$$P_{y_i y_j y_k^*}(\omega^{(l)}, -\omega^{(l)})$$
$$= \sum_{q_1}\sum_{q_2}\sum_{q_3} T_{iq_1}(0) T_{jq_2}(\omega^{(l)}) T^*_{kq_3}(\omega^{(l)}) P_{s_{q_1} s_{q_2} s^*_{q_3}}(\omega^{(l)}, -\omega^{(l)})$$
$$+ \sum_{q_1}\sum_{q_2}\sum_{q_3} H_{iq_1}(0) H_{jq_2}(\omega^{(l)}) H^*_{kq_3}(\omega^{(l)}) P_{e_{q_1} e_{q_2} e^*_{q_3}}(\omega^{(l)}, -\omega^{(l}$$
(AP.1)

By A.1, A.2 and (8)

$$\sum_q T_{iq}(0) T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) P_{s_q s_q s^*_q}(\omega^{(l)}, -\omega^{(l)}) = 0$$
(AP.2)

for all i,j,k=1,...,n; j<k; By (2)

$$\prod_{q=1}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) \det T(0) = 0 \qquad \text{(AP.4)}$$

Then by (4)

$$\prod_{q=1}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) = 0 \qquad \text{(AP.5)}$$

Without loss of generality, suppose $T_{j1}(\omega^{(l)}) T^*_{k1}(\omega^{(l)}) = 0$, then by (2)

$$\prod_{q=2}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) \tilde{T}_{p1}(0) = 0 \qquad p = 1, \ldots, n$$
(AP.8)

where $\tilde{T}_{p1}(0)$ denotes the cofactor of $T_{p1}(0)$.

Suppose $\prod_{q=2}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) \neq 0$, so $\tilde{T}_{p1}(0) = 0$ for all p=1,...,n and

$$\det T(0) = \sum_{p=1}^{n} (-1)^{p+1} T_{p1}(0) \tilde{T}_{p1}(0) = 0$$ which contrudicts with (4). Thus

$$\prod_{q=2}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) = 0 \qquad \text{(AP.9)}$$

Similarly, one can obtain

$$\prod_{q=r}^{n} T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) = 0 \qquad r = 3, \ldots, n.$$
(AP.10)

Thus we derive

$$T_{jq}(\omega^{(l)}) T^*_{kq}(\omega^{(l)}) = 0 \quad j, k, q = 1, \ldots, n.\ j < k \qquad \text{(AP.11)}$$

By (5) we have

$$T(\omega^{(l)}) = P(\omega^{(l)}) \Lambda(\omega^{(l)}) \qquad l = 1, \ldots, L \qquad \text{(AP.12)}$$

Analogous to the derivation of (AP.11), one may obtain by (3), (6) and (9)

$$T_{jq}(\omega^{(l-1)}) T^*_{kq}(\omega^{(l)}) = 0 \qquad \forall j, k = 1, \ldots n;\ j < k;$$
$$l = 2, \ldots L;\ q = 1, \ldots, n$$

(AP.13)

By (AP.12) and (AP.13), we have (7).

Table I. The true value, estimate mean and standard deviation of the filter coefficients

Filter coefficient	True value	Mean	Standard deviation
$a_{11}^{(1)}$	-0.4442	-0.4409	0.0107
$a_{21}^{(0)}$	0.6848	0.6864	0.0141
$a_{21}^{(1)}$	-0.8580	-0.8625	0.0141
$a_{21}^{(2)}$	0.4083	0.4113	0.0116
$a_{31}^{(0)}$	0.2044	0.2016	0.0073
$a_{31}^{(1)}$	-0.4918	-0.4884	0.0097
$a_{31}^{(2)}$	-0.8859	-0.8800	0.0125
$a_{12}^{(1)}$	0.3480	0.3508	0.0153
$a_{22}^{(0)}$	-0.1709	-0.1676	0.0076
$a_{22}^{(1)}$	0.1501	0.1435	0.0136
$a_{22}^{(2)}$	-0.6292	-0.6248	0.0104
$a_{32}^{(0)}$	-0.1485	-0.1541	0.0165
$a_{32}^{(1)}$	-0.3864	-0.3906	0.0116
$a_{32}^{(2)}$	0.2652	0.2712	0.0147
$a_{13}^{(1)}$	-0.4167	-0.4097	0.0162
$a_{23}^{(0)}$	-0.7040	-0.7036	0.0127
$a_{23}^{(1)}$	0.6033	0.6042	0.0118
$a_{23}^{(2)}$	0.8699	0.8714	0.0118
$a_{33}^{(0)}$	-0.0801	-0.0752	0.0126
$a_{33}^{(1)}$	-0.5838	-0.5798	0.0091
$a_{33}^{(2)}$	-0.4397	-0.4375	0.0144