SUBSPACE METHOD FOR BLIND SEPARATION OF SOURCES IN CONVOLUTIVE MIXTURE.

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ABSTRACT

For the convolutive mixture, a subspace method to separate the sources is proposed. It is showed that after using only the second order statistic but more sensors than sources, the convolutive mixture can be itentified up to instantaneou mixture. Furthermore, the sources can be separated by any algorithm for instantaneous mixture (based in generally on the fourth order statistics).

1 INTRODUCTION

A number of efficient second order statistics based methods have been recently developed to solve the so-called blind multichannel equalization problem ([8], [10] and [9]). In these works, the observation is a q–variate signal supposed to be the output of a single input / q–outputs unknown FIR filter driven by a non observable scalar sequence. In the digital communication context, this sequence represents the symbols to be transmitted, while the unknown filter is due to the multi-paths effects.

If the scalar sequence is replaced by a p–variate signal $s(n) = (s_1(n), \ldots, s_p(n))^T$ (with $p < q$) whose components are statistically independent signals, the above problem is nothing but in source separation problem a convolutive mixture. In this context, each component $s_k(n)$ of $s(n)$ is a possibly temporally correlated signal with an unknown spectrum. It has been shown recently ([4], [1]) that the so-called subspace approach of [8] can be generalized to the context of the source separation of convolutive mixtures. In particular, under certain assumptions to be precised below, the unknown FIR $q \times p$ transfer function $H(z)$ can be identified up to a constant mixture matrix from the sole knowledge of the second order statistics of the observations. After this preliminary identification stage, the outputs are filtered by a left inverse of the identified filter, thus providing a p–variate signal from which $s(n)$ can be retrieved by solving source separation problem in instantaneous mixture.

However, this approach is not well suited to adaptive implementation. In this context, it may be more convenient to adapt directly a left inverse of the filter $H(z)$ from the observations. Such a second-order based direct deconvolution approach has been proposed by Gesbert et al [2] in the multichannel blind equalization context. This method is quite attractive : it does not require the knowledge of input spectrum, and is based on the minimization of a simple quadratic cost function, which can be realized adaptively by a LMS algorithm. The purpose of this paper is to indicate how this approach can be adapted to the source separation of convolutive mixture considered here. Finally, one should note that Van der Veen et al. [11] have also proposed an interesting second order based direct deconvolution approach in the source separation context.

2 GENERAL NOTATIONS AND ASSUMPTIONS.

Let us first precise the notations and the assumptions used throughout this paper. We denote by $y(n)$ the q–variate observed signal. It is assumed that :

$$y(n) = \sum_{i=0}^{M} H(i)s(n-i) = [H(z)]s(n) \qquad (1)$$

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where $H(z) \triangleq \sum_{i=1}^{p} H(i)z^{-i} \triangleq [H_1(z), \ldots, H_p(z)]$ is an unknown FIR $q \times p$ (with $p < q$) transfer function and where the non observable inputs $s(n) = (s_1(n), \ldots, s_p(n))^T$ are stationary signals such that for each $k \neq l$, s_k and s_l are statistically independent. We denote by $M_1, \ldots, M_p$ the degrees of the columns[1] $H_1(z), \ldots, H_p(z)$ of $H(z)$, and we assume without restrictions that $M_1 \leq M_2 \leq \ldots \leq M_p$. Moreover, we set $M_p = M$. From now on, we make the following important assumptions on the transfer $H(z)$.

- **H1**: $H(z)$ is irreducible ($\text{Rank}(H(z)) = p, \forall z$).
- **H2**: $H(z)$ is column reduced, i.e. the highest coefficient matrix $H_1(M_1), \ldots, H_p(M_p)$ is a full rank column matrix.

As soon as $p < q$, these assumptions have been shown in [3] (see also [4]) to be realistic. They have the following important consequences in the sequel :

First, assumption **H1** implies the existence of a (non unique) $p \times q$ polynomial matrix $G(z)$ such that $G(z)H(z) = I_p$. In others words, $H(z)$ can causally be left inverted by a polynomial matrix, or equivalently, the source signal $s(n)$ can be perfectly recovered from a finite number of past observations.

On the other hand, let us denote by $T_N(H)$ the so-called $q(N+1) \times (M+N+1)p$ Sylvester matrix associated to $H(z)$

$$\begin{bmatrix} H(0) & \ldots & H(M) & 0 & \ldots & 0 \\ \vdots & \vdots & & & & \\ 0 & \ldots & 0 & H(0) & \ldots & H(M) \end{bmatrix}. \quad (2)$$

Then, it can be shown (see [6], in chapter 6) that under assumptions $\mathbf{H1}, \mathbf{H2}$, $\text{Rank}(T_N(H)) = p(N+1) + \sum_{i=1}^{p} M_i$, as soon as $N \geq \sum_{i=1}^{p} M_i$. One should note that $p(N+1) + \sum_{i=1}^{p} M_i$ is precisely the number of non zero columns of $T_N(H)$. In particular, if all the degrees $(M_i)_{i=1,p}$ coincide with M, then, $T_N(H)$ is full rank column if $N \geq pM$. It admits therefore a left inverse.

In the follow, if we will discuss on two different model parametrizations, corresponding to the cases of equal or not equal degree on the column of $H(z)$

[1] By definition, the degree of a column vector $H_i(z)$ is maximum degrees between all his coefficient.

3 THE PROPOSED APPROACH.

3.1 THE CASE OF EQUAL DEGREES.

In order to simplify what follows, we first present our blind deconvolution scheme in the case where the degrees $(M_i)_{i=1,p}$ all coincide with M. Its formulation is an obvious generalization of the method proposed by Gesbert et al. [2].

Let $Y_N(n)$ and $S_{M+N}(n)$ the random vectors defined by $Y_N(n) = (y^T(n), \ldots, y^T(n-N))^T$ and $S_{M+N}(n) = (S^T(n), \ldots, S^T(n-M-N))^T$. Then, the equation (1) writes in a matrix form

$$Y_N(n) = T_N(H) S_{M+N}(n). \quad (3)$$

Let us choose $N \geq pM$. Then, as mentioned previously, $T_N(H)$ is left invertible, so that it exists a $p(M+N+1) \times q(N+1)$ matrix G for which $GT_N(H) = I_{p(M+N+1)}$, i.e., $GY_N(n) = S_{M+N}(n)$. On the other hand, $GY_N(n+1) = S_{M+N}(n+1)$. So it is obvious that the first $M+N$ block $(p \times q(N+1))$ rows of $GY_N(n)$ are equal to the last $M+N$ block rows of $GY_N(n+1)$. The important point lies on the fact that this last property characterizes the left inverses of $T_N(H)$. More precisely, if G is a $p(M+N+1) \times q(N+1)$ matrix for which

$$(I_{(M+N)p} \;\; 0_p) G Y_N(n) = (0_p \;\; I_{(M+N)p}) G Y_N(n+1)$$

for each n, then

$$GT_N(H) = diag(A, \ldots, A). \quad (4)$$

for some $p \times p$ matrix A. To show this, we denote $B = GT_N(H)$, and remark that B satisfies

$$(I_{(M+N)p} \;\; 0_p)(0_p \;\; B) S_{M+N+1}(n) = (0_p \;\; I_{(M+N)p})(B \;\; 0_p) S_{M+N+1}(n)$$

for each n, where 0_p is a $(M+N)p \times p$ zero matrix. Under very weak assumptions on the input sequences (the inputs should be persistently exciting), this implies that $(I_{(M+N)p} \;\; 0_p)(0_p \;\; B) = (0_p \;\; I_{(M+N)p})(B \;\; 0_p)$. And, it is easily seen that this relation hold if and only if B is block diagonal as in (4). Therefore, it is possible to identify a left inverse of $T_N(H)$ by minimizing the cost function:

$$C(G) = E\|(I_{(M+N)p} \;\; 0_p) G Y_N(n) - (0_p \;\; I_{(M+N)p}) G Y_N(n+1)\|^2 \quad (5)$$

under a constraint ensuring that the matrix A corresponding to the argument G of the minimum through (4) is invertible. In this case, the first p–components of $GY_N(n)$ write as $As(n)$ for an invertible matrix A, from which $s(n)$ can be retrieved by using a source separation algorithm for instantaneous mixtures. For this purpose, we propose to minimize (5) under the constraint $G_0 R_Y G_0^T = I_p$, where G_0 is the first block row ($p \times q(N+1)$) of G, and $R_Y = EY_N(n)Y_N(n)^T$ is the covariance matrix of $Y_N(n)$.

In practice, the above minimization algorithm can be solved adaptively by a classical LMS algorithm, but in which the current estimate $G(n)$ of G is normalized at each step in such a way that the constraint is satisfied (this can be done by calculating a square root of $G(n)\hat{R}_Y(n)G(n)^T$ where $\hat{R}_Y(n)$ is an estimate of R_Y.

For convolutive mixtures involving a causal filter of 3th order, two inputs and four outputs, the minimization of 5 leads to $G\,T_N(H)$ which is the diagonal bloc matrix shown in fig (1). After computing G, the separation of the instantaneous mixture is achieved using a modified version [7] of Jutten-Herault algorithm [5]. It succeeds in separating stationary sources, with about -20 dB of residual crosstalk.

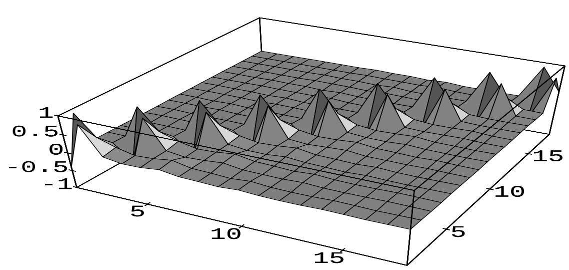

Figure 1: $GT_N(H)$

3.2 THE CASE OF NON EQUAL DEGREES.

We now indicate how to adapt the above procedure to the case where the degrees of the columns of $H(z)$ do not coincide. In order to simplify the notations, we shall present in detail the corresponding scheme in the particular case where the source number $p = 2$. The results corresponding to the most general case will be presented without justification.

In order to treat this problem, it is more convenient to introduce the $q(N+1) \times (M_1 + M_2 + 2(N+1))$ matrix $U_N(H)$ given by

$$U_N(H) = (T_N(H_1), T_N(H_2)).$$

It is clear that $T_N(H)$ and $U_N(H)$ have the same rank. Therefore, if N is chosen greater than $M_1 + M_2$, $U_N(H)$ is full rank column, and admits a left inverse. On the other hand, the equation (1) writes as $Y_N(n) = U_N(H)(s_{1,M_1+N}^T(n), s_{2,M_2+N}^T(n))^T$ where the vectors $s_{i,M_i+N}(n)$ are defined as $S_{M+N}(n)$. The deconvolution approach still consists in identifying a left inverse of $U_N(H)$. Let G be a candidate matrix, and put $G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}$ where G_i is a $(M_i + N + 1) \times q(N+1)$ for $i = 1, 2$. Denote

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = GU_N(H) \qquad (6)$$

where B_{ij} is $(M_i + N + 1) \times (M_j + N + 1)$. Let us characterize the matrices G for which :

$$(I_{(M_i+N)} \;\; 0)G_iY_N(n) = (0 \;\; I_{(M_i+N)})G_iY_N(n+1)$$

for $i = 1, 2$. Replacing Y by its expression in terms of S, and assuming that the inputs are persistently exciting, we get immediately that

$$(I_{(M_i+N)} \;\; 0)(0 \;\; B_{ij}) = (0 \;\; I_{(M_i+N)})(B_{ij} \;\; 0)$$

for $(i, j) = 1, 2$. This implies that $B_{11} = a_1 I_{M_1+N+1}$, $B_{22} = a_2 I_{M_2+N+1}$, $B_{21} = 0$, and that

$$B_{12} = \begin{pmatrix} b_0 & b_1 & \ldots & b_{M_2-M_1} & 0 & \ldots & 0 \\ 0 & b_0 & \ldots & b_{M_2-M_1} & 0 & \ldots & 0 \\ \vdots & & & & & & \\ 0 & \ldots & 0 & b_0 & b_1 & \ldots & b_{M_2-M1} \end{pmatrix}.$$

Denote b the vector $b = (b_0, b_1, \ldots, b_{M_2-M_1})^T$. Therefore, $G_2Y_N(n) = a_2 s_{2,M_2+N}(n)$ and

$$G_1Y_N(n) = a_1 s_{1,M_1+N}(n) + \begin{pmatrix} b^T s_{2,M_2-M_1}(n) \\ b^T s_{2,M_2-M_1}(n-1) \\ \ldots \\ b^T s_{2,M_2-M_1}(n - M_1 - N) \end{pmatrix}.$$

Then, such a matrix G allows to retrieve directly the source signal corresponding to the highest degree column of $H(z)$. But, the extraction of the signal filtered by the lowest order filter needs an additional algorithm. A possible solution consists in first extracting s_2, and then in using a classical substraction algorithm in order to cancel the contribution of signal s_2 into $G_1 Y_N(n)$. Note that this last step is will based on the second order statistics of the outputs. Therefore, if $M_1 < M_2$, it is possible to retrieve s_1 and s_2 by using only the second order statistics of the observations. This is in accordance with the results presented in [4] and [3]. This procedure can be used in an adaptive context. But to the lack of space, the corresponding results will be presented elsewhere.

This approach can be extended to the general case $p > 2$. As above, if $M_1 < M_2 < \ldots < M_p$, the separation of the sources can be achieved by using only the second order statistics of the observations. Generally, if two (or more) filters have the same degree, it leads to a separation of the corresponding sources up to an instantaneous mixtures as in (4). Then the complete separation needs a second step of instantaneous separation involving basically high order statistics.

4 CONCLUSION

In this paper, we proposed a method based on a subspace approach. The method allows the separation of convolutive mixtures of independent sources using mainly second order statistics. A simple instantaneous mixtures, separation of which needs high-order statistics, appears only if filters have the same order. Most of the parameters can be estimated using a simple LMS algorithm.
However, the algorithm is up to now slow, due to large size matrices and LMS method. Moreover, the algorithm requires to know the degrees of the filters. Currently, we study another algorithm based on Gradient Conjugate in order to improve convergence speed and be able to process stationary as well as non stationary signals.

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